Navigating into the realm of Non-Supersymmetric String Theories

MAVROUDI, EIRINI

Navigating into the realm of Non-Supersymmetric String Theories

Finiteness, Stability and

Phenomenological impact

By

Eirini Mavroudi

This Thesis is submitted to the University of Durham, in Fulfilment of the Requirements for the Degree of

Doctor of Philosophy

Durham University

Institute for Particle Physics Phenomenology
Department of Physics
University of Durham
United Kingdom

September 2016

Dedicated to

the people from whom I received unconditional and limitless love,
and who unfailingly support all of my endeavours.

Mom, for raising me up to be who I am.

Sister, for brightening our lives with her special sense of humour.

Mom's siblings, for always supporting me.

Maternal grandparents, for their wise advices and precious guidance.

Me, for making it this far.

Αφιερωμένο

Στους ανθρώπους οι οποίοι με αγαπούν ανιδιοτελώς,
και οι οποίοι υποστηρίζουν ανελλιπώς όλες μου τις προσπάθειες.

Στη μητέρα μου, η οποία με μεγάλωσε.

Στην αδερφή μου, η οποία με τ' αστεία της ομορφαίνει τις ζωές μας.

Στ' αδέρφια της μητέρας μου, για τη συνεχή τους υποστήριξη.

Στους γονείς της μητέρας μου, για τις σοφές συμβουλές
και πολύτιμη καθοδήγηση.

Σ' εμένα, που έφτασα ως εδώ.

Navigating into the realm of
Non-Supersymmetric String Theories

Eirini Mavroudi

Submitted for the degree of Doctor of Philosophy
September 2016

Abstract

For more than two decades, remarkable progress has been made in the construction of supersymmetric Standard Model-like theories from the heterotic string theory. In particular, considerable effort has been invested in studying the abundant phenomenological features of heterotic strings exhibiting $\mathcal{N} = 1$ spacetime supersymmetry. At the same time, their *non-supersymmetric* counterparts have received little attention on the grounds that strings which do not exhibit spacetime supersymmetry admit large one-loop dilaton tadpoles, and are therefore unstable. Nonetheless, in this epoch of data acquisition from high-luminosity experiments, the observational absence of supersymmetry is striking. Consequently, *non-supersymmetric* theories receive a profound interest in the particle physics community.

In this thesis, a class of *non-supersymmetric, tachyon-free, four-dimensional* string models is constructed via a string generalisation of Scherk-Schwarz compactification. Such models demonstrate greatly enhanced finiteness and stability properties, and exhibit some general features on their mass spectra, the behaviour of the one-loop cosmological constant and their interpolation properties. Special attention is paid to how from an exponentially suppressed one-loop cosmological constant, and therefore from an *almost* vanishing dilaton tadpole, finiteness and stability ensue. The existence of such models is characterised by prominent phenomenological features which involve their natural energy scales, particle-charge assignments, and the magnitudes of the associated Yukawa couplings and scalar masses. A radical result is the existence of Standard Model-like theories emerging as the *low energy limit* of non-supersymmetric strings; *there are no light superpartners and supersymmetry is absent at all energy scales*.

Declaration

I hereby declare that the work in this thesis is based on research carried out at the Institute for Particle Physics Phenomenology, the Department of Physics, Durham University, United Kingdom. No material presented in this thesis has previously been submitted by myself in whole or in part for consideration for any other degree or qualification at this or any other University. The research described in this thesis, unless referenced to the contrary in the text, has been carried out in collaboration with my supervisor, Professor Steven A. Abel and Professor Keith R. Dienes. Parts of this thesis therefore have been or will be published as follows:

1. S. Abel, K. R. Dienes and E. Mavroudi
   Towards a Non-Supersymmetric String Phenomenology
   Phys. Rev. D **91** (2015) 12, 126014 `[arXiv:1502.03087 [hep-th]].`

2. S. Abel, K. R. Dienes and E. Mavroudi
   Setting the Stage for a Non-Supersymmetric UV-Complete String Phenomenology
   `[arXiv:1603.05195 [hep-th]].`

3. S. Abel, K. R. Dienes and E. Mavroudi
   GUT precursors and Entwined SUSY: The phenomenology of Stable Non-Supersymmetric Strings
   To be submitted (`arXiv` 2017).

4. S. Abel, B. Aaronson and E. Mavroudi
   On the interpolation from Non-Supersymmetric to Supersymmetric String Theories
   To be submitted (`arXiv` 2016).

September 20, 2016

Acknowledgements

Throughout the course of my PhD studies there have been a number of people who have encouraged me to exceed my expectations, assisted with my financial and academic needs, reinforced my academic knowledge and helped to sustain my normality and to balance my soundness of mind. All these people deserve my heartfelt thanks and sincere gratitude.

To the greatest extent, I would like to express my immense gratitude to my supervisor, Professor Steven A. Abel. In the past four years, working with him has been a highly rewarding experience on both academic and personal levels for which I will always be beholden. I thank him for his tremendously helpful advice, the valuable guidance and the constant support. I specifically appreciate his encouragement and inspiration during the 'dark' stages of research when the derivation of results seemed to be unfeasible. He always took his time discussing problems, ideas and speculations, providing me with all the indispensable means which form the driving force behind the deep comprehensive level that I established on this research. His quick-witted nature, along with his humorous and outgoing personality led to countless vivacious and uninhibited conversations that never failed to prompt my motivation, sustain my interest in the project and fuel my zeal to substantially contribute to the fruition of this work. This thesis would not have been possible without his patience, astuteness, excellent understanding of physics, strong insight into research and expertise. You have been an inspiring role model to me and I genuinely believe that you will succeed into bringing a revolution to the field of non-supersymmetric strings with the continuation of this work. I cordially wish you all the best for the future.

It is my pleasure to thank our collaborator, Professor Keith R. Dienes for the enlightening discussions through our email exchange that has led to the completion of my first published article and to the formation of current and future research ideas. His dynamic research attitude in addition to his outstanding and diverse knowledge on physics are qualities that I have hugely admired and consequentially appreciated. Along with my supervisor, Keith has showed me how fun, illuminating and undoubtedly beneficial it is to be part of an excellent collaboration with mutual respect and great enthusiasm. I look forward to the results of your future collaboration.

I am heartily grateful to my colleagues at the IPPP for making my time here such a delightful experience. Their friendliness, liveliness and support have been the necessary ingredients for providing such a helpful and entertaining environment that I enormously enjoyed. My special thanks go to my fellow office mates in OC216 - Simon Armstrong, Helen Brooks, Dr Danielle Galloni, Petar Petrov, Dr Gunnar Ro and Duncan Walker for the amazingly good company, the stimulating (but not necessarily meaningful and productive) discussions and the shared spirit on both work and non-work related subjects. They never stopped imparting me with healthy doses of laughter, valuable help, patience, and sanity; especially during the times when I complained "I can't get it right again"! Working in the same office as them has most certainly been a fascinating experience characterised by some oh - so - unforgettable moments, such as the birth of the "Triangle Rule"! I have lost count of the number of times that the "Triangle Rule" came to the rescue at the end of the day but I hope this tradition carries on! All my friends in the IPPP, especially Mark Ross-Lonergan, Gilberto Tetlalmatzi-Xolocotzi and Ryan Wilkinson, are amazing people. They are all exceptionally bright and gifted, and I sincerely wish them every luck with their future career pursuits.

A special thank you to Professor Silvia Pascoli and Dr Michael Spannowsky for listening to and understanding my worries, for arranging my funding for my postgraduate studies and for giving me the opportunity to prove my worth. Of course the IPPP wouldn't have been such an efficient place without the support and the services of Mrs. Linda Wilkinson, Mrs. Trudy Forster and the system administrators, therefore thanks are due to these people as well.

September 20, 2016

I am especially indebted to Ben Aaronson, Helen Brooks, Tom Jubb and Duncan Walker for their meticulous care in reading this thesis, their patience and attentive ability not to miss the details. Furthermore, their useful suggestions and remarks improved dramatically the final presentation of this work. All grammatical corrections (commas, exclamation marks, colons, semi-colons, *etc*) should be sent to Ben Aaronson for spending far too long anglicising that which should have been already obvious !!!

I owe my indispensable thanks to Mrs Izabella Eliades for her financial support as a sponsor of both my undergraduate and postgraduate studies. She believed in me as if she was my family and she trusted me to go as far as I could reach. She granted me not only with financial flexibility when I most needed it, but also she inspired me to set high goals and strive to reach them. I will always be greatly beholden to you for making my dreams come true.

Finally, I owe my most heartfelt gratitude to my family for their inestimable love, encouragement and support all these years. They have, without doubt, taught me one too many priceless lessons about life and ultimately it is through these lessons that I acquire the strength to reach my goals. Their constant belief that I am able to do more than I can ever imagine never cease to amaze me. That belief was the catalyst for making this thesis a reality.

For the duration of my doctorate, October 2012- March 2016, the research was funded by a studentship of the Science and Technology Fascilities Council - STFC.

September 20, 2016

Contents

List of Figures

List of Tables

Acronyms

IR Infrared. 81, 191, 192

KK Kaluza Klein. 100, 104, 106–108, 130, 134, 136, 140, 142, 143, 149, 151, 154, 172–179, 190–193, 205, 210, 211

LHC Large Hadron Collider. 10, 11, 27, 28, 36, 207

LIGO Laser Interferometer Gravitational-Wave Observatory. 10, 53

LSP Lightest Supersymmetric Particle. 47, 52

MSSM Minimal Supersymmetric Standard Model. 7–9, 17, 30, 40, 42–50, 52, 121, 124, 129, 148, 220, 221, 223

QCD Quantum Chromodynamics. 27, 55

QFT Quantum Field Theory. 14, 20, 35, 38, 88, 89, 92, 182, 214

QM Quantum Mechanics. 20

RG Renormalisation Group. 30, 40, 169, 182, 210

RGE Renormalisation Group Equation. 5, 29, 31

RNS Ramond-Neveu-Schwarz. 62, 64, 65, 67–70, 74

SB Symmetry Breaking. 24

SM Standard Model. 1–4, 6–8, 10–14, 16–31, 33, 35–37, 39–44, 46–50, 120–122, 128, 129, 132, 133, 148, 154, 158, 159, 164, 166, 167, 169, 172, 182, 183, 185, 192, 193, 196–203, 209, 212, 213, 215–219, 221, 223

SR Special Relativity. 20

SSB Spontaneous Symmetry Breaking. 25, 217, 219, 222

SSSB Spontaneous Supersymmetry Breaking. 16, 47, 48, 51, 52, 101, 128, 182, 183, 185, 203, 211

SUGRA Supergravity. 51, 203–206, 211

SUSY Supersymmetry. 7–19, 38–41, 44–47, 49–52, 62, 64, 73–75, 79, 80, 82, 85, 86, 89, 91–96, 99, 101, 103, 105, 106, 108–111, 119–126, 128, 129, 136, 137, 142, 143, 149, 153, 157–160, 172–176, 180–182, 191, 193, 197, 199, 201, 203, 205–213, 221, 242, 243

UV Ultraviolet. 7, 14, 15, 33, 54, 55, 81, 92–94, 101, 106, 107, 137, 156, 157, 173, 174, 191, 192, 203

VEV Vacuum Expectation Value. 9, 15, 26, 43, 46–49, 51, 76, 97, 137, 138, 148, 186, 217, 221

w.r.t with respect to. 32, 60, 177, 190

WCFFHS Weakly Coupled Free Fermionic Heterotic String. 78

Nomenclature

A The index of the gluons. 215

Λ The cosmological constant. 88–92, 96, 98–101, 108, 109, 112, 116, 117, 149, 154, 171, 180, 181

$\hat{a}$ The index of the quarks' colour charge. 22

$\hat{c}$ The index of the fermion families. 23, 216

$\hat{h}$ Yukawa couplings in the Standard Model Lagrangian. 216

λ The cut-off energy scale. 32–35, 92

λ_H The Higgs coupling in the Standard Model Lagrangian. 217

NS Neveu-Schwarz (or antiperiodic) Boundary Conditions on the worldsheet fermionic fields. 65–67, 69, 71–75, 78, 84, 87, 124, 125, 130, 142, 148, 165, 178, 194, 197, 199, 200, 243

R Ramond (or periodic) Boundary Conditions on the worldsheet fermionic fields. 65–75, 78, 84, 88, 125, 133, 142, 167, 169, 170, 243

$\overline{MS}$ Modified Minimal Subtraction Scheme. 29

$\tilde{\mu}$ An arbitrary energy scale. xiii, 29, 30, 32, 33

a The index corresponding to each vector boson of the $SU(2)$ gauge symmetry. 215

g_0 Dimensionless bare couplings in gauge theories. 32, 33

m Physical mass of the particle states in a theory. 33, 50

m' Dimensionless renormalised mass parameter. 33

m_H The Higgs mass. 28, 33–36, 91, 92, 217, 224

m_t The mass of the top quark. 35, 36

v The vacuum expectation value. 26, 36, 43, 217

$\mathcal{L}$ The Lagrangian of a theory. 24, 33, 215, 217

Eq. Equation. 2, 4, 25, 26, 30, 33, 41, 42, 44–52, 58–61, 63, 64, 66, 67, 70, 71, 85, 86, 88, 90, 92, 96, 101–104, 106, 108–117, 124, 127, 129, 134, 135, 138, 140, 142–145, 147, 148, 151–156, 160, 163, 166, 173–175, 177–179, 185, 187, 193, 196–198, 203, 205, 206, 215, 217, 220, 222, 223, 226–228, 230–232, 234–236, 239, 240, 243, 244

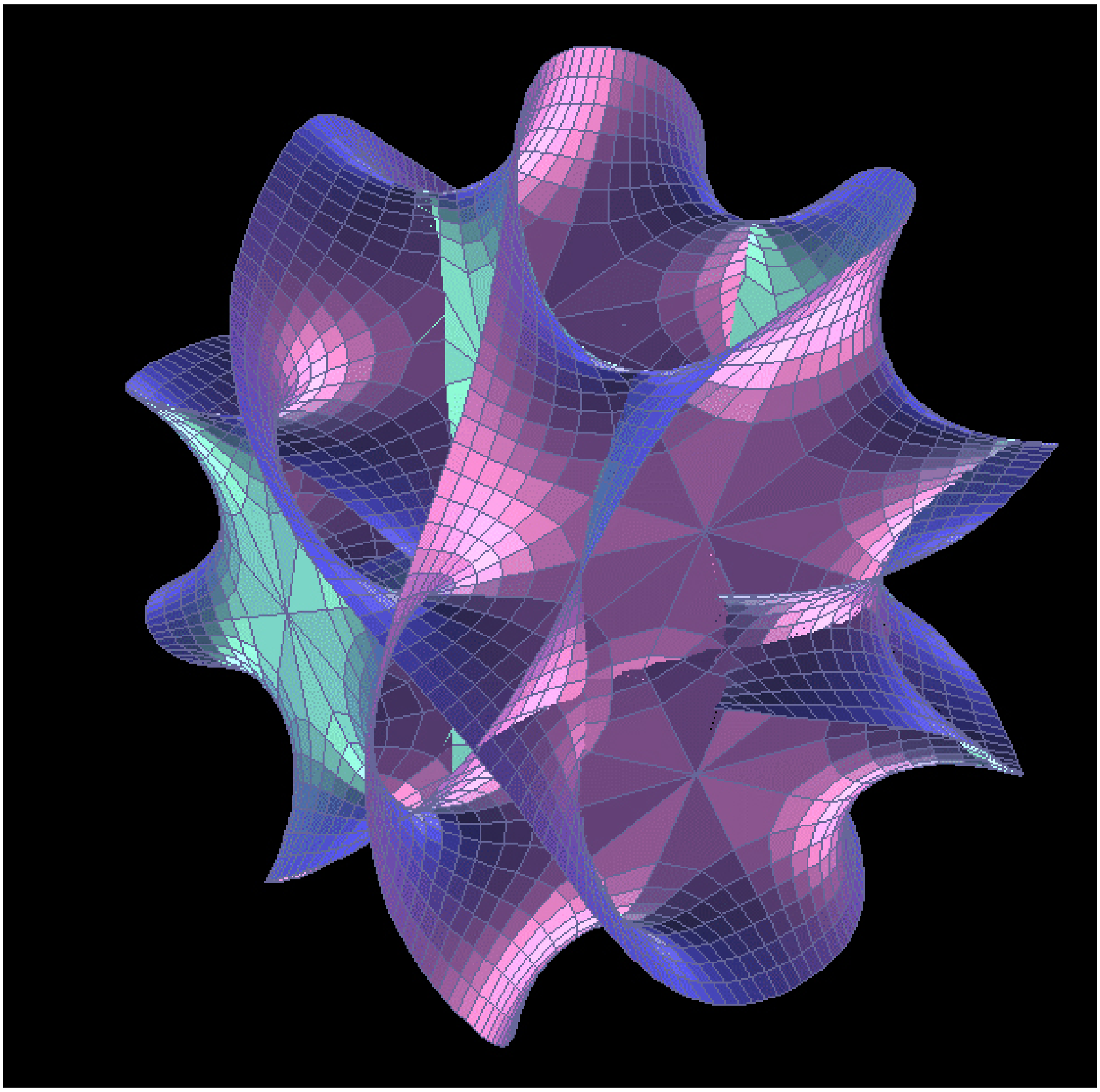
Fig. Figure. 12, 21, 24, 30, 31, 33, 35, 40, 41, 54, 55, 80, 93, 94, 97, 106, 107, 114, 116, 117, 122, 172, 175–177, 180, 212, 218, 219

GMSB Gauge Mediation. 49, 52, 221

mSUGRA Gravity Mediation. 49, 50, 52, 221

Ref. Reference. 2, 5, 6, 9, 10, 12, 14–16, 29, 30, 35, 49, 50, 55, 56, 75, 80, 84, 86, 94, 95, 97, 100–102, 104, 105, 107, 115, 117, 118, 121–124, 127, 129, 132, 135, 140, 143, 145, 146, 149, 151, 173, 176, 179–182, 185, 188, 190–192, 196, 197, 199, 203–205, 221, 232, 237, 244

W Superpotential. 45, 46



A Calabi-Yau manifold in String Theory

"According to String Theory, what appears to be empty space is actually a tumultuous ocean of strings vibrating at the precise frequencies that create the four dimensions you and I call height, width, depth and time."

Roy H. Williams

Chapter 1

Introduction

Knowledge must come through action; you can have no test which is not fanciful, save by trial.

Sophocles

Over the course of time, the irresistible challenge to understand and be able to describe all natural phenomena has inevitably led to a significant evolution in physics. As a result, the history of physics is marked by events when different and seemingly unrelated phenomena were found to be related through theories which had undergone considerable reformulation. A renowned example of such an event is the unification of electricity and magnetism. The theoretical developments, followed by their experimental verification, established the success of connecting various natural phenomena, with the most notable efforts starting in the nineteenth century. That was the critical point, defining the genesis of *unification*, the area in physics which aspires to provide an accurate description of all the fundamental physical forces in terms of a unified set of mathematical relations and theoretical laws, able to agree with experimental results and predict further experimental observations.

The ultimate intellectual achievement of mankind will be the invention of a theory that *unifies all known fundamental forces* - electromagnetic, gravitational, strong and weak - and which will provide physicists with the ability to understand what rules govern the various complex operations of nature. The Standard Model

(SM) is only the first step in the ambitious scheme of constructing this long sought after theory. Originally, the construction of the SM was attempted in 1973 by Georgi and Glashow in an effort to shed some light on the principles underlying the electroweak and strong interactions. By that time, the idea of obtaining a Grand Unified Theory (GUT) had begun to take root and was inevitably influencing the course of theoretical research. The mathematical model proposed in Ref. [1], was the first instance of a GUT which postulates that all the SM gauge interactions are embedded in a single, simple gauge group: the $SU(5)$ GUT group. The embedding relies on the relation $\mathbf{3} \oplus \mathbf{2} = \mathbf{5}$, which is better realised in the form of a block diagonal matrix:

$$\begin{pmatrix} [SU(3)]_{3\times 3} & * \\ * & [SU(2)]_{2\times 2} \end{pmatrix}_{5\times 5} . \tag{1.0.1}$$

This model bears the brunt of a couple of successes at a qualitative level and as many failures at a quantitative level. Focusing first on its phenomenal successes, a brief account is provided as follows:

- Unification in the aforementioned framework calls for the spontaneous breaking of $SU(5)$ into $SU(3) \otimes SU(2) \otimes U(1)$, which is successfully implemented via the celebrated Brout-Englert-Higgs mechanism. The weak hypercharge is generated by a traceless, diagonal matrix which commutes with the $SU(3) \otimes SU(2)$, *i.e.*

  $$Y \equiv diag\left(-\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}, 1, 1\right), \tag{1.0.2}$$

  so as to be consistent with the weak hypercharge values demanded in the SM. A distinct feature of the emerging theory is the prediction for the existence of additional gauge bosons, identified as X and Y, as well as additional fermions. These particles correspond to the off-diagonal blocks in Eq. (1.0.1). There is plenty of information provided with regards to the number of the existing particles. Conversely, there is limited to no information regarding to the properties of the known particles, and most specifically about the gauge bosons that had been discovered. The enlarged particle content is a question unto itself concerning the assignment of the additional gauge bosons, and most importantly of the SM particle spectrum into $SU(5)$ representations. Here is the point at which the first advantage of having an $SU(5)$ GUT comes into play; in this framework all particles of a single SM generation fall nicely into two irreducible representations of $SU(5)$ and the particles within either of the two irreducible representations are related to one another through gauge transformations. For a mathematical clarification, the left-handed particles fit into the five-dimensional vector representation in conjunction with the ten-dimensional antisymmetric tensor representation of $SU(5)$, while the right-handed particles (antiparticles) suit themselves into the conjugate representations. The representations for the left-handed particles are defined as

$$\overline{\mathbf{5}}_i \equiv \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ -\nu_e \end{pmatrix}_L \quad ; \qquad \mathbf{10}^{[ij]} \equiv \begin{pmatrix} 0 & u_3^c & -u_2^c & -u^1 & -d^1 \\ -u_3^c & 0 & u_1^c & -u^2 & -d^2 \\ u_2^c & -u_1^c & 0 & -u^3 & -d^3 \\ u^1 & u^2 & u^3 & 0 & -e^+ \\ d^1 & d^2 & d^3 & e^+ & 0 \end{pmatrix}_L \,, \tag{1.0.3}$$

which are expressed in terms of the $SU(3)$ and $SU(2)$ representations as well as the $U(1)_Y$ values as:

$$\overline{\mathbf{5}} = \left(\overline{\mathbf{3}}, \mathbf{1}, +\frac{1}{3}\right) \oplus \left(\mathbf{1}, \overline{\mathbf{2}}, -\frac{1}{2}\right); \qquad \mathbf{10} = \left(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3}\right) \oplus \left(\mathbf{3}, \mathbf{2}, +\frac{1}{6}\right) \oplus (\mathbf{1}, \mathbf{1}, 1)\,. \tag{1.0.4}$$

A remarkable outcome of this decomposition is that the anomaly from the $\overline{\mathbf{5}}$ representation is equal and opposite to the anomaly from the $\mathbf{10}$ representation, thus anomalies are cancelled and the $SU(5)$ GUT is *anomaly free* [1].

- A noteworthy characteristic of the model is that there is an identification of *five* distinct colour charges; three of these (blue, green, red) are the established colour charges involved in the strong interactions and the other two are novel colour charges involved in the weak interactions. It is notable that there is no need for a sixth colour to be involved in the electromagnetic interactions. Instead, the five colour charges are sufficient for the consistency of this proposed GUT. The colour charge assignment is immensely useful in determining the only coupling constant of the unified theory, the electric charge [2]. To assign the charge it is essential to utilise the weak hypercharge distribution on particles, as defined in Eq. (1.0.2). The charge is then determined by

$$Q = I_3 + \tfrac{1}{2}Y = diag\Big(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 1, 0\Big), \tag{1.0.5}$$

where I_3 is a traceless generator of the $SU(5)$ group. Subsequently, the charge of quarks and leptons is inferred from the $\overline{\mathbf{5}}$ multiplet in conjunction with the fact that the overall electric charge Q is traceless. This yields

$$3Q(d) + Q(e) + Q(\nu) = 0 \xrightarrow{Q(\nu)=0} Q(d) = -\frac{1}{3}Q(e)\,, \tag{1.0.6}$$

which construes *charge quantisation*; a phenomenon which cannot be explained within the SM context.

The spotlights are now turned on the failures of the proposed model. In the following, a brief account of those failures will be provided.

- All gauge interactions exhibit different coupling strengths. The strong force, as befits its name, has the greatest coupling strength of interactions followed by the weak force which has a slightly greater coupling strength than the electromagnetic force. At the level of unified theories this observation is in direct contrast with the concept of unification, which predicts that all unified gauge interactions have a unique gauge coupling, and thus the strength of their interactions is exactly equal. So, what kind of conspiracy could be lurking amidst the gauge interactions that would allow true unification?

- The X and Y gauge bosons permit the occurrence of *proton decay*, *i.e.* they mediate process such as $p \to e^+\pi^0$ that destabilise the proton. However, so far there are no reported findings of a decaying proton. Indeed the proton lifetime is experimentally found to be 10^{31} years; a direct contradiction to the lifetime predicted by this theory. It is only natural to assume that this theory is at fault due to the extra gauge bosons that seem to have vastly different properties compared to the known ones.

Building on this prior work, Georgi, Quinn, and Weinberg addressed these failures by demonstrating that unification is feasible provided that the $SU(5)$ GUT group is spontaneously broken to the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ at an energy scale of the order of 10^{16} GeV [3]. A good scientific strategy for checking the consistency of GUTs with the experimental outcomes is to measure the coupling of each theory when unification occurs. The unified coupling is related to the electromagnetic, strong and weak couplings through a relation that is expressed in terms of the weak mixing angle θ_W. The crucial measurement needed is that of $sin^2\theta_W$, which in the case of this $SU(5)$ GUT model is evaluated to be $sin^2\theta_W \approx 0.20$ - about the same value as that determined by experimental tests. Note that the unified coupling runs according to the Renormalisation Group Equations (RGEs) and hence for its computation one needs to consider a logarithmic scale. Indeed, what is taken into account is the logarithm of the GUT scale. Therefore, a GUT scale of 10^{16} GeV is considered to be in fact close enough to the Planck scale of 10^{19} GeV - the maximum energy scale that nature handles. Furthermore, a GUT with this order of magnitude accords the appropriate framework for the incorporation of gravity, and predicts a proton lifetime of 10^{30} years; a big enough value that it could be considered compatible with the experimental data.

Even though these outcomes contribute to a vast amelioration in the $SU(5)$ GUT, they are still far from what one could define as 'ideal'. The realisation that there is still room for improvement is what prompted the theoretical physicists to attempt the construction of other *four-dimensional GUTs*. In hot pursuit of a desirable theory, an avalanche of ideas emerged in an effort to investigate whether all fermions (and antifermions) could possibly be accommodated into *one* irreducible representation. A prominent idea is that which promotes the lepton number of fermions to a colour charge and in consequence claims the existence of a left-handed antineutrino. As it is argued in Refs. [4, 5], for the fulfilment of this scenario the simplest course of action is to extend the $SU(5)$ to the $SO(10)$ gauge group. There are thirty fermion fields of the $SU(5)$ model plus two heavy lepton fields which are neutral in charge. The thirty-two fermion fields (and antifermions) are accommodated into the reducible representation of $SO(10)$: $\mathbf{16} \oplus \mathbf{16}$. The Pati-Salam ($SU(4) \otimes SU(2) \otimes SU(2)$) representations can be unified into $SO(10)$ as $\mathbf{16} = (\mathbf{4}, \mathbf{2}, \mathbf{1}) \oplus (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ and the $SU(5)$ representations can be unified into $SO(10)$ as $\mathbf{10} \oplus \overline{\mathbf{5}} \oplus \mathbf{1}$. A generation of fermions is then identified as follows [5]:

$$\mathbf{16} \equiv \begin{pmatrix} u_1^c & u_2^c & u_3^c & \nu_e^c \,|\, u^1 & u^2 & u^3 & \nu_e \\ d_1^c & d_2^c & d_3^c & e^+ \,|\, d^1 & d^2 & d^3 & e^- \end{pmatrix}_L , \tag{1.0.7}$$

where the numbers 1, 2, 3 represent the colour charge of fermions. This is the simplest unique extension of $SU(5)$ and the allocation of the fermion fields to two $\mathbf{16}$-dimensional irreducible complex spinor representations leaves the theory anomaly free. It should be noted that the left-handed antineutrino is a SM singlet and therefore in this framework is expected to get a GUT scale mass. This is what inspired the idea of the see-saw mechanism, a subject which would be a diversion from the purposes of this thesis.

Another interesting model that reproduces somehow the $SO(10)$ results is the E_6 model proposed by Ref. [6]. The E_6 group is the *only exceptional group* that admits chiral representations and is big enough to allow the embedding of the $SO(10)$ group. In this case, six quarks and nine leptons of both left- and right-chirality fit into two $\mathbf{27}$-dimensional complex representations of the E_6, as follows [6]:

$$\mathbf{27} \equiv (\overline{\mathbf{3}}, \mathbf{3}, \mathbf{1}) \oplus (\mathbf{3}, \mathbf{1}, \mathbf{3}) \oplus (\mathbf{1}, \overline{\mathbf{3}}, \overline{\mathbf{3}}) , \tag{1.0.8}$$

where the representation is decomposed into representations of the $SU(3)_L \otimes SU(3)_R \otimes SU(3)_c$[1]. A noteworthy characteristic of this model is that even if another representation is chosen for the fermions, the resulting theory will always be anomaly-free. Furthermore, the E_6 group could be embedded in the E_7 group; specifically the $\mathbf{56}$-dimensional of E_7 accommodates the two $\mathbf{27}$ representations of E_6 along with two singlets. This is what enables the unification of all fermions into one irreducible representation, as it happens in the case of the $SO(10)$ GUT.

Despite the promising findings there is still a flood of theoretical questions that

[1]This is the maximal compact subgroup of the E_6.

cannot be addressed in the context of four-dimensional theories [7]. Among these issues are the *gauge hierarchy problem* - the immense ambiguity in stabilising the quantum effects of the GUT scale at the level of the electroweak scale, which take the form of Ultraviolet (UV) divergences - and the fact that the unification of Higgs or gauge bosons with leptons and quarks seems to be an insurmountable hurdle. Throughout the years, the various developments in the field altered the face of GUTs. The most outstanding development is the discovery of Supersymmetry (SUSY). This is the only existing symmetry which allows the grouping of particles with different spins. The fundamental operation of SUSY is to transform fields of integer spins to fields with half-integer spins, *i.e.* to transform bosons to fermions and vice versa. Originally, the idea of SUSY surfaced in the early construction of string theories; a point that is extensively discussed in Chapter 3. As proposed in a celebrated model by Ramond and Neveu-Schwarz [8], SUSY is a substantial symmetry of the two-dimensional worldsheet for the theory to be successful. In 1974, this model gave a big impetus to the expansion of SUSY to a spacetime symmetry which was formulated by Wess and Zumino [9] and led auspiciously to the supersymmetrisation of the SM.

No signs of spacetime SUSY in nature have so far been observed, and nor there have been any hints. It is an undeniable fact of nature that the four-dimensional world is non-supersymmetric. This leads to the conclusion that SUSY must be a symmetry which is broken during the evolution of the universe. Nevertheless, many theoretical physicists still support the idea that SUSY is a necessary ingredient in the description of nature. The primary goal of spacetime SUSY is to amend one of the greatest flaws in the SM, which is none other than the aforementioned gauge hierarchy problem. What makes SUSY a strong contender for this achievement is the fact that shields an Effective Field Theory (EFT), such as the SM, against unwanted UV completions of any kind. As a result, there have been pioneering attempts to construct a supersymmetric theory which would be consistent with current realistic particle physics models. A widely known example of such a theory is the simplest extension of the SM, the Minimal Supersymmetric Standard Model (MSSM). This model demands that for every SM particle there exists a supersymmetric counterpart, differing by a half-integer spin. For the MSSM to serve its theoretical purposes, the masses of the superpartners need to be at the TeV scale.

Besides complementing the SM symmetries, the inclusion of supersymmetric particles has many implications in the structure of GUTs. First, SUSY raises the GUT scale, and thus brings it even closer to the Planck scale, without causing any drastic changes in the relation among the couplings. As a consequence, the predicted value of $\sin^2\theta_W$ is altered such that it is more compatible with the experimental results. This value is a splendid indication that the electromagnetic, strong and weak coupling strengths converge to a single point at the GUT scale, and greatly reinforces the necessity for a unified theory of the fundamental forces. Second, the raised GUT scale lifts the predicted value of proton lifetime into a value that could be consistent with experimental measurements. These appealing outcomes opened the path for the exploration of the SUSY-GUTs.

As a matter of fact, the four-dimensional GUTs fail spectacularly to incorporate gravity - the least comprehensible fundamental force in nature, and thus are rendered as unsuitable frameworks for the unification of *all* fundamental forces. This shortcoming could in a sense be rectified through the SUSY-GUTs; it is established that when spacetime SUSY is promoted to a local gauge theory, gravity appears to venture into the realm of unification and the emerging EFT is termed *supergravity* [10]. The four-dimensional GUTs also fail on another level: Any attempt to unify all three chiral generations into a single irreducible representation of a gauge group has turned out to be futile. From this observation stems another interesting development which is the suggestion of the possible existence of extra spacetime dimensions. All these developments merge exceptionally well in the content of a single framework; *string theory*.

The earlier model of Ramond and Neveu-Schwarz has only worldsheet SUSY. This was succeeded by the breakthrough discovery of spacetime SUSY on strings, formulated by Gliozzi, Olive and Scherk [11]. The subsequent construction of the heterotic string theory formulated by [12] as well as the Green-Schwarz (GS) anomaly cancellation theorem [13] imparted a great boost to the accession of

SUSY-GUTs into string theory and thus to the unification of all fundamental interactions. In this framework, gravity manifests itself through the existence of a massless spin-2 field, the *graviton*, whose interactions are in agreement with general relativity. Henceforth, quantum mechanics and general relativity are brought together forming a consistent quantum theory of gravity. This means that all gauge bosons plus the graviton, the three generations of chiral matter fields and the Higgs boson could be unified into a single irreducible representation of a simple compact gauge group.

There are in total five superstring theories, the Type IA, IB, Type IIA, IIB and the Heterotic. Each one of this is related to another via symmetries that hold at a quantum mechanical level and are called *dualities*. In the modern history of string theory all theories are realised as different limiting cases of a single, grander theory known as the M-theory. Among these, the Type I and Type II theories could not be considered as realistic models of particle physics – where "realistic" is usually taken to mean that the string spectrum bears a resemblance to the MSSM. However, in the heterotic formulation it was demonstrated by Ref. [14] that there exist only two models that could accomplish the purpose of unification, could be considered as realistic and at the same time allow the cancellation of anomalies: the ten-dimensional heterotic $E_8 \otimes E_8$ and $SO(32)$. Of these two options, the former works remarkably well. The starting point of the $E_8 \otimes E_8$ theory is a geometrical space of $\mathbb{R}^4 \otimes \mathbb{K}^6$, where $\mathbb{R}^4$ is the four-dimensional Minkowski space which is necessary to preserve spacetime SUSY in four dimensions and $\mathbb{K}^6$ is a six-dimensional compact manifold [14]. The E_8 group breaks down to a subgroup which is usually one of the GUT groups discussed above. The breaking occurs when the gauge fields on $\mathbb{K}^6$ that lie in other E_8 subgroups are assigned a Vacuum Expectation Value (VEV) so that there remains an unbroken gauge group. The upshot of this is the fact that all the chiral fields fit into the correct representations [14]. Note that due to an abundance of choices for assigning VEVs to the $\mathbb{K}^6$ fields there is in general an ambiguity concerning the total number of chiral generations one could get in the model.

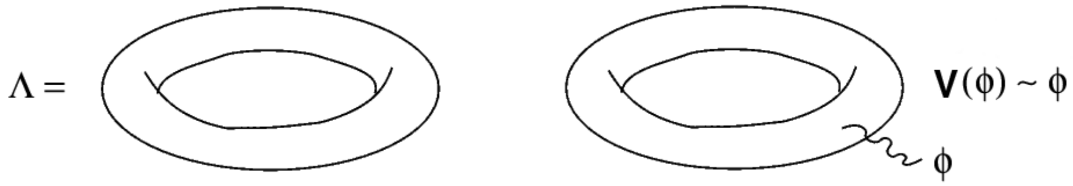
Even though the $E_8 \otimes E_8$ is a ten-dimensional GUT, it yields quite similar results to those of four-dimensional GUTs with regards to the value of $\sin^2\theta_W$, the proton lifetime and the quantum numbers of fermions. A considerable point is that the scale of grand unification in string theories is raised so that the difference to the Plank scale is reduced even more. However, the $E_8 \otimes E_8$ bears many differences to four-dimensional GUTs and a cursory explanation about these differences is given by Ref. [7]. It is therefore inferred that the enrichment of GUTs with SUSY, in addition to the inclusion of string theory, makes the quest for a unified theory of all fundamental interactions appear even more promising.

Over the last three decades, the idea that unification is such an appealing world that it would be a shame to give it up without a fight has been dramatically reinforced. This is due to many interesting developments both on the theoretical and the experimental side of particle physics. First, the proton decay suggested by the models of grand unification is being probed by experiments, such as Super-Kamiokande, with great sensitivity and the experimental value is currently in the range of the theoretical prediction. Second, the precision measurements of $\sin^2\theta_W$ are consistent with the GUTs results. Third is the discovery of neutrino oscillations which imply that neutrinos have masses albeit very small ones. The GUT models predict that neutrinos have masses which are in the range of the experimental results, a prediction expected to be confirmed by the next generation of neutrinoless double beta decay experiments. The most recent splendid finding in particle physics that greatly encourages the case for unification was the discovery of Higgs boson at the Large Hadron Collider (LHC) on July 2012. The Higgs boson is the manifestation of the Higgs field and had been a missing piece in the completion of the SM puzzle. With this discovery, it is confirmed that theory is on the right track and new paths beyond the SM physics are now available for exploration.

Ultimately, the biggest dream of theoretical particle physicists is the formulation of a correct consistent quantum theory of gravity. This dream is now one step closer to reality by virtue of the detection of gravitational waves for the first time, by the Laser Interferometer Gravitational-Wave Observatory (LIGO) collaboration earlier this year. The existence of gravitational waves is postulated in the theory of general relativity and their direct observation took place a century after the waves were predicted by Albert Einstein. The detection of gravitational waves has been the only missing piece from the ingenious formulation of general relativity. The evidence of their existence establishes the superiority of general relativity at a classical level and confirms everything that has been predicted years ago. This discovery is expected to give a big impetus to the elucidation of gravity and consequently to the advancement of quantum gravity theories. On the account of deciphering gravity the future irrefutably shines bright.

The same argument cannot be applied for SUSY, the framework that is supposed to expand the foundations of the SM into a more solid and comprehensive picture of the universe mechanism. The LHC has been tuned on a relentless race in search of SUSY but so far has not buzzed with any signs. An indisputable observation is that SUSY has not been detected amongst the elementary particles or their fundamental interactions at the presently accessible energy scales, nor are there at the present time any signs or even hints of its imminent appearance at higher energies. Indeed, the recent LHC data seem to be giving indications against even the most minimal version of SUSY, therefore implying that it would no longer be capable of addressing the gauge hierarchy problem. Especially after the recent discovery of the Higgs, the issue of stabilising the gauge hierarchy becomes all the more pressing. While many theoretical physicists continue to feel that SUSY lurks somewhere ready to be found, and support the idea that its discovery is only a matter of time, there is increasing room for doubt and thus much interest in alternatives. So this appears to be a good moment to ponder what the implications are for string theories and even more for string phenomenology.

As it is, there have always been distinct choices available to string model-builders: The first one is to assume that nature is fundamentally supersymmetric and that SUSY is broken so as to produce the observable non-supersymmetric world. The second one, is to assume that nature is, by contrast, non-supersymmetric at *all* levels, and then to follow what this path entails. In string theory, non-supersymmetric and supersymmetric theories are radically different, even when they might seem to be closely related. For most of the modern history of string theory, and specifically after the discovery of spacetime SUSY on strings, the first



(a)The one-loop Casimir energy (cosmological constant) Λ.

(b)The one-loop one-point dilaton "tadpole" diagram.

Figure 1.1: In general, the value of the dilaton tadpole is always proportional to Λ. As a result, a non-zero cosmological constant implies a non-vanishing one-loop dilaton tadpole diagram, in turn indicating a linear term $\sim \phi$ in the effective potential. This figure is adapted from Refs. [15, 16].

option has been tremendously favoured over the second option. As a result, string phenomenology has largely consisted of constructing realistic or semi-realistic string models with $\mathcal{N} = 1$ SUSY which is then broken by the operation of some field-theoretic SUSY-breaking mechanism. Meanwhile the second option, that of constructing non-supersymmetric models of which the low energy limit corresponds to the SM, has not been dealt with an overly enthusiastic response. Hence the possibility of developing a genuinely non-supersymmetric string phenomenology for the weakly coupled heterotic string has not attracted the attention it deserves.

One of the major hurdles in constructing a string model without spacetime SUSY is to tackle the issue of dilaton stability. As shall be discussed in Chapter 4, non-supersymmetric strings are generally unstable, giving rise to non-zero dilaton tadpole diagrams. The existence of such tadpole diagrams has proved to be extremely problematic, indicating that such strings are generally formulated on destabilised vacua. At one-loop order, a non-vanishing value of the dilaton tadpole always results in a non-vanishing value for the cosmological constant which in turn implies a non-vanishing dilaton tadpole diagram. This effect is realised by having an extra linear term ϕ appearing in the effective potential of the theory, as sketched in Fig. 1.1. In general, this is what string theorists describe as the *cosmological constant problem*.

By and large, the dilaton tadpoles could be absorbed via the Fischler-Susskind mechanism [17], as in Refs. [18, 19]. However, if such tadpoles are unsuppressed there would be some dire consequences, in the sense that the new resulting background is expected to be very different from the initial one, thereby invalidating the original construction. Thus, in any complete discussion of non-supersymmetric string phenomenology, the challenge of overcoming the instabilities associated with the dilaton tadpoles will play a crucial role. It goes without saying that this realisation is very advantageous as regards to the cosmological constant problem. Indeed, in any non-supersymmetric string theory it is of utmost importance to obtain a value for the cosmological constant that is hierarchically smaller than the generic value. In this way, only those special theories in which the cosmological constant becomes zero to leading order could be deemed to be consistently stabilised.

It is the purpose of this thesis to show that it is possible to overcome the hindrance of non-zero dilaton tadpoles and build *non-supersymmetric perturbative heterotic string models* which are essentially stable. In this thesis, is undertaken a dedicated and systematic exploration of this class of models for which the degree of instability associated with the dilaton tadpole is exponentially suppressed, at least at one-loop. On that account, these models could be considered as standing on equal footing with their SUSY cousins. Moreover, it shall be demonstrated through explicit construction that models within this class exhibit a semi-realistic particle content, with the massless states resembling the SM while their erstwhile superpartners have masses that can be tuned to literally any value, including values at the TeV scale! It is thus shown that the dilaton tadpole and the associated non-supersymmetric instabilities are suppressed while simultaneously retaining an auspicious low-energy phenomenology, all without any light superpartners for the SM particles and hence no remnant of SUSY in the spectrum of the resulting string theory. It is worth emphasising that these are entirely *non-supersymmetric* string models at all scales, including their fundamental Planck scales, and it would therefore be a big mistake to view them as having been supersymmetric at the Planck scale but subsequently subjected to some sort of purely field-theoretic SUSY-breaking mechanism at lower energies.

The existence of such models establishes the beginning of a new framework for the development of an entirely *non-supersymmetric* string phenomenology. The significant advantage of building models in this framework is that the models are UV complete, with finiteness ensured through entirely stringy mechanisms such as the modular invariance and "misaligned SUSY" [20–22]. Both the modular invariance and misaligned SUSY are additional symmetries ingrained in string theory and always guarantee the finiteness of the theory, even when there is no spacetime SUSY on strings. However, they have no equivalent counterparts within Quantum Field Theory (QFT). From this standpoint, it is only natural to wonder whether progress in non-supersymmetric, UV complete phenomenology could provide answers or explanations for possible physics beyond the SM - such as the gauge hierarchy problem for example - which might descend directly from string theory but which nevertheless has no traces of SUSY at any energy scale.

A large part in this thesis is invested in the methods by which suitable non-supersymmetric UV complete strings might be constructed, the criteria for their stabilisation and the low-energy phenomenologies to which they give rise. The string models presented in this work are first constructed in Ref. [15] and have the innovative advantage that their one-loop cosmological constants are exponentially suppressed. Their dilatons are thus essentially stable, at least to one-loop order, rendering these models as suitable platforms upon which to build a study of non-supersymmetric string phenomenology.

The results of Ref. [15] demonstrate that hierarchically separated scales *can* be natural within the context of non-supersymmetric string theories. In this, whatever spacetime SUSY might have otherwise existed has been destroyed through purely string-theoretic steps in the primordial model-construction process. This could occur through particular choices of SUSY-breaking compactifications from ten dimensions that nevertheless respect modular invariance. Such compactifications are often considered to be generalised versions of Scherk-Schwarz compactification. It is important to clarify a critical point regarding the mechanism of Scherk-Schwarz compactification. It is common to use a heuristic language when talking about the various steps taken in the construction of a given string model and one might then refer to a "SUSY-breaking" step. For example, this applies for Scherk-Schwarz compactifications, in which case the terminology is slightly abused so that one often speaks of this as a spontaneous breaking of spacetime SUSY. This terminology is repeatedly used throughout this thesis, therefore it should be stressed that spacetime SUSY is not broken according to any time-ordered or energy-ordered dynamics. A non-supersymmetric self-consistent model is produced from another string model via a set of well-defined procedures. As it shall be demonstrated, it is possible to produce a four-dimensional non-supersymmetric string model from a higher-dimensional (parent) supersymmetric theory. However, the resulting model is not inferior to the original one as it is fundamentally non-supersymmetric at every energy level of the theory.

It is crucial to realise that the set of well-defined procedures involves purely string-theoretic construction methods that ensure the UV-finiteness of the emerging theory with or without SUSY. Specifically, the modular invariance imposes powerful constraints which enable the truncation of the one-loop integrals excluding those regions where the UV divergences reside, thus the finiteness of the theory remains intact. More physically, at the level of the string spectrum, this finiteness is maintained via the "misaligned SUSY". This is a hidden residual symmetry that *always remains in the spectrum of any self-consistent string theory*, even if spacetime SUSY itself is absent [20–22]. It entails a subtle configuration of bosonic and fermionic states throughout the string spectrum and one of its distinct characteristics is the absence of any boson/fermion pairing, either exact or approximate. Finiteness is then achieved through the contribution of all the states in the spectrum which distribute themselves in such a way so as to produce finite amplitudes. Given this, the major accomplishment of the semi-realistic models constructed in Ref. [15] has been to demonstrate that these models can tolerate large scale separations and all the while preserve their UV-finiteness.

This work by no means claims that the string models presented here are completely satisfactory as *bona-fide* models of the universe, or even as phenomenologically acceptable string vacua. Just as with the SUSY string models, these non-supersymmetric heterotic string models generally contain many unfixed moduli whose VEVs ultimately remain to be determined. The main point, however, is to demonstrate that these non-supersymmetric models could be just as viable as the supersymmetric ones, in the sense that the instabilities associated with the absence of spacetime SUSY could be exponentially suppressed. Henceforth, the phenomenology of such non-supersymmetric heterotic strings could be viewed as mirroring that developed over the past thirty years for their traditional supersymmetric counterparts.

However, it would be remiss of me not to mention that even though the past three decades there has been a torrent of work focusing on supersymmetric string model-building, there has likewise been a steady trickle of work focusing on the diverse properties of non-supersymmetric strings. An interest on this work was sparked by the original studies of the ten-dimensional $SO(16) \otimes SO(16)$ heterotic strings [23] – the first known model of non-supersymmetric theory which is completely free of tree-level tachyons. This includes studies of the one-loop cosmological constants of non-supersymmetric strings [20, 21, 24–37], their finiteness properties [20, 21, 38] and their strong/weak coupling duality symmetries [39–42]. The landscapes of such strings have also been studied [43, 44], and all studies of strings at finite temperature are also implicitly studies of non-supersymmetric strings. An early work on this area is found in Refs. [45–49]. Generally, the non-supersymmetric string models studied in previous works were either non-supersymmetric by construction or exhibited a form of Spontaneous Supersymmetry Breaking (SSSB) [19, 24, 50–56] by adapting the Scherk-Schwarz mechanism to string theory [57]. Within this class, there have been constructed a number of potentially viable models which are presented in Refs. [18, 25, 42, 58–63]. Moreover, non-supersymmetric string models have also been studied in various other configurations [64–77], including studies of how the energy scales of different schemes could possibly be related [78–84]. Eventually, moving into more recent times, there has been a continuing progress in the study and progress of phenomenology for non-supersymmetric string theory [15, 85–90].

The rest of this thesis will be devoted to a discussion on the construction of *four-dimensional, tachyon-free, non-supersymmetric* models with a low energy spectrum that resembles either the Pati-Salam or the SM theory. The centre of attention will be on their properties as well as in some future ideas regarding the development of non-supersymmetric string phenomenologies. This thesis is then organised in four main parts. The first part, consisting of **Chapter 2** through **Chapter 3** lays the general groundwork of the study presented, which is independent of the model construction formalism. *Chapter 2* is the preface chapter of the thesis which imparts an overview of the theoretical foundations of the SM and SUSY, with a special emphasis on the MSSM. *Chapter 3* is an overview of the string theory framework. It summarises the mathematical development of string theory to the present and points at some of the possible paths that lie open ahead in the future.

The second part, consisting of **Chapter 4** through **Chapter 5** gives the general theoretical features of all non-supersymmetric string models. It provides a detailed explanation of the expected phenomenological properties of interpolating models - a special class of non-supersymmetric models which interpolate between two higher-dimensional theories at their two endpoints $R \to \infty$ and $R \to 0$, with R being a generic radius of compactification. Moving on then to *Chapter 4* there is an introduction on the formalism used throughout this thesis for the construction of the desired models. This is followed by details regarding the theoretical properties of all such non-supersymmetric, tachyon-free heterotic strings and a general account on the associated finiteness and stability properties. In *Chapter 5* the discussion is focused on a specific class of models, the so-called *interpolating models*. There is presented the link between interpolation and the enhanced stability properties of the models constructed; the one-loop cosmological constants associated with such strings sit at the root of this explanation. Overall, this chapter provides a derivation of the leading and subleading terms that govern the behaviour of the cosmological constant, paying particular attention to the role played by off-shell string states and their contributions.

The stage having thus been set, the third part of this thesis, consisting of **Chapter 6** through **Chapter 8** focuses on the actual construction of semi-realistic non-supersymmetric heterotic string models with suppressed cosmological constants. This is done in several steps, for which a general outline is provided in *Chapter 6*. It is first shown a particular six-dimensional free-fermionic string model from which the ultimate four-dimensional string models emerge upon compactification. For comparison purposes it is also shown a traditional SUSY-preserving $\mathbb{Z}_2$ orbifold compactification, along with a description of the associated theoretical and phenomenological features. Then, in *Chapter 7*, there are presented two different types of SUSY-breaking by applying different Coordinate Dependent Compactifications (CDCs). The first one is a CDC performed on a two-dimensional torus thereby producing a non-chiral $\mathcal{N} = 2 \to \mathcal{N} = 0$ four-dimensional theory, whereas the second one is a CDC performed on $\mathbb{Z}_2$ orbifold thereby producing a chiral $\mathcal{N} = 1 \to \mathcal{N} = 0$ four-dimensional theory. For the latter, an analysis of the emerging low energy spectrum and of the cosmological constant's behaviour is provided. In *Chapter 8* it is then shown how this model can be altered in different ways in order to achieve the desired goal, namely models with exponentially suppressed dilaton tadpole instabilities. Indeed, several models of this type are presented, one whose low-energy spectrum resembles the SM, and others resembling either Pati-Salam-like or GUT-like "unified" extensions thereof.

The fourth and final part of this thesis, consisting of **Chapter 9** through **Chapter 11** then presents the phenomenological properties of models with exponentially suppressed cosmological constants, including their Yukawa couplings and scalar masses. There is also a brief account of the potential implications of having a non-supersymmetric theory which is consistent with the naturalness problem in string theory. In *Chapter 9* there are first discussed two theoretical properties of these models: the behaviour of the degeneracies associated with their physical-state spectra as functions of energy, and the behaviour of their cosmological constants as functions of their compactification radii. For both properties, it is deduced that there are special features which are unique and which reflect the fact that these models exhibit enhanced stability properties relative to typical non-supersymmetric string models. Next discussed are their phenomenological properties, focusing on particle assignments, Yukawa couplings, and scalar masses. In *Chapter 10*, there is a reference to the danger of having extremely large gauge couplings at large volumes in weakly coupled heterotic strings. It is therefore proposed that such dangers could be overcome through models which admit extra states known as “GUT precursors”. In order to demonstrate this, there is shown an example of a ‘stable’ non-supersymmetric SM-like theory which allows a GUT precursor structure. *Chapter 11* ends the narrative with a concise summary of everything presented in this thesis and provides an outline of some avenues for further study in the future.

This thesis also contains five Appendices. The first three provide some additional information on the theoretical background of this work. The next one describes the notation and conventions used in the analytic computation of the partition functions and subsequently of the cosmological constants. The final Appendix presents an alternative way of breaking SUSY, known as *discrete torsion*.

Chapter 2

Theoretical Foundations

The world is a construct of our sensations, perceptions, memories. It is convenient to regard it as existing objectively on its own. But it certainly does not become manifest by its mere existence.

Erwin Schrödinger

2.1 The Standard Model of particle physics

All of the natural phenomena occurring at the microscopic level are understood in terms of the electromagnetic, weak and strong fundamental forces. A complete description of the behaviour of all known subatomic particles can be provided within a single, theoretical framework which incorporates the basic principles of the Quantum Mechanics (QM) and Special Relativity (SR). The relativistic QFT which emerges from this framework is none other than the illustrious SM of elementary particle physics. The SM, as it is discussed below, is only a very tiny fraction of the *optimum GUT* that will unify the gravitational force with the remaining three fundamental forces and will depict the fundamental laws operating at the roots of our world.

In particle physics the concept of *symmetry* plays a crucial role for the successful development of accurate and 'elegant' unified theories. The existence of various symmetries points to the underlying principles of nature and these symmetries are generally classified as either *global* or *local*. Exact symmetries are realised through the latter and require the existence of gauge fields, whereas the former are usually only approximate.

The electromagnetic, strong and weak interactions are all related to local symmetries described by Abelian and non-Abelian gauge theories. Consequently, the SM is a well-defined gauge theory with the non-Abelian symmetry group

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \,, \tag{2.1.1}$$

where C, L and Y denote the colour, weak isospin and electroweak hypercharge respectively. The first gauge group encompasses the strong interactions, while the remaining two encompass the weak and electromagnetic interactions [91]. Specifically, the electromagnetic interactions are associated with a combination of $SU(2)$ and $U(1)$ generators, *i.e.* $Q_{em} = T_3 + Y$.

2.1.1 Technical Overview

The SM is a theory of fields with spin $0, \frac{1}{2}$ and 1; its particle content is shown in Fig. 2.1. The gauge interactions of the particles involved are determined by their spin as well as their mass and quantum numbers (charges). The spin-$\frac{1}{2}$ particles are fermions which are either leptons or quarks. Since all of the observed matter in the universe is made up of leptons and quarks, the SM fermionic fields are defined to be the *matter fields*. As it turns out, there exist sets of fermions which have identical quantum numbers but different masses. Based on their mass hierarchies the fermions are organised in *three families or generations*. Generations with larger masses are unstable and will eventually decay into the lightest, stable generation which forms the ordinary matter in the universe, including us! As shown in Fig. 2.1, each column corresponds to a generation which consists of two quarks and two leptons, distinguished by their charges under the electromagnetic and

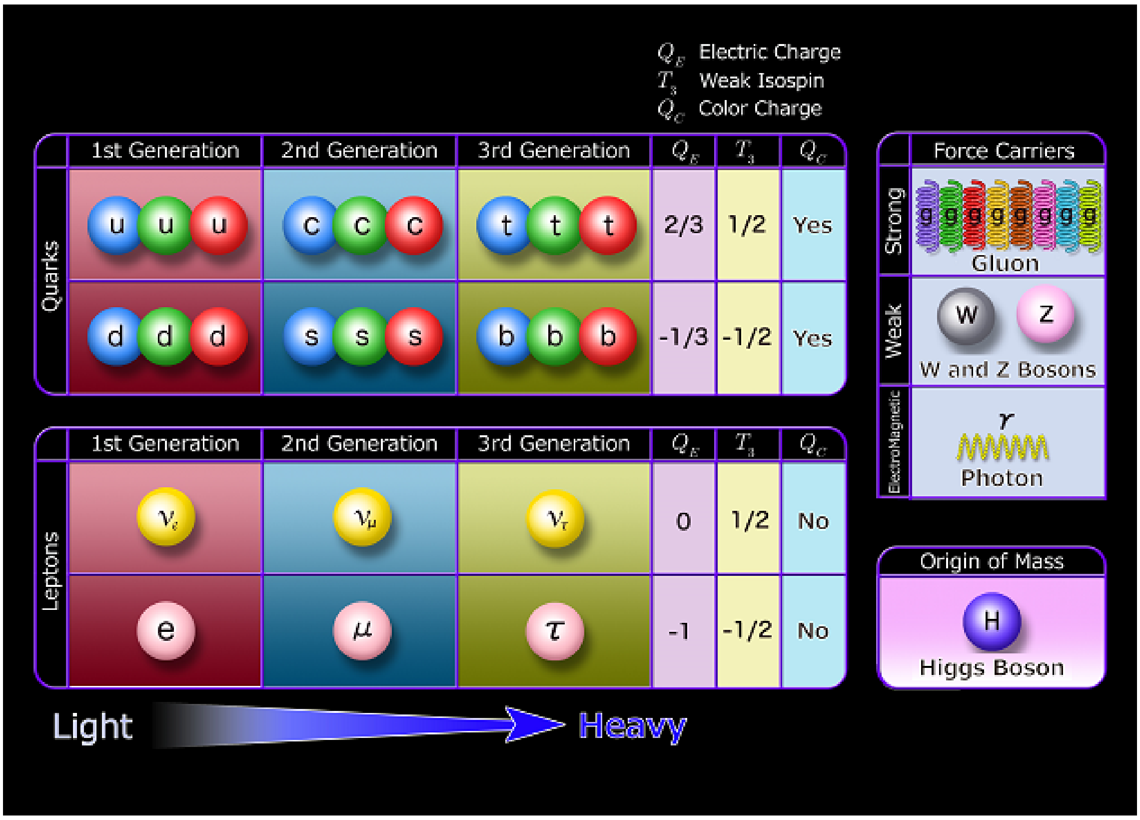


Figure 2.1: Particle content of the Standard Model.

strong interactions. The leptons participate solely in the electromagnetic and weak gauge interactions, therefore they *only* carry an electromagnetic charge (Q_{em}). The quarks participate in all three gauge interactions, carrying an extra charge under the strong interactions, the *colour* charge. Specifically, the quarks come in triplets of colour, *i.e.* they carry an index $\hat{a} = 1, 2, 3$ which in the experimental observations are identified with the blue, green and red colours. Among the leptons, the neutrinos appear to have the most bizarre behaviour: although they participate in the weak interactions they are not charged under any of the three gauge interactions in the SM. In addition, the neutrinos are experimentally found to have masses which are several orders of magnitude lighter than the masses of the other SM fermions.

From a theoretical point of view, the spin-$\frac{1}{2}$ fermions are associated with the two irreducible representations of the $SO(1,3)$ Lorentz group, which are distinguished by what is known as the *chirality*. A massive fermion has two components of different chirality, forming what is known as a Dirac spinor ψ which is therefore expressed as the sum of a left-handed part ψ_L, and a right-handed part ψ_R,

$$\psi = \psi_L + \psi_R\,, \tag{2.1.2}$$

where

$$\psi_L = P_L\psi \qquad \text{with} \qquad P_L = \tfrac{1}{2}(1-\gamma_5) \tag{2.1.3a}$$

$$\psi_R = P_R\psi \qquad \text{with} \qquad P_R = \tfrac{1}{2}(1+\gamma_5)\,. \tag{2.1.3b}$$

The P_L and P_R are projection operators, projecting out the left-handed (or negative) and the right-handed (or positive) chirality states of the fermion, respectively. They satisfy

$$P_LP_L = P_L, \quad P_RP_R = P_R, \quad P_LP_R = P_RP_L = 0 \quad \text{and} \quad P_L + P_R = \mathbb{I}\,. \tag{2.1.4}$$

As a result, each SM matter field is a *chiral* fermion. Although chirality is a definite property of the fermions it is not a physical observable. In the case of massless fermions, chirality is conserved and coincides with the helicity. In the SM, the total number of chiral fermions organised in three generations is 48. Therefore, each generation consists of 16 *chiral fermions* with the left-handed leptons and quarks coming in doublets of the weak isospin $SU(2)$,

$$l_L^{\hat{c}} \equiv \begin{pmatrix} \nu_e^{\hat{c}} \\ e^{\hat{c}} \end{pmatrix}_L = \left(\mathbf{1}, \mathbf{2}, -\tfrac{1}{2}\right) \qquad \text{and} \qquad Q_L^{\hat{a}\hat{c}} \equiv \begin{pmatrix} u^{\hat{a}\hat{c}} \\ d^{\hat{a}\hat{c}} \end{pmatrix}_L = \left(\mathbf{3}, \mathbf{2}, +\frac{1}{6}\right), \tag{2.1.5a}$$

and the right-handed leptons and quarks being $SU(2)$ gauge singlets

$$e_{R\hat{c}} = (\mathbf{1}, \mathbf{1}, 1); \qquad \{\nu_{R\hat{c}} = (\mathbf{1}, \mathbf{1}, 1)\}; \tag{2.1.5b}$$

$$u_{R\hat{c}} = \left(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3}\right); \qquad d_{R\hat{c}} = \left(\overline{\mathbf{3}}, \mathbf{1}, +\frac{1}{3}\right). \tag{2.1.5c}$$

Here $\hat{c} = 1, 2, 3$ is the family index, and the quantities in brackets represent the $SU(3), SU(2)$ representations and $U(1)_Y$ value respectively. Experimentally it is found that only the left-handed components of the fermions participate in the charged current weak interactions. There are no right-handed neutrinos in the SM, but if they exist, they could provide answers for several phenomena that cannot be explained within the SM, including some which are discussed in the next section. Experimental evidence from the neutrino oscillations hint to their existence but they remain yet to be observed.

The spin-1 particles are the gauge bosons whose exchange is associated with the fundamental interactions in the SM. These particles include the photon mediating the electromagnetic interactions, the eight gluons mediating the strong interactions and the $W^{\pm}$, Z^0 bosons mediating the weak interactions, as per Fig. 2.1.

Evidently, our world is not quite as symmetric as the theories of interactions used for its description. The reason for this lies in the fact that nature favours many symmetries which are broken. Symmetries can be *explicitly* broken due to the presence of non-invariant terms in the Lagrangian ($\mathcal{L}$) of the theory. Sometimes, the quantisation of a theory may also lead to an explicit Symmetry Breaking (SB) even if the Lagrangian is invariant. In such a case, the explicit SB occurs in the measure of the Feynman path integral introducing *anomalies*. Only theories in which a *global* symmetry is explicitly broken are regarded as consistent theories. In the SM, the explicit breaking of the gauge symmetries introduces gauge anomalies. The consistency of the SM as a theory is guaranteed by the cancellation of all the gauge anomalies as a consequence of the properly arranged chiral matter fields in each generation.

Symmetries can be also *spontaneously* broken, a dynamical effect which takes place when the ground state of the system is not symmetric, so the system itself breaks that symmetry in a ‘spontaneous’ way. When a *continuous global* symmetry is spontaneously broken by the choice of the vacuum, a massless field known as *Goldstone field* appears for each broken generator of the symmetry. The Goldstone modes carry the same quantum numbers as the corresponding broken symmetry generators. This implies that if a global symmetry with bosonic generators is spontaneously broken the resulting massless Goldstone particles will be neutral bosons; the so-called *Goldstone bosons*. The choice of the true vacuum of the theory is equivalent to choosing a suitable gauge for the quantisation of that theory.

The Goldstone bosons which can, in principle, transform the chosen vacuum state into any other degenerate vacuum state, now influence transformations into states which are inconsisted with the original chosen gauge. This leads to the conclusion that the Goldstone bosons are in fact 'unphysical'.

On the other hand, the Spontaneous Symmetry Breaking (SSB) of a *gauge* (or *local*) invariance results in the well-known Higgs effect, where after a gauge coupling the Degrees of Freedom (d.o.f) of a Goldstone boson associated with the broken generator of the symmetry provide an extra helicity state, a longitudinal component for the spin-1 local gauge boson which now becomes massive[1]. The scalar fields responsible for the spontaneous breaking of such a gauge symmetry are known as *Higgs fields*.

The fundamental interactions described by the symmetry group in Eq. (2.1.1) are only valid when the gauge bosons and chiral matter fields are massless and the gauge invariance is preserved. However, a series of accurate and successful experimental results revealed that the weak force interactions are short range and are mediated by the $W^{\pm}$ and Z^0 bosons which are massive. The chiral fields also get a mass at low energies, while the gluons and the photon remain massless. These results directly contradict SM gauge invariance, so the logical explanation is that SSB must occur. Indeed, the electroweak symmetry is spontaneously broken, preserving only the unbroken group

$$SU(3)_C \otimes U(1)_{em} \,. \tag{2.1.6}$$

The spontaneous breaking of the electroweak symmetry in the vacuum is due to the self-interactions of the scalar Higgs field (Φ) which permeates the entire universe and when excited takes the form of a scalar (spin-0) particle, the famed Higgs boson (H). The Higgs field is a complex doublet with $Y = \frac{1}{2}$ under $SU(2)_L \otimes U(1)_Y$ and a singlet under $SU(3)_C$.

Having introduced all the ingredients of the SM, it is now possible to write the

[1]A massless spin-1 gauge boson, *i.e.* the photon has only two d.o.f which are its polarisation states. A massive spin-1 gauge boson has three d.o.f - helicity states.

complete Lagrangian of the theory which is rather simple:

$$\mathcal{L}^{SM} = \mathcal{L}^{SM}_{gauge\ bosons} + \mathcal{L}^{SM}_{fermion\ KT} + \mathcal{L}^{SM}_{fermion\ masses} + \mathcal{L}^{SM}_{Higgs}\,, \tag{2.1.7}$$

with

$$\mathcal{L}^{SM}_{gauge\ bosons} = -\tfrac{1}{2}Tr\ [F_{\mu\nu}F^{\mu\nu}] \tag{2.1.8a}$$

$$\mathcal{L}^{SM}_{fermion\ KT} = \overline{\Psi}_{L\hat{c}}\gamma^{\mu}D_{\mu}\Psi_{L\hat{c}} \tag{2.1.8b}$$

$$\mathcal{L}^{SM}_{fermion\ masses} = -\tfrac{1}{2}\Psi^{T}_{L\hat{c}}\,\mathbf{Ch}\,\Phi\Psi_{L\hat{c}} + h.c \tag{2.1.8c}$$

$$\mathcal{L}^{SM}_{Higgs} = |D_{\mu}\Phi|^2 - \mu^2\Phi^{\dagger}\Phi + \tfrac{1}{2}\lambda_H(\Phi^{\dagger}\Phi)^2\,. \tag{2.1.8d}$$

The $\mathbf{C}$ and $\mathbf{h}$ in the third term are the charge conjugation matrix and the Yukawa coupling matrix respectively. The explicit form of the terms in Eq. (2.1.8) is given in the Appendix A.1. All coupling constants are dimensionless and as it is noted above there is no direct mass term for any of the leptons, quarks or vector bosons. The masses of all the SM particles, except those of neutrinos, are generated via the Higgs mechanism in which case the arrangement of all the renormalisable interactions causes the neutral component of the Higgs field to acquire a VEV, v = 246.22 GeV [92]. This outcome sets the scale of the electroweak symmetry breaking [93].

$SU(3)_C$ has eight generators in total that remain unbroken. Hence the corresponding gauge fields, the gluons remain massless. The $SU(2)_L \otimes U(1)_Y$ group has four generators in total and the Higgs field has initially four *real*, scalar d.o.f. When the electroweak symmetry breaks spontaneously (only for $\mu^2 > 0$) there is a generator that remains unbroken and which is associated with the $U(1)_{em}$ gauge symmetry. Its corresponding gauge field is none other than the massless photon. The remaining three generators correspond to the gauge fields which mix with one neutral and two charged massless Goldstone d.o.f (G^0 and $G^{\pm}$ respectively). The unphysical Goldstone bosons can be removed by a transformation to the *unitary gauge*. In this gauge, their d.o.f become the longitudinal components of the Z^0 and $W^{\pm}$ physical gauge bosons respectively. At the same time two d.o.f of the Higgs field are absorbed by the $W^{\pm}$ and one by the Z^0 gauge bosons which now become massive. The fourth d.o.f of the Higgs field becomes a new fundamental spin-0 particle; this is the physical Higgs boson H. In the unitary gauge the Higgs field is parametrized as

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} . \tag{2.1.9}$$

The Higgs boson is a singlet under $SU(3)_C$ and is neutral under electromagnetic interactions, therefore it does not couple at tree level to the massless gluons and photon. The chiral matter fields become massive when the Higgs doublet field couples to them through Yukawa interactions. However, the masses of neutrinos are not generated by Yukawa interactions but instead through another process. The most prevalent scenario is that of the see-saw mechanism. The masses of the particles in the SM are beyond the scope of this thesis, however, for the enthusiastic readers the formulas for the computation of their values are presented in the Appendix A.2.

2.1.2 Phenomenological Overview

Over the past few decades, the SM has been subjected to a series of experimental tests at energies up to several hundred GeV and even after the results of the first LHC run, it passes all tests with flying colours. At present it is the theory that provides the best phenomenological understanding of the fundamental world with a great accuracy. Undeniably, the rich structure of the theory has led to different areas of research such as Quantum Chromodynamics (QCD), flavour physics which is mainly focused on the Cabbibo-Kobayashi-Maskawa (CKM) model of flavour mixing and Charge-Parity (CP) violation, and of course Higgs physics.

For more than 40 years, it was predicted that the missing piece required to complete the SM puzzle was the Higgs boson, responsible for the origin of mass. As stated in [94–96], without the Higgs boson the SM no longer exhibits perturbative unitarity at high energies because the longitudinal scattering amplitude of the $W^{\pm}$ and Z^0 bosons would be allowed to grow proportionate to the increase in the centre-of-mass energy. Furthermore, the longitudinal components of the radiative corrections to the self-energies of the gauge bosons would certainly exhibit unpredictable logarithmic divergences [93].

The hunt for the elusive Higgs boson, finally came to fruition in July of 2012 at the LHC. The Higgs boson is measured by both ATLAS and CMS experiments to have a combined mass of m_H = 125.09 ± 0.24 GeV [97]. Although at the first stages of its discovery physicists seemed to be suspicious whether it actually was the SM Higgs boson, evidence shows that it is as predicted: the possibility of the boson found at the LHC having spin-1 is excluded and the experimental results exclude also the possibility of it having spin-2, confirming that it is in fact a spin-0 particle. Moreover, flavour calculations and CP violation measurements regarding the Higgs are consistent with SM predictions [98].

Consequently, the long-awaited discovery of the Higgs boson validates the Higgs mechanism and confirms the SM to be one of the greatest triumphs of 20th century particle physics. With this discovery, the experimental verification that m_H < 1 TeV establishes that indeed the WW scattering cross-section does not violate the perturbative unitarity as predicted in [94–96]. Most importantly, it can be seen as the first step into finding and understanding the properties of the so significant Higgs field. Nonetheless, the multitude of experimental results strongly imply that the SM is a gauge theory with the potential to form part of a consistent theory all the way to the Planck scale.

Currently, all the LHC results indicate a rich and diverse phenomenology regarding the Higgs physical properties, interactions with fundamental particles and CP violation measurements. It does not come as a surprise that these outcomes, in conjuction with other results from the first LHC run [98], point the way to Beyond Standard Model (BSM) physics, with the SM being an EFT at the low energies (< 1 TeV). Certainly, with the discovery of the Higgs boson, the door to BSM physics research is now wide open and the experimental results from the second LHC run are expected to shed light on this.

Despite the astounding success of the SM, phenomenological observations leave no room for doubt that there are still many theoretical problems to be addressed where the SM fails spectacularly to provide answers. Given the strong experimental indications, it is widely thought that the SM is just a small part of a bigger picture; it is simply *the low energy limit of a more fundamental theory*. There are numerous issues that render the SM a highly insufficient theory, and short descriptions of some of the most important ones are provided below.

Gauge Coupling Unification

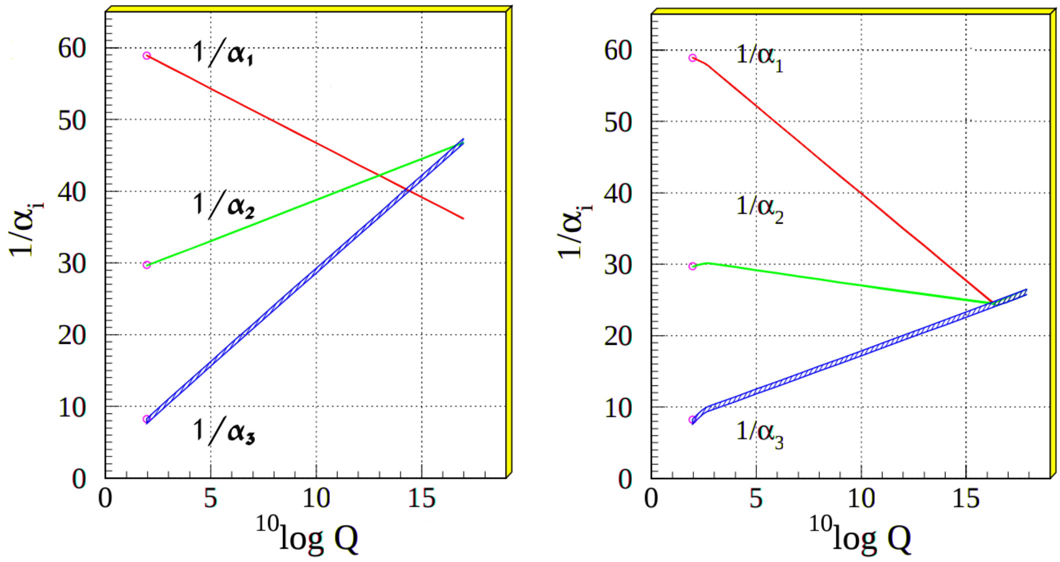
A fortuitous occurrence regarding the foundations of the SM is that each of the gauge interactions is accompanied by a different coupling constant (or coupling strength), with all three being independent and seemingly unrelated. In this case, the crucial idea which arises due to the effect of higher-order quantum corrections in the gauge boson propagators is that of *running coupling constants* - a terminology used to describe the variation of the coupling strength as a function of a typical energy scale $\tilde{\mu}$. A mathematical account of the running of the gauge couplings is provided by the specification of a renormalisable scheme. Variation of the coupling strengths is then solely determined by the particle content and their couplings inside the higher-order loops of the gauge bosons. The value of this variation is expressed by a set of RGEs. The *one-loop* RGE for the SM gauge couplings, as computed in the $\overline{MS}$ are

$$\frac{d\alpha_i}{dt} = \frac{1}{4\pi} b_i \alpha_i^2 , \qquad t = log\left(\frac{\tilde{\mu}}{\tilde{\mu}_0}\right); \quad i = 1, 2, 3 , \tag{2.1.10}$$

where $\alpha_i \equiv \frac{1}{4\pi} g_i^2$ and $\tilde{\mu}_0$ is a very high energy scale which is arbitrarily chosen. For the SM the coefficients, as given in Ref. [99] are

$$b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{22}{3} \\ -11 \end{pmatrix} + N_{FAM} \begin{pmatrix} \frac{4}{3} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix} + N_{HIGGS} \begin{pmatrix} \frac{1}{10} \\ \frac{1}{6} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{41}{10} \\ -\frac{19}{6} \\ -7 \end{pmatrix} , \tag{2.1.11}$$

with $N_{FAM} = 3$ being the number of chiral matter families and $N_{HIGGS} = 1$ being the number of Higgs doublets. The evolution of the inverse of the coupling strengths



(a)Standard Model (b)MSSM $M_{SUSY} \sim 1\,TeV$

Figure 2.2: The two-loop Renormalisation Group (RG) evolution of the three gauge coupling constants g_i with energy scale $Q = \tilde{\mu}$ (GeV) in the SM and MSSM. The evolution is calculated according to the formulas in Ref. [100], p.199 assuming that the masses of the superpartners are in the range of the TeV scale. The red, green and blue lines correspond to the running of the electromagnetic, weak and strong couplings respectively. Shown is the inverse gauge coupling strength α_i which is parametrized as $\alpha_i^{-1} = 4\pi g_i^{-2}$. The line thickness is a representation of the errors in the coupling constant measurements.

as a function of the logarithm of energy is described by the solution to Eq. (2.1.10):

$$\frac{1}{\alpha_i(\tilde{\mu}^2)} = \frac{1}{\alpha_i(\tilde{\mu}_0^2)} - 4\pi b_i ln\left(\frac{\tilde{\mu}^2}{\tilde{\mu}_0^2}\right). \tag{2.1.12}$$

Based on experimental data, a graphical representation of this result was originally presented in the renowned paper of Ref. [99]. A modified form of the original graph is adopted by Ref. [101] and is shown in Fig. 2.2. A significant aspect of the interpretation of the behaviour of each gauge interaction's coupling strength is the self-interaction of gauge fields that yield an antiscreening effect and hence a negative contribution in the overall value of the gauge couplings. On the other hand, the matter multiplets produce a counter effect - screening - which yields a positive contribution. The $SU(3)$ coupling is affected by asymptotic freedom to a much greater extent than the other couplings. This is due to the numerical superiority of gluons which are responsible for an antiscreening effect that outweighs the screening effect from the quarks. Therefore, the strong coupling strength decreases as the energy scale increases. In the case of the $SU(2)$ the antiscreening effect is slightly stronger than screening. The weak coupling strength therefore decreases at higher energy scales but at a much slower rate than the strong coupling. The $U(1)$ group is free from gauge field self-interactions so nothing hinders the screening effects from completely taking over. This leads to an increasing electromagnetic coupling strength at increasing energy scales. Finally, one has to take into account the Higgs fields which are able to influence the running of the weak coupling at one-loop. They are also able to influence the electromagnetic coupling but to a lesser extent. In both cases, the coupling strengths increase with energy.

At one-loop the evolution is a straight line. At two-loops there is no observable deviation from the straight line plots because the effects are relatively small and therefore unable to cause any significant alterations. The directions of the gauge couplings provide an indication that the couplings could possibly meet at a single point. This indication fuels the idea for the existence of a GUT in which all three gauge couplings become equal at a unification point. A good scientific strategy is to check whether such claims have a reasonable ground by using the RGEs to calculate the running of each gauge coupling at very high energy scales. It is found that all three gauge couplings meet at an energy of $\mathcal{O}(10^{16})$ GeV, something that forms strongly suggestive evidence for the existence of a theory at very high energy scales which incorporates greatly enhanced gauge symmetries and allows the unification of gauge couplings.

For a notable change in the logarithmic energy scale of gauge couplings it is necessary to have an enormous change in the energy scale itself. Indeed, the proposed unification scale of $\mathcal{O}(10^{16})$ GeV, is consistent with the large difference required between the SM and the GUT energy scales. Nevertheless, Fig. 2.2a demonstrates that within the SM the unification of gauge couplings is an impracticable dream.

The Hierarchy problem

On gauge theories there is an imposing, order of magnitude, restriction at every energy scale $\tilde{\mu}$ which is a physical parameter or a set of physical parameters $\alpha_i(\tilde{\mu})$. These parameters are allowed to be very small provided that the $\alpha_i(\tilde{\mu}) = 0$ replacement would enhance the symmetry of the theory. This enhancement is what is defined to be the "*naturalness*", a property of the gauge theories. Naturalness arises in the case that a not well comprehended theory with strong interactions yields a mass spectrum with various symmetry properties at energy scales $\tilde{\mu}>\tilde{\mu}_0$. If at the $\tilde{\mu}=\tilde{\mu}_0$ scale some of the parameters in the uncomprehended theory are determined to be several orders of magnitude smaller than that energy scale, then it must be because of some form of a symmetry [102].

The basic framework for naturalness assumes the existence of a fundamental energy scale (λ) which is the real cut-off of the theory. It is generally assumed that the cut-off λ is the Planck scale, $\mathcal{O}(10^{19})$ GeV. The basic parameters of gauge theories with such a cut-off are a set of dimensionless bare couplings (g_0) and masses (m_0). The dimensionless bare masses are renormalised with respect to (w.r.t) the cut-off energy scale as

$$m' = \frac{m_0}{\lambda}\,. \tag{2.1.13}$$

According to the principle of naturalness, the physical properties of a theory at low energies must be stable when the coupling, g_0 and the renormalised masses are subjected to very small variations. Since the 'light' physical mass spectrum of the theory emerges at energies many orders of magnitude smaller than the fundamental scale, there is clearly a large difference between the physical and the fundamental scale. In order to ensure that the quantum effects of the higher energy scale preserve the stability of the theory at low energies the parameters must be adjusted in such a way so as to account for the quantum effects. The most common quantum effect which is in conflict with naturalness occurs when a particle receives a self-energy which is quadratic in λ. If the mass corrections are of the form

$$m^2 = m_0^2 + \delta m = m_0^2 + \lambda^2 g_0^2\,, \tag{2.1.14}$$

then the bare mass parameter from Eq. (2.1.13) must obey

$$m'^2 = \frac{m^2}{\lambda^2} - g_0^2\,. \tag{2.1.15}$$

If m is a physical mass of $\mathcal{O}(10^2)$ GeV and $\lambda \sim \mathcal{O}(10^{19})$ GeV then

$$m'^2 = 10^{-34} - g_0^2\,. \tag{2.1.16}$$

What Eq. (2.1.16) means is that for the naturalness to manifest itself, the renormalisable mass m' must be adjusted in such a way so as to produce a result of the order of $\sim 10^{-34}$. If m' is taken to be instead of the order of $\sim g_0$, then the physical mass will come out to be $m \sim \mathcal{O}(10^{19})$ GeV [103].

Such a scenario which involves unnatural renormalisable mass parameters only occurs in theories with scalar particles, such as the SM. The only fundamental scalar particle in the SM is the Higgs boson and as deduced from $\mathcal{L}_{Higgs}$ in Eq. (A.1.7) of Appendix A.1, the mass-squared $[(m_H)^2]$ is up to a constant coefficient a parameter of $\mathcal{L}^{SM}$. Since the Higgs boson is established as a light scalar at energy scales $\tilde{\mu} \gg m_H$, there arises the question as to whether a symmetry exists that protects its mass when the renormalised Higgs mass parameter is very small, *i.e.* $m_{H,0} \to 0$. This is due to the naturalness criterion which distinctly states that without a symmetry to protect the mass of the Higgs, the light scalar is left exposed to a sensitivity in the UV region of the theory. This can be verified analytically when the Higgs mass parameter is renormalised from a fermionic loop as seen in Fig. 2.3. For the renormalisation the relevant terms in the SM Lagrangian are the last three components of Eq. (2.1.7). Computation of the fermionic loop yields the correction to the Higgs mass,

$$\begin{aligned}(\delta m_H)^2 &\sim -\hat{h}_f^2 \int^{\lambda} d^4p\ \mathrm{Tr}\left[\frac{1}{(\not{p} - m_\Psi)^2}\right] \\ &\sim -\hat{h}_f^2 \lambda^2\,. \end{aligned} \tag{2.1.17}$$

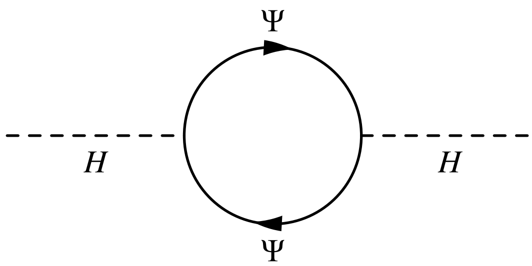


Figure 2.3: Higgs mass renormalisation from a fermion loop.

It is obvious that the Higgs mass diverges quadratically! From a theoretical perspective, the quadratic singularities which are sensitive to the cut-off scale λ are not cancelled, suggesting that the Higgs mass is of the order of the Planck mass. This is not consistent with the experimental data which verify that the Higgs boson is a light scalar with a mass $m_H \approx 125$ GeV - about 15 orders of magnitude smaller than the Planck mass. Due to the nature of this elementary particle, there is currently no experimental verification of a mechanism which could protect its mass by providing a natural way to cancel the quantum corrections. There is, however, compelling evidence that there are other symmetries underlying the fundamental interactions.

The faux pas of theoretical particle physics to explain this huge discrepancy between prediction and the experimental measurement leads to the troublesome result:

$$m^2_{H,\,bare} = m^2_{H,0} + \delta m^2_H + \text{counter term}\,, \tag{2.1.18}$$

where $m_{H,0} = \frac{1}{\lambda} m_H$ is the renormalised mass parameter. The counter term accounts for the additional radiative corrections that the mass of the Higgs boson acquires due to the symmetry (or other mechanism) that protects the m_H from becoming very large. The consequence of adding a counter term is that for the theoretical and experimental values to be agreeable the counter term must be 'fine tuned' so that it will cancel the quadratically divergent contributions to δm^2_H. Moreover, this adjustment must be made at every order in perturbation theory.

In general the Higgs mass parameter is affected by heavy particles, especially

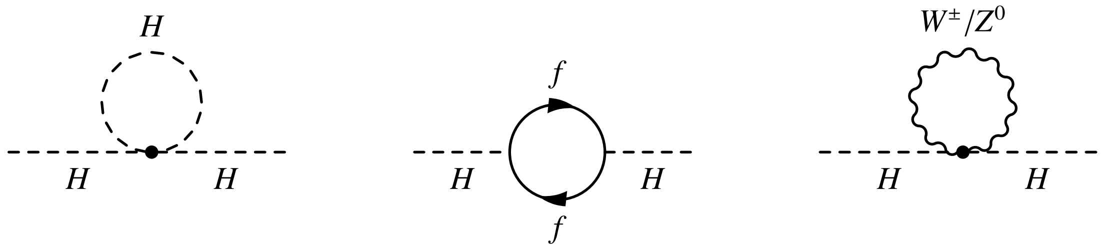


Figure 2.4: The Feynman diagrams for the one-loop quantum divergences to the Higgs boson mass in the SM.

by fermions, such as the top quark which has a mass $m_t = 172.3 \pm 1.3$ GeV [104]. Aside from fermions, the Higgs boson mass is also renormalised from scalar loops. Although the resulting corrections to m_H are quite different, the dominant term still causes the physical value of m_H to diverge quadratically. Therefore the heavy particles that couple to the Higgs induce very large radiative corrections to the Higgs mass parameter which in turn demand a very large fine tuning to justify the small m_H. This problem is the result of introducing a physical significance to the cut-off energy scale λ [105] with the light Higgs scalar being unable to survive in the presence of heavy states at much higher energy scales. This disparity between the theory and experiment is known as the *hierarchy problem* in the SM.

For the avid readers, an explicit calculation of the one-loop quantum divergences to the Higgs boson mass due to heavy scalars and fermions is presented in Appendix A.3 based on the work in Ref. [106]. In the SM, these divergences are induced as depicted in Fig. 2.4.

Gravity

The most significant hole in the SM is the lack of gravitational interactions. Gravity is by far the weakest fundamental force in nature, but unlike the other three fundamental forces that are related to local symmetries, gravity has a global effect and thus is a universal force. Gravity is thought to have its own force-carrier, a spin–2 particle known as the 'graviton', which has not been observed yet. QFT is not consistent with the framework needed to describe the gravitational interactions. The impenetrable barrier to the attempt to incorporate gravity in the SM is the fact that all theories turn out to be non-renormalisable. In a more precise way, any attempt for renormalising gravity results in an infinite scattering cross-section for interactions. This requires a very fine tuning at each order in the interaction so as to yield a finite result [107]. At the microscopic scale, the gravitational effects are almost negligible, in contrast to the macroscopic scale where they dominate.

Dark Matter and Dark Energy

Cosmological observations lead to the staggering conclusion that there is an unknown form of matter which does not interact with the electromagnetic force and hence does not absorb, reflect or emit light. It is thought to be extremely weakly coupled and has only been detected by the gravitational effects that has on the visible matter. Visible matter constitutes only 5% of the universe while the dark matter constitutes the 27%. There are many conjectures about the nature of dark matter, but there is one definite fact: it cannot possibly be a part of the SM! Another mystery of the universe, which has no place in the SM is dark energy which is distributed evenly throughout spacetime and is associated with the vacuum in space. The even distribution of dark energy causes global gravitational effects on the universe resulting in a repulsive force. This force is believed to drive the accelerating expansion of the universe. The existence of dark energy is experimentally confirmed and constitutes 68% of our universe.

Electroweak Vacuum

In the SM the electroweak vacuum is measured to be $v = 246.22$ GeV. However the measured values of m_t and m_H, indicate that the effective potential of the theory is shifted to a more unstable state. This is due to the dominant contribution of the top quark in the renormalisation of the Higgs self-coupling. With these results, the electroweak vacuum is in principle regarded to be unstable in the SM. With more accurate results from the second LHC run the stability of the vacuum may be improved but it is not guaranteed. BSM physics is therefore unavoidable for dealing with this problem.

Neutrino Masses

In the SM, neutrinos are assumed to be massless due to a global chiral lepton number symmetry [108]. This is in stark disagreement with observations of neutrino mixing and oscillations, which can only be possible if neutrinos are massive. Indeed, experimental results have revealed the masses of neutrinos to be small but not negligible. The undeniable fact that neutrinos are light elementary particles raises many questions: why is their mass many orders of magnitude smaller than that of the other chiral matter fields, do they have Dirac or Majorana masses, are baryon and lepton numbers conserved *etc*. A model that demonstrates how neutrinos may carry such a small mass was proposed by [109] and is widely known as the see-saw mechanism. This is a natural way of obtaining small neutrino masses provided that there exist heavy singlet neutrinos ν_R. Such a mechanism cannot be accommodated in the SM framework, leading once again to the labyrinthine paths of BSM physics.

Baryon Asymmetry

The observed lack of antimatter is in direct contrast with the abundance of ordinary matter in the universe. It is unknown exactly why matter dominates over antimatter, especially since the Big Bang should have created an equal amount of each, but the answer for this asymmetry is believed to lie with CP violation [110].

2.2 Supersymmetry

In 1967, a new kind of geometrical symmetry, supersymmetry, was proposed in the paper 'All possible Symmetries of the S-Matrix', by Coleman and Mandula. Therein, they state that there are only certain symmetries of the S-matrix: the C, P, T symmetries and Poincaré invariance, postulating the Coleman-Mandula theorem [111].

Theorem 2.2.1 *In a theory with non-trivial scattering in more than* $1+1$ *dimensions, the only possible conserved quantities that transform as tensors under the Lorentz* *group (i.e. without spinor indices) are the energy-momentum vector P_μ, the generators of Lorentz transformations $J_{\mu\nu}$, as well as possible scalar 'internal' symmetry charges Z_i which commute with P_μ and $J_{\mu\nu}$.*

Hence the most general algebra of the bosonic symmetry commuting generators of the S-matrix is obtained by the direct sum of a Poincaré algebra and a compact Lie algebra which contains internal symmetries. Furthermore, this algebra generates symmetries of the S-matrix consistent with relativistic QFT. Haag, Lopuszański and Sophnius proved that the only additional symmetry of the S-matrix is the unique extension of this general algebra by including fermionic generators with anticommutation relations. This forms the Super-Poincaré algebra [112]. In any number of dimensions, the Super-Poincaré - or supersymmetry - algebra has the form:

$$\begin{aligned}
\{Q_\alpha{}^A, \overline{Q}_{\dot\beta B}\} &= 2\sigma_{\alpha\dot\beta}{}^m P_m \, \delta^A{}_B \\
\{Q_\alpha{}^A, Q_\beta{}^B\} &= \{\overline{Q}_{\dot\alpha A}, \overline{Q}_{\dot\beta B}\} = 0 \\
[P_m, Q_\alpha{}^A] &= [P_m, \overline{Q}_{\dot\alpha A}] = 0 \\
[P_m, P_n] &= 0\,,
\end{aligned} \tag{2.2.1}$$

where the Greek indices $(\alpha, \beta, \ldots, \dot\alpha, \dot\beta, \ldots)$ are in the range $(1, 2)$ and denote two-component Weyl spinors. The Latin indices $(m, n, \ldots)$ are in the range $(1, \ldots, 4)$ and denote Lorentz four-vectors. Finally, the indices $(A, B, \ldots)$ are in the range $(1, \ldots, \mathcal{N} \geq 1)$. The supersymmetric algebra has $\mathcal{N} = 1$, while the *extended* supersymmetric algebras have $\mathcal{N} > 1$.

The anticommutating generators of the SUSY transformations are the Q and $\overline{Q}$ fermionic operators. A SUSY transformation is responsible for generating a bosonic state from a fermionic one and vice versa:

$$Q\,|fermion\rangle = |boson\rangle\,, \qquad Q\,|boson\rangle = |fermion\rangle\,. \tag{2.2.2}$$

Since the SUSY generators carry a spin angular momentum, it is concluded that SUSY is a spacetime symmetry. However, the SUSY operation is applied in a mathematical space which extends the ordinary Minkowski spacetime $\mathbb{R}^4$ by Grassman variables $(\theta^\alpha, \bar{\theta}_{\dot{\alpha}})$,

$$z^A = (x^\mu, \theta^\alpha, \bar{\theta}_{\dot{\alpha}}), \tag{2.2.3}$$

where $(\theta^\alpha)^* = \bar{\theta}^{\dot{\alpha}}$.

The commuting and anticommuting generators transform in the spinor representation of the Lorentz group. This implies that fermions and bosons form different spin states of a single fundamental supersymmetric entity, related by their masses and their couplings. The irreducible representations of the Super-Poincaré algebra must contain several irreducible representations of the Poincaré algebra in such a way that particles are grouped together. These groupings are known as *superfields* or *supermultiplets* and contain particles differing in spin by $\frac{1}{2}$, related by the action of the anticommutating generators. Furthermore, the commutation relations between the SUSY generators and the generators of gauge transformations imply that particles which fall in the same supermultiplet must also fall in the same representation of the gauge group. As a result, particles that are in the same supermultiplet carry the same quantum numbers such as electric charge, weak isospin and colour d.o.f. So for each ordinary fermion there exists a corresponding particle in the supermultiplet which is interpreted as a boson superpartner and similarly for each ordinary boson there is a corresponding fermion superpartner.

Supersymmetry is the only complex mathematical structure extended beyond the SM framework which has been developed for many years. The primary theoretical goal of SUSY is to provide a complete solution for the stability of the electroweak vacuum and the hierarchy problem. Over the last decades, research has unveiled that SUSY has the potential to address many other issues that are beyond the realm of SM. In the context of cosmology, SUSY predicts the existence of supersymmetric particles which form good candidates for being dark matter particles. In the context of particle theory, SUSY theories accommodate the unification of the couplings of the weak, strong and electromagnetic interactions at high energies, reinforcing the theoretical idea that the three interactions originate from a single fundamental theory [99].

In a supersymmetric generalisation of the SM, such as the MSSM discussed presently, the particle content of the theory is enlarged due to the inclusion of the superpartners which are assumed to appear at an energy scale of $\mathcal{O}(1)$ TeV. The superpartners contribute to the screening and antiscreening effects in such a drastic way that the RG evolution of the three gauge couplings is modified and is now based on a new set of coefficients:

$$b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_{FAM} \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + N_{HIGGS} \begin{pmatrix} \frac{3}{10} \\ \frac{1}{2} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{33}{5} \\ 1 \\ -3 \end{pmatrix}, \tag{2.2.4}$$

with $N_{FAM} = 3$ and $N_{HIGGS} = 2$.

The RG evolution of the gauge couplings in the MSSM model is demonstrated in Fig. 2.2b. It is notable that the running of the strong coupling is much weaker than the one demonstrated in Fig. 2.2a. This occurrence is mainly due to the dominating one-loop b_i contributions. Similarly, the running of the weak coupling has opposite direction while the running of the electromagnetic coupling is faster than in the SM case. Finally, there is a contribution from all the Higgs fields and their supersymmetric counterparts. However, their contribution to the running gauge couplings in comparison with the contribution of the fermions and gauge bosons is quantitatively minor. The results of Fig. 2.2b provide a splendid evidence for the likelihood of a GUT with the masses of superpartners at the range of 1 TeV.

To illustrate how SUSY addresses the hierarchy problem it is adequate to consider the simplest supersymmetric Lagrangian with a superfield that contains a scalar field S and a two-component Majorana spinor ζ

$$\mathcal{L} = -\partial_\mu S^* \partial^\mu S - i\bar{\zeta}\bar{\sigma}^\mu \partial_\mu \zeta - \tfrac{1}{2} m(\zeta\zeta + \overline{\zeta\zeta}) - cS\zeta\zeta - c^* S^* \overline{\zeta\zeta} - |mS + cS^2|^2, \tag{2.2.5}$$

where $\sigma^\mu \equiv (\mathbf{1}, -\vec{\sigma})$, $\vec{\sigma}$ are the Pauli matrices, c is an arbitrary coupling constant and m is the mass which is the same for both the scalar and fermion. This Lagrangian is invariant under the SUSY transformations and the scalar self-interactions

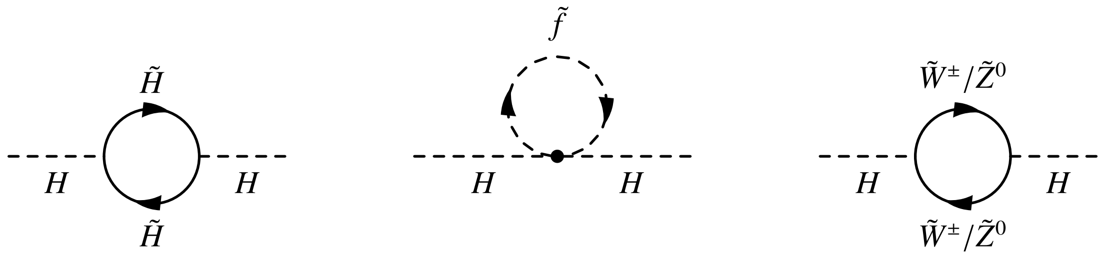


Figure 2.5: The Feynman diagrams for the one-loop quantum divergences to the Higgs boson mass due to the supersymmetric partner particles. The contribution to the Higgs boson mass from these diagrams exactly cancels the contribution induced from the diagrams of Fig. 2.4, thus solving the hierarchy problem.

yield a positive definite potential,

$$V(S, S^*) = |mS + cS^2|^2 \geq 0\,. \tag{2.2.6}$$

This is a general feature of all theories in which SUSY is an *exact symmetry*. This means that the strengths of the interactions between the proposed superpartners is expected to be identical to the strengths of the interactions between the various ordinary partners. As a result, the theory has its minimum at $\langle V \rangle = 0$. It is straightforward to deduce from Eq. (2.2.5) that both the boson and fermion interactions have the same couplings. So SUSY relates particles that differ by a spin-$\frac{1}{2}$ but have identical quantum numbers and masses. Revisiting Eqs. (A.3.2) and (A.3.3) of Appendix A.3, it is obvious that the quadratic divergences from a fermion loop cancel against those from a scalar loop, provided that the scalar and fermion interactions have the same coupling. This is precisely what occurs in supersymmetric theories. Therefore, theories with spacetime SUSY are automatically free from the quadratic divergences and it is inferred that if there existed supersymmetric partners for the SM particles then the one-loop corrections to the Higgs boson mass would be attainable as depicted in Fig. 2.5.

However, it has been observed that there is a difference between the masses of ordinary bosons and fermions. A striking experimental observation is that there are no candidate supersymmetric fermion partners with the same mass and quantum numbers as the scalar particles in the SM, and vice versa. Therefore, if supersymmetry exists it must be a *broken* symmetry.

2.2.1 The Minimal Supersymmetric Standard Model

The supersymmetric extension of the SM that respects the same $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge symmetries is known as the MSSM. The MSSM is constructed by introducing a chiral superfield for every fermion (χSF) and a vector superfield (VSF) for every gauge boson of the SM. Each postulated superfield contains the observed SM particle along with the corresponding superpartner. Note that the MSSM is constructed as an $\mathcal{N} = 1$ theory, which means that the superpartners of the fermions are necessarily scalar particles whereas the superpartners of the gauge and Higgs boson(s) are spin-$\frac{1}{2}$ fermions. The superfields $\hat{l}$ and $\hat{Q}$ thus consists of an $SU(2)_L$ doublet of left-handed leptons and quarks (as presented in Eq. (2.1.5)) along with their scalar partners, the left-handed sleptons and squarks which are also in $SU(2)_L$ doublets,

$$\tilde{l}_{L\hat{c}} \equiv \begin{pmatrix} \tilde{\nu}_{e\hat{c}} \\ \tilde{e}_{\hat{c}} \end{pmatrix}_L \qquad \text{and} \qquad \tilde{Q}^{\hat{a}}_{L\hat{c}} \equiv \begin{pmatrix} \tilde{u}^{\hat{a}}_{\hat{c}} \\ \tilde{d}^{\hat{a}}_{\hat{c}} \end{pmatrix}_L . \tag{2.2.7a}$$

Similarly, the superfields $\hat{E}$, $\hat{U}$ and $\hat{D}$ contain the right-handed, gauge singlets leptons and quarks along with their scalar partners, the right-handed sleptons and squarks,

$$\tilde{e}_{R\hat{c}}; \quad \tilde{u}_{R\hat{c}}; \quad \tilde{d}_{R\hat{c}} . \tag{2.2.7b}$$

In the MSSM all the gauge bosons acquire a Majorana fermion partner, the gaugino. In consistency with this particle partnering, the superfield $\hat{B}$ contains the $U(1)_Y$ gauge field (B^μ) and its fermionic counterpart ($\tilde{b}$ - bino). Similarly, the superfields $\hat{G}^A$ contain the gluon fields ($G^{A\mu}$) and gluinos ($\tilde{g}^A$) whereas the superfields $\hat{W}^a$ contain the $SU(2)_L$ gauge fields ($W^{a\mu}$) and their fermionic counterparts ($\tilde{\omega}^a$ - winos). The particle content of the MSSM is summarised in Tables 2.1 and 2.2.

One important feature of the MSSM is the existence of *two* Higgs doublet fields (H_u, H_d) and the corresponding superpartners - the Higgsinos ($\tilde{H}_u, \tilde{H}_d$) which are

Superfield	Spin−0	Spin−$\frac{1}{2}$	$SU(3)$	$SU(2)_L$	$U(1)_Y$
$\hat{Q}$	$(\tilde{u}_{L\hat{c}}, \tilde{d}_{L\hat{c}})$	$(u_{L\hat{c}}, d_{L\hat{c}})$	3	2	$\frac{1}{6}$
$\hat{U}$	$\tilde{u}_{R\hat{c}}$	$u_{R\hat{c}}$	$\overline{3}$	1	$-\frac{2}{3}$
$\hat{D}$	$\tilde{d}_{R\hat{c}}$	$d_{R\hat{c}}$	$\overline{3}$	1	$\frac{1}{3}$
$\hat{l}$	$(\tilde{e}_{L\hat{c}}, \tilde{\nu}_{L\hat{c}})$	$(e_{L\hat{c}}, \nu_{L\hat{c}})$	1	2	$-\frac{1}{2}$
$\hat{E}$	$\tilde{e}_{R\hat{c}}$	$e_{R\hat{c}}$	1	1	1
$\hat{H}_u$	(h_u^+, h_u^0)	$(\tilde{h}_u^+, \tilde{h}_u^0)$	1	2	$\frac{1}{2}$
$\hat{H}_d$	(h_d^0, h_d^-)	$(\tilde{h}_d^0, \tilde{h}_d^-)$	1	2	$-\frac{1}{2}$

Table 2.1: Chiral superfields of the MSSM with their particle content and representations under the SM gauge group. There are three copies of the quark and lepton superfields, one for each chiral generation. The superscripts $\pm, 0$ indicate the Q_{em} of the Higgs and Higgsino fields.

$SU(2)_L$ doublets of Majorana fermion fields, as it is shown in Table 2.1. The Higgsino superpartner of the SM Higgs doublet contributes to the triangle of $SU(2)_L$ and $U(1)_Y$ gauge anomalies. Although the SM fermions have the correct quantum numbers to guarantee the cancellation of these anomalies, the contribution from the Higgsino remains uncancelled. Since the MSSM is a gauge theory and hence cannot have anomalies, these contributions must also cancel. This is achieved by adding a second Higgs doublet field with the opposite $U(1)_Y$ quantum numbers to the SM Higgs doublet field. The fermionic partner of the second Higgs doublet will contribute to the gauge anomalies and will precisely cancel the contribution from the first Higgsino, leaving the theory anomaly free [113]. The presence of two Higgs doublets is also necessary for giving masses to both the up-type and down-type quarks. Specifically, once the electroweak symmetry is spontaneously broken the neutral component of the H_d acquires a VEV $\langle h_d^0 \rangle = v_d$, giving mass to the down-type quarks, while the neutral component of the H_u acquires a VEV $\langle h_u^0 \rangle = v_u$, giving mass to the up-type quarks. A detailed description of the MSSM Higgs sector is provided in Appendix B.

Due to the extended Higgs sector in the MSSM, the phenomenology of Higgs scalar particles is quite diverse. At this point it is worth noting that in MSSM theories the particles that contribute to the cancellation of quadratic divergences in the

Superfield	Spin$-\frac{1}{2}$	Spin-1	$SU(3)$	$SU(2)_L$	$U(1)_Y$
$\hat{G}^A$	$\tilde{g}^A$	$G^{A\mu}$	8	1	0
$\hat{W}^a$	$\tilde{\omega}^a$	$W^{a\mu}$	1	3	0
$\hat{B}$	$\tilde{b}$	B^μ	1	1	0

Table 2.2: Vector superfields of the MSSM with their particle content and representations under the SM gauge group.

Higgs boson mass renormalisation are the SM particles along with the gauginos, Higgsinos and sfermions. Revisiting Eqs. (A.3.2)-(A.3.4) of Appendix A.3, if one identifies the scalar and fermion couplings as equal, *i.e.* $g_F^2 = g_S$, as SUSY requires, and $m_1 = m_2$ then the total contribution to the Higgs boson mass renormalisation becomes

$$(\delta m_H^2)_{total} = \frac{g_S}{4\pi^2}\left[m_f^2\, ln\left(\frac{\lambda}{m_f}\right) - m_1^2\, ln\left(\frac{\lambda}{m_1}\right)\right] + O\left(\frac{1}{\lambda^2}\right). \tag{2.2.8}$$

If the difference between the fermion and the scalar masses is relatively small then the overall cancellation has a very small net value, free from quadratic divergences. From this perspective, the theory obeys the 'naturalness' requirements. Because of this, the theory supports the requirement that supersymmetric particles must have masses below the 1 TeV energy scale if SUSY is to be the theory that addresses the hierarchy problem.

Having introduced all the ingredients of the MSSM, it is now possible to write a supersymmetric renormalisable Lagrangian for chiral superfields $\hat{\Phi}_i$ and vector superfields $\hat{V}^A$:

$$\mathcal{L}^{SUSY} = \mathcal{L}^{SUSY}_{Kinetic\ Energy} + \mathcal{L}^{SUSY}_{interactions} + \mathcal{L}^{SUSY}_{superpotential} \tag{2.2.9}$$

The component fields of $\hat{\Phi}_i$ are the SM chiral fermion fields ψ_i and their SUSY partners φ_i, as listed in Table 2.1. The component fields of $\hat{V}^A$ are the SM gauge fields and their SUSY partners λ_A, as listed in Table 2.2. The explicit form of Eq. (2.2.9) is then given by:

$$\mathcal{L}^{SUSY}_{Kinetic\ Energy} = \sum_i \Big\{ (D_\mu \varphi)^\dagger_i (D^\mu \varphi)_i + \tfrac{1}{2} i\, \psi_i \sigma^\mu (D_\mu \overline{\psi})_i - \tfrac{1}{2} i\, (D_\mu \psi)_i\, \sigma^\mu \overline{\psi}_i \Big\}$$
$$+ \sum_A \Big\{ -\tfrac{1}{4} F^A_{\mu\nu} F^{\mu\nu\, A} + \tfrac{1}{2} i\, \lambda^A \sigma^\mu (D_\mu \overline{\lambda}^A) - \tfrac{1}{2} i\, (D_\mu \lambda^A)\, \sigma^\mu \overline{\lambda}^A \Big\} \quad (2.2.10a)$$
$$\mathcal{L}^{SUSY}_{interactions} = -\sqrt{2} i \sum_{ij,A} g \overline{\psi}_i \overline{\lambda}^A T^A_{ij}\, \varphi_j + \sqrt{2} i \sum_{ij,A} g \varphi^\dagger_i T^A_{ij}\, \psi_j\, \lambda^A$$
$$- \tfrac{1}{2} \sum_A (g \varphi^\dagger_i T^A_{ij}\, \varphi_j + k^A)^2 \quad (2.2.10b)$$
$$\mathcal{L}^{SUSY}_{superpotential} = \mathcal{L}_W = - \sum_i \left| \frac{\partial W}{\partial \varphi_i} \right|^2 - \tfrac{1}{2} \sum_{ij} \left\{ \frac{\partial^2 W}{\partial \varphi_i \varphi_j} \psi_i \psi_j - \frac{\partial^2 W^\dagger}{\partial \varphi^\dagger_i \partial \varphi^\dagger_j} \overline{\psi}_i \overline{\psi}_j \right\}. \quad (2.2.10c)$$

The covariant derivative D is the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge invariant derivative; for the fermion fields it is defined in Eqs. (A.1.3)-(A.1.5) of Appendix A.1. The strengths of interactions are determined by the gauge couplings and g stands for the relevant gauge coupling. The last term of Eq. (2.2.10b), $g\varphi^\dagger_i T^A_{ij} \varphi_j + k^A = D^A$ is defined as the D-term while the first term of Eq. (2.2.10c), $\frac{\partial W}{\partial \varphi_i} = F_i$ is defined as the F-term. Both terms play a crucial role on the breaking of SUSY. Due to the lack of a proper kinetic term for each of the fields corresponding to the D- and F-terms in the supersymmetric Lagrangian, both fields are *auxiliary*. However, the sum of D- and F-terms forms the positive definite potential of the theory,

$$V(\varphi_i, \varphi^\dagger_j) = \sum_i \left| \frac{\partial W}{\partial \varphi_i} \right|^2 + \tfrac{1}{2} \sum_A (g \varphi^\dagger_i T^A_{ij} \varphi_j + k^A)^2 . \quad (2.2.11)$$

There is no freedom in constructing the $\mathcal{L}^{SUSY}_{Kinetic\ Energy}$ and $\mathcal{L}^{SUSY}_{interactions}$ since all the parameters they carry are non adjustable. However, there is freedom in constructing the superpotential, W which is an analytic function of the chiral superfields of Table 2.1 *only*. The W contains terms up to three chiral superfields (four or more result in non-renormalisable interactions). It is not allowed to contain the complex conjugates of the superfields neither any derivative interactions. The most general gauge invariant MSSM superpotential is

$$W_{MSSM} = \epsilon_{ij} \Big[y_e\, \hat{l}^i \hat{H}^j_d \hat{E} + y_d\, \hat{Q}^i \hat{H}^j_d \hat{D} + y_u\, \hat{Q}^j \hat{H}^i_u \hat{U} \Big] - \epsilon_{ij}\, \mu \hat{H}^i_d \hat{H}^j_u , \quad (2.2.12)$$

with the two-dimensional Levi-Civita symbol: $\epsilon_{12} = -\epsilon_{21} = 1$, $\epsilon_{11} = \epsilon_{22} = 0$.

The term $\mu \hat{H}_d^i \hat{H}_u^j$, often called the μ-term, gives mass terms for the Higgs bosons once the electroweak symmetry is spontaneously broken. The mass is determined from the first term of the $\mathcal{L}_W$ and μ is considered to be the Higgs mass parameter. The coefficients y_e, y_d, y_u are the supersymmetric equivalents of Yukawa couplings since their corresponding terms produce the usual Yukawa interactions from the remaining terms of the $\mathcal{L}_W$ in Eq. (2.2.10c). Due to the fact that these coefficients are determined by the fermion masses and the VEVs acquired by the neutral components of the scalar Higgs doublets, they are not free parameters of the theory.

Apart from the terms in Eq. (2.2.12), there are more gauge invariant contributions in the superpotential,

$$W_{\not R} = \epsilon_{ij} \left[\frac{1}{2} y\, \hat{l}^i \hat{l}^j \hat{E} + y'\, \hat{l}^i \hat{Q}^j \hat{D} + \mu'\, \hat{l}^j \hat{H}_u^i \right] + y'' \hat{U} \hat{D} \hat{D} \,. \tag{2.2.13}$$

However, these terms contribute to a lepton number violation (the first three) and baryon number violation (the last one). Consequentially, this leads to proton decay, mediated at tree-level through the exchange of down-type squark [114]. This result is avoided in the SM due to gauge symmetries, hence the lepton and baryon number conservation is accidental. To strategically avoid the lepton and baryon number violating terms in MSSM theories, an additional symmetry is required, R-parity which forbids any violating contributions *ad hoc*. It is a discrete $\mathbb{Z}_2$ symmetry - a multiplicative quantum number through which the SM particles are clearly distinguished from their supersymmetric partners. It is defined as

$$R = (-1)^{3B+L+2S} = \begin{cases} +1 & \text{SM particles} \\ -1 & \text{SUSY particles} \end{cases} \tag{2.2.14}$$

where B, L and S stand for the baryon, lepton and spin quantum numbers of the particles respectively. Assuming that R-parity is conserved, its multiplicative nature requires an even number of supersymmetric particles produced in any interaction. Hence all SUSY particles are expected to be produced in pairs with their SM partners. Another consequence of this assumption is that a supersymmetric particle will undergo a chain decay until it decays to a final state producing the Lightest Supersymmetric Particle (LSP). The LSP is expected to be stable and neutral. Hence, as in the case of neutrinos, the LSP is expected to interact with ordinary matter via the weak force and is not expected to be observed by the experimental detectors.

2.2.2 Supersymmetry Breaking

Although it is well known that SUSY is a broken symmetry, the mechanism underlying its breaking is not well understood. It is, however, well-known that if the SUSY breaking scale is at the order of the GUT scale or the string scale then the minimal theory that emerges as the low energy limit EFT is the SM.

Revisiting the Lagrangian of the unbroken MSSM, in Eq. (2.2.9), it is easy to deduce that a breaking of SUSY will lift the masses of all the currently massless particles, resulting in the SM particles and their corresponding superpartners having an equal mass. This is in direct contrast with the mechanisms of nature that are inclined to favour non-equal but larger masses for the superpartners of the SM particles. Following the breaking of the electroweak symmetry, the most convenient way to break SUSY is *spontaneously*, by giving some scalar fields VEVs. The spontaneous breaking causes the ground state $|0\rangle$ to be no longer invariant under SUSY, *i.e.* $Q_\alpha|0\rangle \neq 0$.

Global SUSY breaking

In globally supersymmetric theories, the Hamiltonian operator is defined in terms of the generators of the theory according to the algebra in Eq. (2.2.1):

$$\mathcal{H} = P^0 = \frac{1}{4}(Q_1Q_1^\dagger + Q_1^\dagger Q_1 + Q_2Q_2^\dagger + Q_2^\dagger Q_2)\,. \tag{2.2.15}$$

A significant property of globally supersymmetric theories, as manifest in Eq. (2.2.15), is that the Hamiltonian operator is bounded from below such that for any state $|s\rangle$ the result is $\langle s|\mathcal{H}|s\rangle \geq 0$. In the unlike scenario of unbroken SUSY, the vacuum state has 0 energy and obeys $\mathcal{H}|0\rangle = 0$. Conversely, in the occurrence of SSSB the vacuum energy is shifted and therefore implies that $\langle 0|\mathcal{H}|0\rangle > 0$. The new vacuum is strictly restricted to have a positive energy

$$\langle 0|\mathcal{H}|0\rangle = \frac{1}{4}(||Q_1^\dagger|0\rangle||^2 + ||Q_1|0\rangle||^2 + ||Q_2^\dagger|0\rangle||^2 + ||Q_2|0\rangle||^2) > 0\,, \tag{2.2.16}$$

as a direct consequence of the requirement that the Hilbert space of the theory must have a positive norm. This can only be true iff the minimum of the scalar potential is also taken to be strictly positive, $V_{min} > 0$. Since Eq. (2.2.11) is the sum of the D- and F-terms, then for the above requirement to be satisfied it is necessary that either $V_{F\,min} > 0$ or $V_{D\,min} > 0$ or some combination of both must be positive.

In the event that the condition $V_{F\,min} > 0$ is met then an F-term breaking occurs of which the canonical example is the O' Raifeartaigh model [115]. Although this procedure requires that at least one scalar field φ gets a VEV, it is not phenomenologically successful as some of the scalar superpartners of the chiral matter fields end up having a lower mass contrary to the expectation of a higher mass. In general, the phenomenological results of this procedure, at tree level only, are summarised by

$$\mathrm{Str}\ \ (M^2) \equiv \sum(-1)^S(2S+1)\,m_S^2 = 0\,, \tag{2.2.17}$$

where m_S is the mass associated with the real component of a spin-S field and the *supertrace* is a sum over all component fields. Even though some superpartners are lighter then the SM particles some others superpartners are heavier so the net result in the average mass is as implied by Eq. (2.2.17). Loop corrections result in a shift in the masses of the superpartners, violating the tree-level result while at the same time opening the possibility of SSSB mediation by loop effects from a hidden sector of a greater theory into the visible sector, the MSSM sector.

On the other hand, if the condition $V_{D\,min} > 0$ is met then a D-term breaking occurs. Contrary to the F-term breaking, the scalar field φ is not strictly required to get a VEV. This is due to the Fayet-Iliopoulos term k^A, which is present only for $U(1)$ gauge fields and means that the potential is always positive. The first scenario is that of the scalar field acquiring a non-vanishing VEV. The general consequence of this scenario is that instead of achieving a SSSB there is a spontaneous breaking of a gauge symmetry resulting in the SM particles and their superpartners getting the same mass [116]. The second scenario is a prevention of the scalar fields from acquiring a VEV by giving them large masses through $U(1)$ gauge invariant superpotential terms. The MSSM has only the $U(1)_Y$ gauge group and as it turns out, the superpotential in Eq. (2.2.12) does not contain any $U(1)_Y$ gauge invariant terms that can induce large masses in the scalar fields. Hence the D-term breaking in the MSSM is also phenomenologically unsuccessful unless it occurs in a hidden sector with a different $U(1)$ group and then mediated to the MSSM sector.

The MSSM is a theory at the electroweak scale thus is considered as an effective low energy theory which cannot accommodate with phenomenological viability the spontaneous breaking of SUSY. It is, therefore, expected that a complete theory exists which also includes gravity, with its unknown or unobserved sector introduced as a *hidden sector*. The breaking of SUSY is assumed to occur spontaneously in a *hidden sector* by some other fields at high energy scales. For a detailed discussion of the hidden sector see Ref. [101]. The breaking is then mediated to the MSSM - the *visible sector* of the theory via messenger fields. There are many speculations on how the breaking is mediated to the visible sector. Among these the most popular are the GMSB [117] and mSUGRA [118]. It is beyond the scope of this thesis to discuss these in depth but more details are available in the relevant references.

Due to the ambiguity in characterising the hidden sector, the terms implementing the SUSY breaking are inserted by hand. These terms are basically mass terms for the SM particles' superpartners, *i.e.* the scalar components of the chiral multiplets and the gauginos of the vector multiplets in the supersymmetric Lagrangian. The mass terms are defined to be 'soft' as they preserve the cancellation of the quadratic divergences. The terms allowed in the soft breaking Lagrangian are determined by dimensional analysis. The mass dimension of the correction to the Higgs mass-squared is $[\delta m_H^2] = 2$ and the couplings g_F and g_S in Eqs. (A.3.2)-(A.3.4) are dimensionless. To prevent terms of the form $g_{S,F}\lambda^2$ appearing in $[\delta m_H^2] = 2$, thus hindering the established cancellation of quadratic divergences, the operators in the soft breaking Lagrangian must have dimension three or less, whereas their corresponding couplings must have dimension one or more respectively. This means that the only possible soft operators allowed are mass terms for the scalars and gauginos as well as bilinear and trilinear scalar mixing terms, provided that the gauge invariance remains intact. So the general form of soft SUSY breaking terms which respect the MSSM gauge symmetry and R-parity is given by the Lagrangian [114]:

$$\begin{aligned}\mathcal{L}_{soft}^{MSSM} = &- m_{H_u}^2 H_u^\dagger H_u - m_{H_d}^2 H_d^\dagger H_d - B\,(H_u H_d + h.c) - \tfrac{1}{2}(M_1\,\overline{\tilde{b}}\tilde{b} + M_2\,\overline{\tilde{\omega}^a}\tilde{\omega}^a + M_3\,\overline{\tilde{g}}\tilde{g} + h.c) \\ &- \mathbf{M}_{\tilde{Q}}^2\,(\tilde{u}_{L\hat{c}}^\dagger \tilde{u}_{L\hat{c}} + \tilde{d}_{L\hat{c}}^\dagger \tilde{d}_{L\hat{c}}) - \mathbf{M}_{\tilde{l}}^2\,(\tilde{e}_{L\hat{c}}^\dagger \tilde{e}_{L\hat{c}} + \tilde{\nu}_{L\hat{c}}^\dagger \tilde{\nu}_{L\hat{c}}) - \mathbf{M}_{\tilde{u}}^2\,\tilde{u}_{R\hat{c}}^\dagger \tilde{u}_{R\hat{c}} - \mathbf{M}_{\tilde{d}}^2\,\tilde{d}_{R\hat{c}}^\dagger \tilde{d}_{R\hat{c}} - \mathbf{M}_{\tilde{e}}^2\,\tilde{e}_{R\hat{c}}^\dagger \tilde{e}_{R\hat{c}} \\ &- (\mathbf{A}_u\,\tilde{Q}_{L\hat{c}}\,H_u\,\tilde{u}_{R\hat{c}}^\dagger - \mathbf{A}_d\,\tilde{Q}_{L\hat{c}}\,H_d\,\tilde{d}_{R\hat{c}}^\dagger - \mathbf{A}_e\,\tilde{l}_{L\hat{c}}\,H_d\,\tilde{e}_{R\hat{c}}^\dagger + h.c)\,. \end{aligned} \tag{2.2.18}$$

In Eq. (2.2.18), each of the $\mathbf{M}_{\tilde{Q}}$, $\mathbf{M}_{\tilde{l}}$, $\mathbf{M}_{\tilde{u}}$, $\mathbf{M}_{\tilde{d},\,\mathbf{M}_{\tilde{e}}}$ is a 3×3 matrix in family space with complex, hermitian entries. Similarly, each of the $\mathbf{A}_u$, $\mathbf{A}_d$, $\mathbf{A}_e$ is a 3×3 matrix in family space that has a mass dimension $[m]$. The MSSM introduces 105 new parameters in addition to the 19 that exist in the SM, all of which are analysed in Ref. [119]. A positive of this is the knowledge that a measurement of these parameters might actually reveal the particulars needed to infer the theory behind SUSY breaking. On the negative side, however lies the difficulty in fully examining this huge parameter space. It might be indeed a challenge to determine so many parameters but it is certainly not an impossibility thanks to some assumptions that dramatically restrict the parameter space. At a GUT scale:

- the gauginos have three real mass parameters which are assumed to be exactly equal,
- the sfermion masses and trilinear couplings are diagonal, real and universal for all three generations, and
- the Higgs mass parameters are real.

These assumptions are well explained in Ref. [119] and are derived by theoretical arguments that have almost no phenomenological consequences regarding the MSSM Higgs sector. As a result, the parameter space is reduced down to just *five*! It is worth emphasising that this constrained form of the MSSM requires only the GUT scale in contrast to the mSUGRA models of Refs. [118] that require a Planck scale.

As per the discussion in Section 2.1.1, regardless of the mechanisms that govern the breaking of global SUSY, the spontaneous breaking implies the existence of a massless Goldstone mode which shares the same quantum numbers as the broken generator of the symmetry. In supersymmetric theories the broken generators are the fermionic charges Q_α so the resulting massless Goldstone particles will be neutral Weyl fermions; the so-called *Goldstinos*. Following the SSSB, the Goldstinos are in fact the fermionic components of the supermultiplets whose auxiliary fields D^A or F_i acquire a VEV. There is an effective Lagrangian associated with the Goldstino field [114], from which one is able to derive the Goldstino interactions with other boson-fermion pairs in a supermultiplet [120,121]. The effective Lagrangian applies for every theory with global SUSY irrespective of how the breaking occurs.

Local SUSY breaking

A topic of great interest is the construction of supersymmetric GUTs which calls for the inclusion of gravity in the SUSY framework. On this ground, SUSY must be promoted to a local symmetry which means that the SUSY fields depend on a spacetime parameter. The locally supersymmetric EFT that stems from this realisation is the vaunted theory of *supergravity* [10]. In Supergravity (SUGRA) theories, the symmetries of spacetime are unified with Einstein's general relativity in a set of equations that come under the guise of local SUSY transformations. The superpartner of the spin-2 graviton is the spin$-\frac{3}{2}$ *gravitino*, which carries both a spinor index α and a spacetime index μ and is therefore symbolised as ψ_μ^α. Following the definition of Eq. (2.2.14), the gravitino has an odd R-parity. At the four-dimensional level of an unbroken local SUSY, both the graviton and gravitino are massless and each has two d.o.f which are spin helicity states. The spontaneous breaking of SUSY changes this picture quite drastically: The Goldstino associated with the broken fermionic generator of SUSY supplies its two d.o.f to the gravitino. These absorbed d.o.f consequently become two extra helicity states $\left(\pm\frac{1}{2}\right)$ that constitute the longitudinal component of the gravitino. As a result, the gravitino now becomes massive with a total of four helicity states. This effect is the supersymmetric analogy to the ordinary Higgs effect so it doesn't come as a surprise that scientists define it as the *super-Higgs* mechanism [122]. All the while, the graviton remains completely unaffected by the super-Higgs effects.

In general, the gravitino mass is estimated as

$$m_{\frac{3}{2}} \sim \frac{1}{\lambda}, \tag{2.2.19}$$

where λ is the SUSY breaking scale. One look at the result of Eq. (2.2.19) would naively trick someone into believing that an estimation for the mass of gravitino is straightforward. The reality is far more complex; the gravitino mass is a subject of theoretical dispute. Revisiting the brief discussion on SSSB mediation, it is worth emphasising that different methods of mediation postulate different high energy scales which in turn yield different values for the gravitino mass. The mSUGRA models, which postulate a Planck scale, predict a gravitino mass of at least $\mathcal{O}(100)$ GeV. This value is comparable to the masses predicted for the MSSM particles. The GMSB models postulate a much smaller energy scale than the Planck scale which in turn predict a gravitino mass much smaller than that of the MSSM sparticles. This leaves no room for doubt that in the GMSB models, the gravitino is considered to be the LSP.

Chapter 3

A glimpse into the world of string theory

Gravity must be caused by an Agent acting constantly according to certain laws, but whether this Agent be material or immaterial I have left to the consideration of my readers.

Isaac Newton

3.1 A quantum theory of gravity?

As alluded to Section 2.1.2 of Chapter 2, there is an elephant in the room when one attempts to unify *all* fundamental interactions in nature into a single consistent mathematical framework; this is of course the gravitational force. A century ago Albert Einstein put forward the theory of general relativity, a revolutionary theory that irrevocably transformed the Newtonian definition of gravity, taking it to an entirely different level. One consequence of gravity's redefinition has been the prediction of gravitational waves whose detection was confirmed by the LIGO instruments on the 14th of September, 2015. Indeed, this is an incontrovertible evidence of general's relativity validity as the true theory of gravity on a classical level and makes the issue of understanding gravity on a quantum level all the more pressing. The word 'understand' is used in the sense that one needs to know what Lagrangian interprets the interaction of the gravitational force with matter and how could provide quantitative results for making predictions.

The major stumbling block when one attempts to include gravity within the framework of quantum gauge theories is the phenomenal incompatibility between quantum mechanics and general relativity. The former describes remarkably well the world at microscopic scales whereas the latter is the *par excellence* theory that describes the world at macroscopic scales. In principle, it is understandable how quantum mechanical effects could be included into gravity as long as the energy of interactions is below the Planck scale. The clash between the two becomes prominent at energies of the order of Planck scale where the perturbative non-renormalisability of gravitational interactions becomes manifest. This contrasts with the established *renormalisability of the gauge interactions* for which it is known how to include quantum effects at *all* energy scales. Given this background, the main differences between the gravitational and renormalisable gauge forces are: The graviton ($g_{\mu\nu}$) is a spin-2 particle whereas all the existent gauge bosons are spin-1; the gravitational coupling constant G_N has a negative mass dimension, $[G_N] = M_{Pl}^{-2}$ whereas the gauge couplings are dimensionless. This is a very important point since it turns out to be a courtesy of the gravitational coupling's dimensionality the fact that gravity is perturbatively non-renormalisable. This means that in the regime of Planck scale, the gravitational interactions, *i.e.* the scattering processes involving gravitons, become so large that they give rise to UV infinities. The associated Feynman diagram is depicted in Fig. 3.1. These infinities cannot be absorbed by a finite number of parameters (local counter terms such as masses and couplings) as required by the processes of renormalisation. It is instead required an infinite number of parameters to absorb the UV divergences which in turn demand measurement, thus rendering the general relativity unquantifiable at energies comparable with the Planck scale. Therefore, it is precisely in this regime that the need for a quantum theory of gravity becomes mandatory.

As was mentioned, there is some degree of understanding how to include quantum effects with gravity but the failure to formulate a complete quantum theory

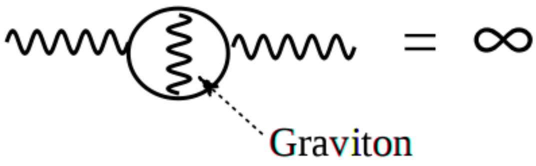


Figure 3.1: The gravitational force is mediated by gravitons whose scattering processes, like the self-interaction presented in this figure, lead to UV divergences at energies of the Planck scale's order. As a result, the quantisation of gravity at these energies is plagued by non-renormalisability. The figure is adapted from a review on "Recent developments in String Theory", extended version of Ref. [123].

of gravity gives strong indications that there is a deeply rooted inconsistency between the two. Nature has emitted unequivocal signals that in order to bypass the mathematical complexities of gravity, a theory with new methods and concepts is required. The most significant feature of this theory must be the definition of gravity as a *consistent quantum theory*. Conceptually, a promising framework for this theory is one based on perturbative expansions, with the emerging theory being *string theory* [124]. Originally, string theory was descended from the "dual resonance models" of hadrons in the pre-QCD era. However, the theory has so many alluring features that over the years it has been established as the most privileged contender for being a consistent *perturbative* theory of quantum gravity.

The issue of non-renormalisability is tackled within the formalism of perturbative string theory, in which there is a set of rules that govern the computation of on-shell amplitudes in on-shell background. In this context, as the point particles are replaced by strings, the individual lines of Feynman diagrams are expanded into strips and the worldlines are replaced by worldsheets of strings. Consequently, the infinitely many gravitational interaction vertices of a Feynman diagram are replaced by a finite number of basic three-point interactions, as depicted in Fig. 3.2. This modification guarantees the UV finiteness of string amplitudes [125]. In addition to the quantum corrections imposed on classical physics, there are now stringy corrections which are related to the finite size of the strings. This size is essentially the string length l_s which takes over the role of the fundamental scale. Except from having finite integrals in the computation of UV divergences there are other benefits that stem from making use of finite objects. Firstly, the string length

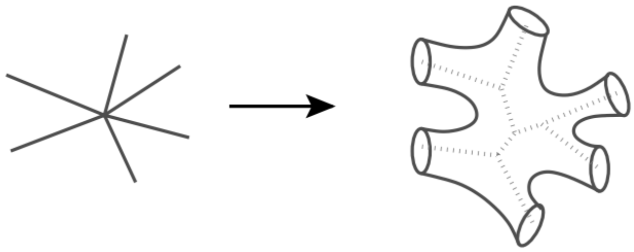


Figure 3.2: An interaction vertex in field theory is replaced by a strip or tube of three-point interactions. The figure is adapted from Ref. [125].

becomes the sole unknown parameter in the Lagrangian of the theory hence there is no need to introduce new parameters to absorb the divergences. This could conceptually give a great predictive power to the theory since this is the only parameter that needs to be measured and set by the experiment. Secondly, the string is the only fundamental object and has vibrational excitation modes which correspond to different elementary particles. Since it necessarily includes a massless spin-2 particle, gravity is automatically included in the picture *in toto*. Furthermore, there are certain symmetry transformations, known as *dualities*, that allow the mutual exchange of quantum and stringy corrections.

The rich structure of string theory induced developments in a range of topics such as non-perturbative dualities, gauge theories at strong couplings, algebraic geometry, entropy of black holes, holography and AdS/CFT correspondence and the theory of branes. However, it is beyond the bounds of possibility to access the energy scale of the gravitational coupling with the current technological means available. Even though in theory it is possible to detect the point-like oscillations of the strings that correspond to elementary particles, the existence of strings cannot be confirmed experimentally and thus all of the attractive features of the theory cannot be explored in depth. Nevertheless, it still remains a huge and rapidly expanding growing field and currently provides favourable guidelines for the quantisation of gravity.

3.2 Strings in D-dimensional spacetime

To start with, strings are one-dimensional extended fundamental objects - loops known as *closed strings* or lines known as *open strings* - that propagate in a D-dimensional background spacetime. The strings sweep out a two-dimensional worldsheet Σ, which is a curved surface embedded in the D-dimensional spacetime. This is parametrized by $\sigma^\alpha = (\tau, \sigma)$, with τ being the timelike coordinate and σ being spacelike. To define the embedding of the worldsheet into the Minkowski background spacetime there exists a set of scalar fields, defined as $X^\mu(\tau, \sigma)$; with $\mu = 0, \ldots, D-1$. These fields describe how the string propagates and oscillates in the given spacetime. The worldsheet of a freely propagating closed string has the topology of a cylinder and the boundary conditions imposed on $X^\mu(\tau, \sigma)$ state that the fields are periodic in σ on the cylinder:

$$X^\mu(\tau, \sigma) = X^\mu(\tau, \sigma + 2\pi)\,; \qquad \sigma \in (0, 2\pi]\,. \tag{3.2.1}$$

On the other hand, the topology of a freely propagating open string is a strip and the boundary conditions imposed on the fields could be either Dirichlet (**D**) or Neumann (**N**) at both ends of the string. The physical interpretation of **N** boundary conditions specifies that there is no spacetime momentum transfer from the string ends to the background spacetime. With **D** boundary conditions, one end of the string is fixed on a dynamical object which absorbs the flow of spacetime momentum. Such an object is called *Dirichlet brane* (**D**-brane). If p of the special components of the fields have **N** boundary conditions at one end of the string while the remaining $D - p - 1$ components have **D** boundary conditions then the other end of the string is attached to a **Dp**-brane. Note that the fundamental strings discussed here are different objects from branes, which are higher dimensional objects. The particles that correspond to the excitations of an open string are able to propagate only on the world-volume of the D-brane. However, the particles that correspond to the excitations of a closed string are free to propagate throughout the D-dimensional spacetime. For the remaining part of this thesis the discussion encompasses *only closed strings,* hence branes are not relevant objects for the work presented in the upcoming chapters.

3.2.1 The bosonic string

The dynamics of the freely propagating bosonic string in D-dimensional Minkowski spacetime is governed by the *Nambu–Goto* action, which is proportional to the area of the worldsheet:

$$S_{NG} = -T \int_\Sigma d^2\sigma \sqrt{-det\ \gamma}\,. \tag{3.2.2}$$

Here the constant of proportionality T is the tension (or mass per unit length) of the string and is defined as $T = (2\pi\alpha')^{-1}$; α' is the fine structure constant and the length of the string is given by $l_s = \sqrt{\alpha'}$. The induced metric on the worldsheet is the pull-back of the flat metric on Minkowski space,

$$\gamma_{\alpha\beta} = \eta_{\mu\nu}\frac{\partial X^\mu}{\partial\sigma^\alpha}\frac{\partial X^\nu}{\partial\sigma^\beta} = \begin{pmatrix} \dot{X}^2 & \dot{X}\cdot X' \\ X'\cdot\dot{X} & X'^2 \end{pmatrix}, \tag{3.2.3}$$

where $\dot{X}^\mu = \frac{\partial X^\mu}{\partial\tau}$ and $X'^\mu = \frac{\partial X^\mu}{\partial\sigma}$. So the Nambu-Goto action for a relativistic string is reduced down to

$$S_{NG} = -T \int_\Sigma d^2\sigma \sqrt{-\dot{X}^2X'^2 + (\dot{X}\cdot X')^2}\,. \tag{3.2.4}$$

For the quantisation of the string, the square root in Eq. (3.2.4) poses a complication. However, the quantisation procedure is simplified in another form of the string action, which gets rid of the square root by introducing the auxiliary field $h_{\alpha\beta}(\tau,\sigma)$. This is the *Polyakov* action given by

$$S_P = -\frac{1}{4\pi\alpha'}\int d^2\sigma \sqrt{-det\ h}\ h^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X^\nu g_{\mu\nu}\,. \tag{3.2.5}$$

What this action describes is that the D scalar fields $X^\mu(\tau,\sigma)$ are coupled to two-dimensional gravity, with the field $h_{\alpha\beta}(\tau,\sigma)$ being a dynamical metric on the worldsheet. Note that the action in Eq. (3.2.5) holds for a general background with metric $g_{\mu\nu}$ and that at the classical level is equivalent to the Nambu-Goto action. The Polyakov action by itself is invariant under a set of symmetries, both global and local. These symmetries are related to:

1. Global spacetime Poincaré transformations:

$$X^\mu \mapsto a^\mu{}_\nu X^\nu + b^\mu; \quad \eta_{\mu\nu} a^\mu_\rho a^\nu_\sigma = \eta_{\rho\sigma} \tag{3.2.6}$$

   in which the parameters $a^\mu{}_\nu$, b^μ are constants.

2. Local reparametrization of the worldsheet: $\sigma^\alpha \mapsto \tilde{\sigma}^\alpha(\tau,\sigma)$. This is a gauge symmetry which reflects the fact that since the worldsheet coordinates σ^α have no physical meaning, there is a redundancy in the description. The fields X^μ transform as worldsheet scalars, while $h_{\alpha\beta}$ transforms as a two-dimensional metric,

$$\begin{aligned} X^\mu(\sigma) &\mapsto \tilde{X}^\mu(\tilde{\sigma}) = X^\mu(\sigma) \\ h_{\alpha\beta}(\sigma) &\mapsto \tilde{h}_{\alpha\beta}(\tilde{\sigma}) = \frac{\partial\sigma^\gamma}{\partial\tilde{\sigma}^\alpha}\frac{\partial\sigma^\delta}{\partial\tilde{\sigma}^\beta}\, h_{\gamma\delta}(\sigma)\,. \end{aligned} \tag{3.2.7}$$

3. Local Weyl invariance on the metric:

$$h_{\alpha\beta} \mapsto e^{2\omega(\tau,\sigma)} h_{\alpha\beta}\,. \tag{3.2.8}$$

   This implies the tracelessness of the energy-momentum tensor as defined in Eq. (3.2.10), *i.e.* $h^{\alpha\beta}T_{\alpha\beta} = 0$, and signals the conformal invariance of the action. In contrast with the other two symmetries, the Weyl invariance is a symmetry that does not appear in the Nambu-Goto action. It is considered to be another gauge symmetry of Polyakov action that reflects the fact that local dilations are an additional redundancy and the $\omega(\tau,\sigma)$ is not a physical field, hence no d.o.f are associated with it.

If one sets $g_{\mu\nu} = \eta_{\mu\nu}$ and chooses a flat metric on the worldsheet in Minkowski coordinates, *i.e.* $h_{\alpha\beta} = \eta_{\alpha\beta}$, the Polyakov action is reduced to a theory of D free scalar fields. As a result, the equations of motion for X^μ are simplified to the equation of a free wave,

$$S_P = -\frac{1}{4\pi\alpha'}\int d^2\sigma\ \partial_\alpha X\cdot\partial^\alpha X \quad \Rightarrow \quad \partial_\alpha\partial^\alpha X^\mu = 0\,. \tag{3.2.9}$$

There is also an equation of motion associated with the metric $h_{\alpha\beta}$. The variation of the action in Eq. (3.2.5) w.r.t to the metric gives rise to the stress-energy tensor

$$T_{\alpha\beta} = -\frac{4\pi}{\sqrt{-det\, h}}\frac{\delta S_P}{\delta h^{\alpha\beta}} \overset{flat\, metric}{=} \partial_\alpha X\cdot\partial_\beta X - \tfrac{1}{2}\eta_{\alpha\beta}\eta^{\rho\sigma}\partial_\rho X\cdot\partial_\sigma X\,, \tag{3.2.10}$$

which satisfies $T_{\alpha\beta} = 0$. In a more explicit form:

$$T_{01} = \dot{X}\cdot X' = 0 \tag{3.2.11a}$$

$$T_{00} = T_{11} = \tfrac{1}{2}(\dot{X}^2 + X'^2) = 0\,. \tag{3.2.11b}$$

From this it is deduced that the equations of motion of the classical string are the equations of a free wave that obey the constraints of Eq. (3.2.11). These can be easily solved in terms of the lightcone coordinates on the worldsheet, $\sigma^{\pm} = \tau \pm \sigma$, such that

$$\partial_+\partial_- X^\mu = 0\,; \tag{3.2.12}$$

with a general solution

$$X^\mu(\tau,\sigma) = X^\mu_L(\sigma^+) + X^\mu_R(\sigma^-)\,, \tag{3.2.13}$$

where the X^μ_L and X^μ_R functions are the left- and right-moving waves which propagate through space at the speed of light. These functions obey the constraints of Eq. (3.2.11) and in the case of *closed strings* they are expanded in Fourier modes as follows:

$$X^\mu_L(\sigma^+) = \tfrac{1}{2}x^\mu + \tfrac{1}{2}\alpha' p^\mu \sigma^+ + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\,\tilde{\alpha}^\mu_n\, e^{-in\sigma^+} \tag{3.2.14a}$$

$$X^\mu_R(\sigma^-) = \tfrac{1}{2}x^\mu + \tfrac{1}{2}\alpha' p^\mu \sigma^- + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\,\alpha^\mu_n\, e^{-in\sigma^-}\,. \tag{3.2.14b}$$

The x^μ and p^μ are the position and momentum of the centre of mass of the string respectively. In addition, the Fourier modes in Eq. (3.2.14) obey $\alpha^\mu_n = (\alpha^\mu_{-n})^*$ and $\tilde{\alpha}^\mu_n = (\tilde{\alpha}^\mu_{-n})^*$ as a consequence of the fact that X^μ must be real. Note that Eq. (3.2.13) is invariant under $\sigma \to \sigma + 2\pi$ whereas the X^μ_L and X^μ_R individually are not. Revisiting the constraints of Eq. (3.2.11), it is found that

$$(\partial_+ X)^2 = (\partial_- X)^2 = 0\,, \tag{3.2.15}$$

which in turn gives

$$\partial_+ X^\mu = \sqrt{\frac{\alpha'}{2}} \sum_n \tilde{\alpha}_n^\mu \, e^{-in\sigma^+} \tag{3.2.16a}$$

$$\partial_- X^\mu = \sqrt{\frac{\alpha'}{2}} \sum_n \alpha_n^\mu \, e^{-in\sigma^-}\,. \tag{3.2.16b}$$

A noteworthy result that plays a key role in the string quantisation is that the zero modes for both the left- and right-moving waves are identified as being the same, with $\tilde{\alpha}_0^\mu = \alpha_0^\mu = \sqrt{\frac{\alpha'}{2}} p^\mu$. Going back to Eq. (3.2.15), the constraints are written as

$$(\partial_+ X)^2 = \alpha' \sum_n \tilde{L}_n \, e^{-in\sigma^+} = 0\,; \qquad \tilde{L}_n = \tfrac{1}{2} \sum_m \tilde{\alpha}_{n-m} \cdot \tilde{\alpha}_m \tag{3.2.17a}$$

$$(\partial_- X)^2 = \alpha' \sum_n L_n \, e^{-in\sigma^-} = 0\,; \qquad L_n = \tfrac{1}{2} \sum_m \alpha_{n-m} \cdot \alpha_m\,. \tag{3.2.17b}$$

As stated, the $\tilde{L}_n$ and L_n are the sum of the oscillator modes and for the classical string they further obey the condition

$$\tilde{L}_n = L_n = 0 \qquad \forall\, n \in \mathbb{Z}\,. \tag{3.2.18}$$

In this case, the $\tilde{L}_0$ and L_0 have a pivotal role as they provide expressions both for the Hamiltonian ($\mathcal{H}$) and the effective mass of a classical bosonic closed string in terms of the excited oscillator modes (m_S):

$$\mathcal{H} = 2(\tilde{L}_0 + L_0)\,; \tag{3.2.19}$$

$$m_S^2 = \frac{4}{\alpha'} \sum_{n>0} \tilde{\alpha}_n \cdot \tilde{\alpha}_{-n} = \frac{4}{\alpha'} \sum_{n>0} \alpha_n \cdot \alpha_{-n} \qquad \textit{or}$$

$$m_S^2 = \frac{2}{\alpha'} \sum_{n>0} (\tilde{\alpha}_n \cdot \tilde{\alpha}_{-n} + \alpha_n \cdot \alpha_{-n})\,. \tag{3.2.20}$$

The result of Eq. (3.2.20) is the so-called *mass-shell level-matching condition* which comes in handy for the quantisation of the string. It is worth mentioning that the elements of the set $\{L_n, \tilde{L}_n\}$ form a *commuting* algebra, known as the *Witt* algebra:

$$\begin{aligned} [L_n, L_m] &= (n-m)\, L_{n+m} \\ [\tilde{L}_n, \tilde{L}_m] &= (n-m)\, \tilde{L}_{n+m}\,. \end{aligned} \tag{3.2.21}$$

These elements are physically interpreted as the infinitesimal generators of an additional set of gauge transformations that leave the action of the bosonic string theory invariant. This additional symmetry combines both reparametrizations and Weyl rescalings into a new symmetry, the *conformal* symmetry.

After presenting the main properties and the basic structure of the classical bosonic string theory with closed strings, the next step is to proceed with its quantisation. Although the quantisation of the bosonic string deviates from the scope of this thesis, it is still relevant for the consistency of the remaining sections in this chapter. Therefore, an account of this topic is given in Appendix C.

3.2.2 The superstring

The upshot of studying the bosonic string theory is the realisation that it falls short as a consistent, unified quantum theory of gravity due to two missing ingredients; stability and fermions. First, as it is discussed in Appendix C, the bosonic string is plagued by unphysical tachyonic states implying that the vacuum of the bosonic string theory is unstable. Second and most important, is the striking absence of fermions from the physical mass spectrum of the theory. In string theory, the incorporation of fermions is described by a set of fermionic fields $\Psi^\mu(\tau, \sigma)$ and requires SUSY. There are two approaches on this construction: one approach requires worldsheet SUSY and is known as the Ramond-Neveu-Schwarz (RNS) formalism whereas the other requires spacetime SUSY in the ten-dimensional Minkowski background and is known as the GS formalism. In both formalisms, the resulting theories are termed as *superstring* theories.

In this work, the only formalism used is the RNS in which D free fermionic fields are added to the D-dimensional bosonic string, so that the overall number of bosonic and fermionic d.o.f matches. The fermionic fields are two-component spinors which reside on the worldsheet and transform as vectors under a Lorentz transformation on the D-dimensional background spacetime. The action of a superstring theory is found by adding the Dirac action for D massless fermionic fields to the bosonic action:

$$S = -\frac{1}{2\pi}\int d\tau\, d\sigma\ \partial_\alpha X^\mu \partial^\alpha X_\mu - \frac{1}{2\pi}\int d\tau\, d\sigma\ \overline{\Psi}^\mu \rho^\alpha \partial_\alpha \Psi_\mu\,. \tag{3.2.22}$$

Here $\alpha = 0, 1$, $\overline{\Psi}^\mu = (\Psi^\mu)^\dagger i\rho^0$ and ρ^α is the two-dimensional representation of Dirac matrices that satisfies the Clifford algebra $\{\rho^\alpha, \rho^\beta\} = 2\eta^{\alpha\beta}$. In this case, the components of the matrices are all real, hence this is a *Majorana* representation of Dirac matrices, defined as

$$\rho^0 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad \text{and} \qquad \rho^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{3.2.23}$$

The fermionic fields are in fact Weyl-Majorana spinors which carry a spacetime index μ. Hence, the reality condition could be imposed on them such that $\overline{\Psi}^\mu = (\Psi^\mu)^T C$ with C being a 2×2 charge conjugation matrix. In terms of lightcone coordinates, the fermionic part of the superstring action defined in Eq. (3.2.22) is given by

$$S_F = \frac{i}{2}\int d\sigma^+ d\sigma^-\, (\psi^\mu_+\, \partial_- \psi_{+\mu} + \psi^\mu_-\, \partial_+ \psi_{-\mu}) \tag{3.2.24}$$

where ψ^μ_+, ψ^μ_- are the explicit components of the two-dimensional Weyl-Majorana spinors. The fermionic action yields two equations of motion: one that describes a left-moving wave and one that describes a right-moving wave. They are determined respectively as

$$\partial_+ \psi^\mu_- = 0 \qquad \text{and} \qquad \partial_- \psi^\mu_+ = 0\,. \tag{3.2.25}$$

This implies that the left- and right-handed fermions are functions of the σ^+ and σ^- coordinates respectively. Similar to the case of the bosonic string theory, the timelike spinors $\psi^0_{L,R}$ would yield negative norm states that need a new local symmetry to be removed. This symmetry is the super-reparametrization invariance on the worldsheet. In addition to global Poincaré symmetry, the action in Eq. (3.2.22) has an additional global symmetry. This is the worldsheet SUSY whose action on the worldsheet fields is given by

$$\begin{aligned} \delta X^\mu &= \bar{\epsilon}\,\Psi^\mu \qquad\qquad \epsilon = \begin{pmatrix} \epsilon_- \\ \epsilon_+ \end{pmatrix}; \\ \delta\Psi^\mu &= \rho^\alpha \partial_\alpha X^\mu \epsilon\,, \end{aligned} \tag{3.2.26}$$

where ϵ is an infinitesimal Majorana spinor with real, constant components. As a result, there are two conserved currents associated with the global symmetries,

$$J^\alpha_A = -\tfrac{1}{2}\,(\rho^\beta\rho^\alpha\Psi_\mu)_A\,\partial_\beta X^\mu \tag{3.2.27}$$

$$T_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu + \tfrac{1}{4}\,\overline{\Psi}^\mu \rho_\alpha\,\partial_\beta\Psi_\mu + \tfrac{1}{4}\,\overline{\Psi}^\mu \rho_\beta\,\partial_\alpha\Psi_\mu\,. \tag{3.2.28}$$

The stress-energy tensor ($T_{\alpha\beta}$) arises from the translational symmetries, and the supercurrent (J^α_A) arises from SUSY. In terms of lightcone coordinates, these are expressed as

$$j_+ = \psi^\mu_+\,\partial_+ X_\mu \quad \text{and} \quad j_- = \psi^\mu_-\,\partial_- X_\mu\,, \tag{3.2.29}$$

$$\begin{aligned} T_{++} &= \partial_+ X_\mu\,\partial_+ X^\mu + \frac{i}{2}\psi^\mu_+\,\partial_+\psi_{+\mu} \\ T_{--} &= \partial_- X_\mu\,\partial_- X^\mu + \frac{i}{2}\psi^\mu_-\,\partial_-\psi_{-\mu} \\ T_{-+} &= T_{+-} = 0\,. \end{aligned} \tag{3.2.30}$$

The RNS superstring theory has a number of constraints imposed on quantisation, which are given by

$$j_+ = j_- = T_{++} = T_{--} = 0\,. \tag{3.2.31}$$

Likewise to the bosonic string, the constraints in Eq. (3.2.31) imply that the superstring theory has superconformal invariance which is used to fix the lightcone gauge and remove the unphysical states from the quantised theory.

In order to solve the equations of motion, boundary conditions are required.

The boundary condition for the closed string leads to two sets of fermionic modes, one that is left- and one that is right-moving. The periodicity conditions in accordance with the Lorentz invariance are

$$\psi^{\mu}_{\pm}(\tau,\sigma) = \pm\,\psi^{\mu}_{\pm}(\tau,\sigma+\pi)\,. \tag{3.2.32}$$

The positive sign implies periodic or Ramond (**R**) boundary conditions whereas the negative sign implies antiperiodic or Neveu-Schwarz (**NS**) boundary conditions. Either one of the two boundary conditions could be imposed on the left- and right-movers, therefore there are two different mode expansions for the fermionic movers. The left-movers could be expanded either as

$$\mathbf{R}:\psi^{\mu}_{+}(\tau,\sigma) = \sum_{n\in\mathbb{Z}} \tilde{d}^{\mu}_{n}\, e^{-2in(\tau+\sigma)} \qquad \text{or} \qquad \mathbf{NS}:\psi^{\mu}_{+}(\tau,\sigma) = \sum_{r\in\mathbb{Z}+\frac{1}{2}} \tilde{b}^{\mu}_{r}\, e^{-2ir(\tau+\sigma)}\,, \tag{3.2.33}$$

and the right-movers could be similarly expanded either as

$$\mathbf{R}:\psi^{\mu}_{-}(\tau,\sigma) = \sum_{n\in\mathbb{Z}} d^{\mu}_{n}\, e^{-2in(\tau-\sigma)} \qquad \text{or} \qquad \mathbf{NS}:\psi^{\mu}_{-}(\tau,\sigma) = \sum_{r\in\mathbb{Z}+\frac{1}{2}} b^{\mu}_{r}\, e^{-2ir(\tau-\sigma)}\,. \tag{3.2.34}$$

Note that $b^{\mu\dagger}_{r} = b^{\mu}_{-r}$ and $d^{\mu\dagger}_{n} = d^{\mu}_{-n}$. Since a state is formed by the tensor product of a right-mover with a left-mover, there are four choices and hence four sectors in the superstring theory: the **NS-NS** and the **R-R** sectors that consist of spacetime bosonic states, as well as the **NS-R** and **R-NS** sectors that consist of spacetime fermionic states.

3.3 Quantising the RNS superstring theory

In the same manner as to the bosonic string theory, the RNS superstring theory can be quantised either via the covariant or the lightcone quantisation methods. In this section, only the covariant quantisation is presented in detail and lightcone quantisation will only be briefly mentioned.

Promoting both X^{μ} and Ψ^{μ} to operator valued fields one obtains the canonical equal-time commutation and anticommutation relations respectively. This means that the modes $\alpha, \tilde{\alpha}$ derived from the bosonic fields obey the algebraic relations in Eq. (C.2.10), and the modes $b, \tilde{b}, d, \tilde{d}$ derived from the fermionic fields become operators and obey the following algebraic relations:

$$\begin{aligned}
\{\hat{\tilde{b}}^\mu_r, \hat{\tilde{b}}^\nu_s\} &= \{\hat{b}^\mu_r, \hat{b}^\nu_s\} = \eta^{\mu\nu}\delta_{r+s,\,0} \\
\{\hat{\tilde{d}}^\mu_m, \hat{\tilde{d}}^\nu_n\} &= \{\hat{d}^\mu_m, \hat{d}^\nu_n\} = \eta^{\mu\nu}\delta_{m+n,\,0} \\
\{\hat{\tilde{b}}^\mu_r, \hat{\tilde{d}}^\nu_n\} &= \{\hat{\tilde{b}}^\mu_r, \hat{d}^\nu_n\} = \{\hat{b}^\mu_r, \hat{d}^\nu_n\} = 0\,.
\end{aligned} \tag{3.3.1}$$

Superstrings have two distinct sectors in the spectrum: the **R** and the **NS** sectors, thus for each sector there is a corresponding oscillator ground state defined by

$$\hat{a}^\mu_m \left|0\right\rangle_{\mathbf{R}} = \hat{d}^\mu_m \left|0\right\rangle_{\mathbf{R}} = 0 \qquad \forall\, m > 0\,, \tag{3.3.2}$$

$$\hat{a}^\mu_m \left|0\right\rangle_{\mathbf{NS}} = \hat{b}^\mu_r \left|0\right\rangle_{\mathbf{NS}} = 0 \qquad \forall\, m, r > 0\,. \tag{3.3.3}$$

The excited states are then constructed by acting on the ground states with the creation operators as demonstrated for the bosonic string theory in Appendix C. The ground state of the **NS** sector is *unique* and gives rise to a spin-0 spacetime boson. Note that the oscillators $\hat{a}^\mu_m$ and $\hat{b}^\mu_r$ behave as spacetime vectors under Lorentz transformations, hence the excited states in the **NS** sector, created by acting on the vacuum by negative mode oscillators, all give rise to massless and massive spacetime bosons. Contrary to this, the ground state of the **R** sector is *degenerate* and gives rise to a spin-$\frac{1}{2}$ spacetime spinor of dimension $2^{\frac{D}{2}}$. The fermionic coordinates, $\hat{d}^\mu_m$, have zero-modes which obey the anticommutation relations

$$\{\hat{d}^\mu_0, \hat{d}^\nu_0\} = \delta_{0,\,0}\,\eta^{\mu\nu} = \eta^{\mu\nu}\,. \tag{3.3.4}$$

This is a representation of the Dirac algebra and allows the set of degenerate states to be written in the following form

$$\sqrt{2}\, i\, d^\mu_0 \left|\alpha\right\rangle = \gamma^\mu_{\alpha\beta} \left|\beta\right\rangle, \tag{3.3.5}$$

where γ^μ is a **D**-dimensional matrix representation of $\hat{d}^0$ and α, β are spinor indices. Similar to before, the oscillators $\hat{\alpha}^\mu_m$ and $\hat{d}^\mu_m$ transform as spacetime vectors hence the excited states in the **R** sector, created in the usual way, all give rise to massless and massive spacetime fermions.

The elements forming the set $\{\hat{L}_m, \hat{\hat{L}}_m\}$, $m \in \mathbb{Z}$, are generators of the RNS superstring theory, known as the *super-Virasoro* generators. There are two sets of them; each one is associated with the mode expansions corresponding to the two different sectors of the theory. The super-Virasoro generators consist of the mode expansions of the stress-energy tensor and the supercurrent, as defined in Eq. (3.2.29), and are determined by adding to the corresponding Virasoro generator of the bosonic part the Virasoro generator of the fermionic part:

$$\begin{aligned} \hat{\hat{L}}_m &= \frac{1}{\pi}\int_{-\pi}^{\pi} d\sigma\ e^{im\sigma}T_{++} = \hat{\hat{L}}^{(b)}_m + \hat{\hat{L}}^{(f)}_m \\ \hat{L}_m &= \frac{1}{\pi}\int_{-\pi}^{\pi} d\sigma\ e^{im\sigma}T_{--} = \hat{L}^{(b)}_m + \hat{L}^{(f)}_m\,. \end{aligned} \tag{3.3.6}$$

The generators from the bosonic part are defined as in Eq. (C.1.8) whereas the fermionic part yields two Virasoro generators and two mode expansions of the supercurrent.

- **NS** sector: The super-Virasoro operators for a closed superstring are expressed as

$$\hat{\hat{L}}^{(f)}_m = \tfrac{1}{2}\sum_{r\in\mathbb{Z}+\frac{1}{2}}\left(r+\frac{m}{2}\right) : \hat{\hat{b}}_{m-r}\cdot\hat{\hat{b}}_r : \tag{3.3.7a}$$

$$\hat{L}^{(f)}_m = \tfrac{1}{2}\sum_{r\in\mathbb{Z}+\frac{1}{2}}\left(r+\frac{m}{2}\right) : \hat{b}_{m-r}\cdot\hat{b}_r : \tag{3.3.7b}$$

and the modes of the supercurrent are given by

$$\hat{\hat{G}}_r = \frac{\sqrt{2}}{\pi}\int_{-\pi}^{\pi} e^{ir\sigma}j_+ = \sum_n \hat{\hat{\alpha}}_{-n}\cdot\hat{\hat{b}}_{r+n} \tag{3.3.8a}$$

$$\hat{G}_r = \frac{\sqrt{2}}{\pi}\int_{-\pi}^{\pi} e^{ir\sigma}j_- = \sum_n \hat{\alpha}_{-n}\cdot\hat{b}_{r+n}\,; \tag{3.3.8b}$$

where $m, n \in \mathbb{Z}$, and : : indicates normal ordering. The zero-mode operators are then defined as follows

$$\hat{\hat{L}}_0 = \tfrac{1}{2}\hat{\hat{\alpha}}_0^2 + \hat{\hat{N}}\,; \qquad \hat{\hat{N}}_{NS} = \sum_{n=1} \hat{\hat{\alpha}}_{-n}\cdot\hat{\hat{\alpha}}_n + \sum_{r=\frac{1}{2}} r\,\hat{\hat{b}}_{-r}\cdot\hat{\hat{b}}_r$$
$$\hat{L}_0 = \tfrac{1}{2}\hat{\alpha}_0^2 + \hat{N}\,; \qquad \hat{N}_{NS} = \sum_{n=1} \hat{\alpha}_{-n}\cdot\hat{\alpha}_n + \sum_{r=\frac{1}{2}} r\,\hat{b}_{-r}\cdot\hat{b}_r\,. \tag{3.3.9}$$

The eigenvalues of the number operators $\hat{N}$ and $\hat{\hat{N}}$ determine the mass-squared value for the excited states of the theory, which is given by

$$\alpha' m_{SS}^2 = \hat{N}_{NS} - \tfrac{1}{2} = \hat{\hat{N}}_{NS} - \tfrac{1}{2}\,. \tag{3.3.10}$$

As usual, the elements of the set $\{\hat{L}_m, \hat{\hat{L}}_m, \hat{G}_r, \hat{\hat{G}}_r\}$, where $m \in \mathbb{Z}$ and $r \in \mathbb{Z} + \frac{1}{2}$ form the *super-Virasoro algebra* which consists of the following *general* algebraic relations:

$$[\hat{L}_m, \hat{L}_n] = (m-n)\,L_{m+n} + \frac{D}{8}\,m\,(m^2-1)\,\delta_{m,-n}$$
$$[\hat{L}_m, \hat{G}_r] = \left(\frac{m}{2} - r\right) G_{m+r}$$
$$\{\hat{G}_r, \hat{G}_s\} = 2\,L_{r+s} + \frac{D}{2}\left(r^2 - \tfrac{1}{4}\right)\delta_{r,-s}\,. \tag{3.3.11}$$

The central charge is denoted by D and is equal to the dimension of the background spacetime for the RNS superstring theory.

- **R** sector: The super-Virasoro operators for a closed superstring are expressed as

$$\hat{\hat{L}}_m^{(f)} = \tfrac{1}{2}\sum_n \left(n + \frac{m}{2}\right) :\hat{\hat{d}}_{m-n}\cdot\hat{\hat{d}}_n: \tag{3.3.12a}$$
$$\hat{L}_m^{(f)} = \tfrac{1}{2}\sum_n \left(n + \frac{m}{2}\right) :\hat{d}_{m-n}\cdot\hat{d}_n: \tag{3.3.12b}$$

and the modes of the supercurrent are given by

$$\hat{\hat{F}}_m = \frac{\sqrt{2}}{\pi}\int_{-\pi}^{\pi} e^{im\sigma} j_+ = \sum_n \hat{\hat{\alpha}}_{-n}\cdot\hat{\hat{d}}_{m+n} \tag{3.3.13a}$$

$$\hat{F}_m = \frac{\sqrt{2}}{\pi}\int_{-\pi}^{\pi} e^{im\sigma} j_- = \sum_n \hat{\alpha}_{-n}\cdot\hat{d}_{m+n}\,; \tag{3.3.13b}$$

where $m,\, n \in \mathbb{Z}$. The mass-squared value for the excited states in this sector of the theory is determined by

$$\alpha' m_{SS}^2 = \hat{\hat{N}}_R = \hat{N}_R\,, \tag{3.3.14}$$

where the number operators are now defined as

$$\hat{\hat{N}}_R = \sum_{n=1} \hat{\hat{\alpha}}_{-n}\cdot\hat{\hat{\alpha}}_n + \sum_{r=\frac{1}{2}} r\,\hat{\hat{d}}_{-r}\cdot\hat{\hat{d}}_r\,; \qquad \hat{N}_R = \sum_{n=1} \hat{\alpha}_{-n}\cdot\hat{\alpha}_n + \sum_{r=\frac{1}{2}} r\,\hat{d}_{-r}\cdot\hat{d}_r\,. \tag{3.3.15}$$

In the **R** sector, the *super-Virasoro algebra* is different because it is now formed by the elements of the set $\{\hat{L}_m, \hat{\hat{L}}_m, \hat{F}_n, \hat{\hat{F}}_n\}$ $(m,\, n \in \mathbb{Z})$ and the *general* algebraic relations are:

$$\begin{aligned}
[\hat{L}_m, \hat{L}_n] &= (m-n)\,L_{m+n} + \frac{D}{8}m^3\delta_{m,-n}\\
[\hat{L}_m, \hat{F}_n] &= \left(\frac{m}{2}-n\right)F_{m+n}\\
\{\hat{F}_m, \hat{F}_n\} &= 2\,L_{m+n} + \frac{D}{2}m^2\delta_{m,-n}\,.
\end{aligned} \tag{3.3.16}$$

Having presented the basic elements of the quantised closed superstring, the next step is to examine the conditions imposed on the physical states ($|\phi\rangle$) emerging from each sector of the RNS superstring theory.

- **NS** sector: The mass-shell conditions satisfy

$$\hat{\hat{L}}_m\,|\phi\rangle = \hat{L}_m\,|\phi\rangle = 0 \qquad m > 0\,, \tag{3.3.17a}$$

$$\hat{\hat{G}}_r\,|\phi\rangle = \hat{G}_r\,|\phi\rangle = 0 \qquad r > 0\,, \tag{3.3.17b}$$

$$(\hat{\hat{L}}_0 - z_{NS})\,|\phi\rangle = (\hat{L}_0 - z_{NS})\,|\phi\rangle = 0\,, \tag{3.3.17c}$$

where z_{NS} is a constant which arises due to the normal ordering ambiguity of $\hat{L}_0$, $\hat{\bar{L}}_0$.

- **R** sector: The mass-shell conditions satisfy

$$\hat{\bar{L}}_m |\phi\rangle = \hat{L}_m |\phi\rangle = 0 \qquad m > 0\,, \tag{3.3.18a}$$

$$\hat{\bar{F}}_n |\phi\rangle = \hat{F}_n |\phi\rangle = 0 \qquad n \geq 0\,, \tag{3.3.18b}$$

$$(\hat{\bar{L}}_0 - z_R) |\phi\rangle = (\hat{L}_0 - z_R) |\phi\rangle = 0\,, \tag{3.3.18c}$$

where z_R is a constant like the z_{NS}. In general, the two constants are not equal as they depend on the zero-mode super-Virasoro generators which are different for each sector.

The last condition in Eqs. (3.3.17) and (3.3.18) implies that the mass of the physical states arising from a closed superstring is given by

$$\alpha' m_{SS}^2 = \hat{\bar{N}}_{NS,R} - z_{NS,R} = \hat{N}_{NS,R} - z_{NS,R}\,. \tag{3.3.19}$$

In the covariant quantisation there are unphysical states emerging through the same procedure as discussed for the bosonic string. It is possible to eliminate these states from the mass spectrum of the theory if the constants are evaluated as $z_{NS} = \frac{1}{2}$ and $z_R = 0$, where the central charge is found to be $D = 10$ [126]. These constraints imply that the background spacetime of a covariantly quantised superstring free from unphysical states and manifestly Lorentz invariant is *ten-dimensional*. Furthermore, a superstring that lives in ten dimensions is free of any conformal anomalies [127].

Unitarity in RNS superstring theory is manifest in the lightcone quantisation. Recall that in the bosonic theory, the conformal symmetry of the action is a residual bosonic symmetry which allows the lightcone gauge condition defined in Eq. (C.2.6) to be imposed on the theory. This condition is also valid in the RNS superstring theory, however, the theory has a superconformal symmetry. This means that there is an additional residual fermionic symmetry which allows an extra condition to be imposed on the lightcone gauge. As a result, the lightcone gauge in RNS superstring theory requires

$$X^+(\tau,\sigma) = x^+ + p^+\tau \qquad \text{and} \qquad \Psi^+(\tau,\sigma) = 0\,. \tag{3.3.20}$$

Each sector of the theory contributes a different set of states to the physical mass spectrum and the lightcone formalism provides an easy way to understand it. As deduced from Eq. (3.3.19), the mass-squared of the physical states in the NS and R sectors is determined respectively as

$$\alpha' m_{NS}^2 = \sum_{n=1} \hat{\tilde{\alpha}}_{-n}\cdot\hat{\tilde{\alpha}}_n + \sum_{r=\frac{1}{2}} r\,\hat{\tilde{b}}_{-r}\cdot\hat{\tilde{b}}_r - \tfrac{1}{2} = \sum_{n=1} \hat{\alpha}_{-n}\cdot\hat{\alpha}_n + \sum_{r=\frac{1}{2}} r\,\hat{b}_{-r}\cdot\hat{b}_r - \tfrac{1}{2}$$
$$\alpha' m_{R}^2 = \sum_{n=1} \hat{\tilde{\alpha}}_{-n}\cdot\hat{\tilde{\alpha}}_n + \sum_{r=\frac{1}{2}} r\,\hat{\tilde{d}}_{-r}\cdot\hat{\tilde{d}}_r = \sum_{n=1} \hat{\alpha}_{-n}\cdot\hat{\alpha}_n + \sum_{r=\frac{1}{2}} r\,\hat{d}_{-r}\cdot\hat{d}_r\,. \tag{3.3.21a}$$

Hence it is demonstrated that the mass spectrum surviving the super-Virasoro constraints consists of states such as the ones presented below.

- The NS ground state is defined to be annihilated by all positive mode oscillators,

$$\hat{\tilde{\alpha}}_n^i\,|0;k^\mu\rangle_{NS} = \hat{\alpha}_n^i\,|0;k^\mu\rangle_{NS} = 0\,; \qquad \hat{\tilde{b}}_r^i\,|0;k^\mu\rangle_{NS} = \hat{b}_r^i\,|0;k^\mu\rangle_{NS} = 0\,, \tag{3.3.22}$$

  along with

$$\hat{\tilde{\alpha}}_0^\mu\,|0;k^\mu\rangle_{NS} = \hat{\alpha}_0^\mu\,|0;k^\mu\rangle_{NS} = \sqrt{2}\alpha' k^\mu\,|0;k^\mu\rangle_{NS}\,. \tag{3.3.23}$$

  The mass-squared value of the ground state is given by $\alpha' m_{NS}^2 = -\frac{1}{2}$ which implies that the ground state gives rise to a tachyon. At this point the existence of a tachyon jeopardizes the stability of the theory.

- The NS first excited state consists of one left- and one right-moving oscillator mode. However, there are two oscillator modes, the bosonic α_{-1}^i and the fermionic $b_{-\frac{1}{2}}^i$, so there are two options for creating this state. It turns out however, that the first excited state in the NS sector is formed by acting on the vacuum state with the lowest frequency oscillator mode such that

$|\Omega^{ij}\rangle = b^i_{-\frac{1}{2}} \tilde{b}^j_{-\frac{1}{2}} |0; k^\mu\rangle$. These states are formed by the tensor product between two *massless* spacetime vectors acting on a scalar ground state. Thus, they give rise to *massless spacetime vector* particles that fill out the representations of the little group $SO(8)$. The particles have eight d.o.f which are transverse directions or polarizations, as required for a massless vector living in ten dimensions.

- The **R** ground state is similarly defined to be annihilated by all mode oscillators with $n > 0$,

$$\hat{\tilde{\alpha}}^i_n |0; k^\mu\rangle_R = \hat{\alpha}^i_n |0; k^\mu\rangle_R = 0 \,; \qquad \hat{\tilde{d}}^i_r |0; k^\mu\rangle_R = \hat{d}^i_r |0; k^\mu\rangle_R = 0 \,, \tag{3.3.24}$$

along with

$$\hat{\tilde{F}}^\mu_0 |0; k^\mu\rangle_R = \hat{F}^\mu_0 |0; k^\mu\rangle_R = 0 \,. \tag{3.3.25}$$

This condition implies that $\Gamma_\mu k^\mu |0; k^\mu\rangle_R \equiv \not{k} |0; k^\mu\rangle_R = 0$, which is the Dirac equation in the momentum representation. From this result one could surmise that the **R** ground state is a *massless* ten-dimensional spinor with 32 independent components and thus gives rise to *massless* spacetime fermions. However, the fermionic fields on the worldsheet of the superstring theory are Weyl-Majorana spinors. Therefore, it is acceptable to impose the Majorana reality condition on the **R** ground state. This reduces the number of independent components down to 16, but the 16-component Weyl-Majorana spinor has to also obey the Dirac equation. As a result, the number of independent components is further reduced down to eight and hence this state carries eight d.o.f and corresponds to an irreducible spinor of $SO(8)$.

- The **R** first excited state similarly consists of a left- and a right-moving oscillator mode. Unlike what happens in the **NS** sector, the **R** first excited states are obtained by acting on the vacuum state with either the bosonic oscillator mode α^i_{-1} or the fermionic oscillator mode d^i_{-1}. These states are the tensor product between two spacetime vectors acting on a spinor ground state. Thus, they give rise to massive *spacetime spinors*.

There are some immediate problems with the above spectrum. For one thing as explained in Section C.2 of Appendix C, the existence of tachyon is a hindrance, thus the tachyon must be projected from the spectrum. Another observation is that even though the bosonic d.o.f for the first **NS** excited state are equal to the fermionic d.o.f for the **R** ground state, they are not level matched as required by spacetime SUSY. The local SUSY manifests itself in the form of a massless gravitino, hence the absence of SUSY from the spectrum is problematic. Moreover, the spectrum is inconsistent with the worldsheet modular invariance, a symmetry that guarantees the absence of global anomalies [127].

All is not lost because the consistency of the theory at one-loop implies that there is an additional condition imposed on the states. This is the Gliozzi, Scherk and Olive (GSO) projection defined as

$$(-)^F = -1\,, \tag{3.3.26}$$

where F is the fermion number operator:

$$F_{NS} = \sum_{r=\frac{1}{2}} \hat{\hat{b}}^i_{-r}\cdot\hat{\hat{b}}^i_r = \sum_{r=\frac{1}{2}} \hat{b}^i_{-r}\cdot\hat{b}^i_r\,; \qquad F_R = \sum_{n=1} \hat{\hat{d}}^i_{-n}\cdot\hat{\hat{d}}^i_n = \sum_{n=1} \hat{d}^i_{-n}\cdot\hat{d}^i_n\,. \tag{3.3.27}$$

The primary achievement of the GSO projection is the elimination of the tachyon from the **NS** sector. For the projection to be satisfied, the F_{NS} must be equal to an odd number. Hence, the only states in the **NS** sector that survive this additional constraint are those states created by an odd number of $\hat{\hat{b}}$ and $\hat{b}$ oscillator excitations. In the case of the **R** sector, the GSO projection acts as a spacetime chirality and satisfies the anticommutation relation

$$\{(-)^F, d^\mu_0\} = 0\,. \tag{3.3.28}$$

Hence the operator $(-)^F$ is identified with the ten-dimensional γ^{11} matrix and in the absence of oscillators the GSO projection satisfies

$$(-)^F = \gamma^{11}(-)^{\sum_{n=1} d^\mu_{-n}\, d^\mu_n}\,. \tag{3.3.29}$$

Hence, the states in the **R** sector that survive this condition are those created either by an even or an odd number of $\hat{\hat{d}}$ and $\hat{d}$ oscillator excitations depending on the chirality of the spinor ground state. Spinors with positive chirality satisfy $\gamma^{11}\Psi^{\mu} = \Psi^{\mu}$ whereas those with negative chirality satisfy $\gamma^{11}\Psi^{\mu} = -\Psi^{\mu}$. This now guarantees that at the massless level of the theory there are two ground states, a vector boson with eight d.o.f along with a Weyl-Majorana spinor with the same number of d.o.f. The two ground states form two different real, eight-dimensional representations of $SO(8)$ and produce all the massless states in the four sectors, **NS**-**NS**, **NS**-**R**, **R**-**NS**, **R**-**R** of the closed RNS superstrings. A summary of the massless spectrum is provided in Table 3.1.

Sector	$SO(8)$ Representation	Massless Fields
NS − NS	$\mathbf{8}_v \otimes \mathbf{8}_v = \mathbf{35}_S \oplus \mathbf{28}_A \oplus \mathbf{1}$	$g_{\mu\nu}, B_{\mu\nu}, \phi$
NS − R	$\mathbf{8}_v \otimes \mathbf{8}_s = \mathbf{8}_s \oplus \mathbf{56}_s$	Ψ_μ, λ
R − NS	$\mathbf{8}_s \otimes \mathbf{8}_v = \mathbf{8}_s \oplus \mathbf{56}_s$	Ψ'_μ, λ'
R − R	$\mathbf{8}_s \otimes \mathbf{8}_s = p - forms$	**R − R** *bosons*

Table 3.1: The massless spectrum of the closed ten-dimensional RNS superstring where the massless fields are identified according to the irreducible $SO(8)$ representations. The subscripts v and s denote the vector and spinor representations. The **NS − NS** states denote the 2-index symmetric traceless, the antisymmetric and trace combinations that correspond to the spin-2 graviton, 2-form and spin-0 dilaton respectively. The fermionic states Ψ_μ, Ψ'_μ are spin-$\frac{3}{2}$ gravitino fields, while the λ, λ' are spin-$\frac{1}{2}$ dilatino fields. All fermions that fall in the same representations have the same helicity. This spectrum coincides with the one obtained in Type IIA, IIB $\mathcal{N} = 2$ supergravity theories.

Having the same number of bosons and fermions in the massless level of the theory is a direct consequence of spacetime SUSY and is most commonly expressed as *Bose-Fermi degeneracy*. More generally, the Bose-Fermi degeneracy holds at each mass level of the RNS superstring theory [126].

3.4 The vacuum landscape of the RNS superstring theory

For the formulation of a consistent quantum field theory of superstrings, the spacetime background must have ten dimensions. However, following the discovery of anomaly cancellation by [13], in terms of the weak coupling perturbation theory there exist five different consistent superstring theories. Three of them, known as *Type I SO(32), Type IIA, Type IIB*, were introduced by [128] building on the earlier work of Ref. [11]. The other two, known as $SO(32)$ *Heterotic and* $E_8 \otimes E_8$ *Heterotic* were formulated by [12]. The Type I $SO(32)$ is a theory that contains both open and closed strings, and $\mathcal{N} = 1$ SUSY. The Type II theories are based on closed strings with $\mathcal{N} = 2$ and a $U(1)$ gauge group. Depending on the relative chirality between the left- and right-movers different states in both the **NS** and **R** sectors are projected out yielding the different Type II superstring theories with different mass spectra, theoretical and phenomenological properties. The heterotic strings form the central piece for the completion of this thesis, therefore they are discussed in more detail in Chapter 4.

The five superstring theories are in fact five different solutions to a unique underlying theory [129]. Specifically, they are just perturbative expansions of the unique theory about five different, consistent quantum vacua which are related to each other through symmetries, known as *dualities* [130, 131],

$$\mathbf{T} : R \mapsto \frac{1}{R}\,; \qquad \mathbf{S} : g_S \mapsto \frac{1}{g_S}\,. \tag{3.4.1}$$

T-duality is an exact quantum symmetry of perturbative closed strings and relates one string theory compactified on a circle of radius R to another string theory compactified on a circle of radius R^{-1}. The two theories are called *dual* and are physically equivalent, in the sense that the worldsheet quantum field theory is invariant under the rewriting of the radius R theory in terms of the dual string coordinates. Type II superstring theories are related to each other via T-duality, and the same also holds for the two Heterotic superstring theories. So, a Type

IIA theory formulated in a nine-dimensional Minkowski spacetime times a circle of radius R is equivalent to a Type IIB theory formulated in a nine-dimensional Minkowski spacetime times a circle of radius R^{-1}. In ten dimensions both theories are two limiting cases of a continuum of consistent vacua that are connected by tuning the value of the radius R. The value of the radius is determined by the VEV of a modulus field and ranges from zero to infinity.

S-duality is another symmetry of perturbative closed strings and relates one string theory with a strong coupling to another string theory with a weak coupling. Type I $SO(32)$ superstring theory is related to the $SO(32)$ Heterotic superstring theory and Type IIB is related to itself in a similar manner. The string coupling constant g_S is determined by the VEV of the dilaton. A direct implication of S-duality is that two theories are continuously connected by varying the coupling constant from zero to infinity. Furthermore, the strong coupling expansion of one theory is determined by the weak coupling expansion of the other.

Even though it is not a primary goal to elaborate on dualities, there is a remarkable upshot: all superstring theories are different manifestations of the same "master entity". This is on par with the expectations from a fundamental unified theory.

Chapter 4

Non-supersymmetric heterotic strings

The important thing in science is not so much to obtain new facts as to discover new ways of thinking about them.

Sir William Lawrence Bragg

4.1 The general framework

The heterotic string is an oriented closed string obtained by the tensor product of the left- and the right-moving modes. All of the right-moving modes are superstring modes in ten spacetime dimensions and all of the left-moving modes are bosonic string modes in 26 spacetime dimensions[1]. The additional 16 dimensions for the right-moving modes are compactified on an internal momentum lattice which bears gauge charges. The gauge charges are the momenta p_R^I; $I = (1, \ldots, 16)$, which under the one-loop modular invariance form an even self-dual lattice ($\vec{p}_R \cdot \vec{p}_R \in \mathbb{Z}$ and $\vec{p}^2 \in 2\mathbb{Z}$) in 16 dimensions.

[1]This assignment is consistent with the KLT notation.

The massless spectrum of the heterotic string consists of the states in the **NS-NS** and **R-NS** sector, as given in Table 3.1. This constitutes the particle content of $\mathcal{N} = 1$ superstring theory in ten dimensions. The bosonic string modes contribute to the formation of massless vector bosons in the spectrum. According to anomaly cancellation, either an $SO(32)$ or an $E_8 \otimes E_8$ gauge symmetry is required for the heterotic theory to be consistent. Hence there are only two self-dual lattices in 16 dimensions, one generated by the roots of $SO(32)$ and another generated by the roots of $E_8 \otimes E_8$.

There are two possible ways to express the additional 16 bosonic fields. The first one is to keep the fields as bosons. The other, more common way, is to express each bosonic field in terms of a Weyl-Majorana fermion $\bar{\lambda}^A$, where A denotes the d.o.f of the fermionic modes. Based on this approach, the action of the heterotic string in the lightcone gauge is given by

$$S_H = \frac{1}{4\pi\alpha'} \int d^2\xi \left(\partial_\alpha X^\mu \partial^\alpha X_\mu + 2i\,\psi^\mu\,\partial_+ \psi_{-\mu} + 2i \sum_{A=1}^{32} \bar{\lambda}^A \partial_- \bar{\lambda}^A \right). \qquad (4.1.1)$$

The heterotic string is the only perturbative closed string theory that provides a satisfactory description of the observable world and produces amazingly viable phenomenological models in four dimensions. Since the holy grail of string phenomenology is to extract the massless particle content of a theory constructed at the string scale and to examine its properties, heterotic strings are the best candidates for this purpose. Of particular phenomenological interest is the $E_8 \otimes E_8$ heterotic string, because the gauge group encompasses all the gauge groups related to the GUT scheme.

4.1.1 Weakly Coupled Free Fermionic Heterotic Strings

This study focuses on weakly coupled, perturbative non-supersymmetric heterotic strings formulated in the free fermionic formalism. Such a framework is notably termed the "Weakly Coupled Free Fermionic Heterotic String (WCFFHS)" and was pioneered around the same time by two independent groups, ABK [132, 133] and KLT [134, 135]. In WCFFHS, all the worldsheet d.o.f - other than the string coordinates - are represented by free complex worldsheet fermions, which are two-dimensional Weyl-Majorana spinors. This implies that the ten bosonic modes for the left-movers and the ten bosonic modes for the right-movers are expressed in terms of non-interacting fermionic modes as follows:

$$\partial_\alpha X^\mu \, \partial^\alpha X_\mu \to i\,\psi^{*\mu}\,\partial_\alpha\psi_\mu + i\,\bar{\psi}^{*\mu}\,\partial_\alpha\bar{\psi}_\mu - h\,\psi^*\psi\,\bar{\psi}^*\bar{\psi}\,, \tag{4.1.2}$$

where h is the Thirring coupling. In this formalism, the compactification of dimensions is performed on tori with radii equal to the self-dual radius. The Thirring coupling is a function of the radius of compactification, R. At the self-dual radius this coupling is taken to zero [136], therefore the worldsheet fermions ψ and $\bar{\psi}$ are defined as *free fermions*. The bosonic coordinates compactified in d dimensions undergo fermionization, *i.e.* from each bosonic coordinate that is compactified on a flat torus originate two left- and two right-moving *real* free fermions [134]. As a result, for d compactified bosonic coordinates there are $2d$ left- and $2d$ right-moving additional real free fermions.

The left- and right-moving fermions that correspond to the compact directions have enhanced symmetries. The local reparametrization invariance and worldsheet SUSY, which are essential to reconcile the Lorentz invariance with unitarity, are inherent to the quantised action for the heterotic string. However, the two-dimensional field theory that is defined on the string worldsheet of a consistent string model and that naturally admits a massless graviton, must be invariant under conformal symmetries. Conformal invariance of a compactified heterotic string theory follows if the worldsheet d.o.f consist of the two string coordinates, and $N_L = \frac{1}{2}(32 + 2d)$ left-moving and $N_R = \frac{1}{2}(8 + 2d)$ right-moving complex worldsheet fermions. Note that the numbers 32 and 8 are the original numbers of *real* left- and right-moving worldsheet fermions respectively in the ten-dimensional heterotic theory. Thus, the complex free-fermions are paired as

$$f \equiv \{f_R; f_L\} \equiv \{f_{i_R}; f_{i_L}\}\,, \tag{4.1.3}$$

where $i_R = 1, \ldots, N_R$ and $i_L = 1, \ldots, N_L$. Remarkably, the number of left- and right-moving free fermions is such that they provide all the necessary d.o.f so as to ensure the cancellation of conformal anomalies. The free-fermionic construction [132, 133], [134, 135] serves as the anchor underpinning the models presented in this work. In general, the rest of this section explores the salient features of this construction for the special case of heterotic strings in six uncompactified spacetime dimensions, which have $N_R = 8$ and $N_L = 20$.

For the entire construction of a string model, it is important to understand and implement the necessary ingredients that specify both the local and the global structure. Having presented the ingredients which specify the local structure, the next step is to present those which specify the global structure of the model. This is achieved by assigning boundary conditions to the fields which live around the non-contractible loops of a given worldsheet topology. Conveniently, the boundary conditions assigned to the integer-spin fields are fixed by the worldsheet metric. On the other hand, the boundary conditions assigned to the half-integer spin fields could be trivial. In particular, the four real fermions originating from a single bosonic coordinate are allowed to have different boundary conditions, which are restricted by the underlying symmetries responsible for the consistency of the theory. Hence the criteria for the consistent assignment of boundary conditions are that the conformal invariance, the worldsheet SUSY (as treated in detail in Refs. [134, 136]) and, most importantly, the reparametrization invariance, must be preserved. The latter criterion is of considerable importance because apart from local reparametrizations, there is a special class of discrete reparametrizations not connected to the identity. These are known as *modular transformations*, which unlike local reparametrizations, play a crucial role in the boundary conditions of the worldsheet fermions. Specifically, *modular invariance* is found to be a fundamental symmetry of all closed string models.

Considering the worldsheet topology to be that of a two-dimensional torus, there are non-contractible loops in two directions (σ_1 and σ_2), as depicted in Fig. 4.1, around which the boundary conditions of the complex free fermions must be specified. The modular transformations that leave the worldsheet action invariant are expressed in terms of the one-loop modular parameter $\tau = \tau_1 + i\tau_2$ and are

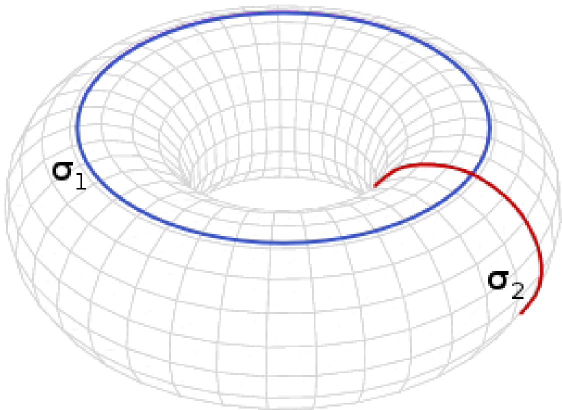


Figure 4.1: The topology of the string worldsheet is a two-dimensional torus with non-contractible loops in two directions. The two directions that yield the non-contractible loops are denoted by the σ_1 and σ_2 coordinates. The boundary conditions of free fermions are determined by the set of phases these fermions acquire when they are parallel transported around the two non-contractible loops.

generated by

$$\sigma_1 \equiv S : \tau \mapsto -\frac{1}{\tau} ; \qquad \sigma_2 \equiv T : \tau \mapsto \tau + 1 . \tag{4.1.4}$$

The S transformation corresponds to interchanging the σ_1 and σ_2 coordinates, which in physical terms means the interchanging of the UV with the Infrared (IR) limit of the string theory. The T transformation corresponds to cutting the torus at some constant σ_2 slice, rotating one end through 2π and then reconnecting it back to the other end.

The complex free fermions are parallel transported around the (σ_1, σ_2) non-contractible cycles of the one-loop worldsheet, acquiring a set of phases which determine their respective boundary conditions,

$$\begin{aligned} S &: f_{i_{R/L}} \to -e^{-2\pi i v_{i_{R/L}}} f_{i_{R/L}} \\ T &: f_{i_{R/L}} \to -e^{-2\pi i u_{i_{R/L}}} f_{i_{R/L}} . \end{aligned} \tag{4.1.5}$$

For toroidal compactifications the phases are collected in two sets of vectors for each of the two non-contractible cycles of the torus,

$$\begin{aligned} v &\equiv \{v_R; v_L\} \equiv \{v_{i_R}; v_{i_L}\} \\ u &\equiv \{u_R; u_L\} \equiv \{u_{i_R}; u_{i_L}\} , \end{aligned} \tag{4.1.6}$$

where $v_{i_R}, v_{i_L}, u_{i_R}, u_{i_L} \in [-\frac{1}{2}, \frac{1}{2})$. The allowable values these phases can take are constrained within the given range by modular invariance as well as the conditions imposed on the left-movers by worldsheet SUSY. Specifically, modular invariance restricts the values of phases to rational numbers, and further requires that the sets of $\{v\}$ and $\{u\}$ vectors must be equivalent. As a result, there exists only one set of such specified phases and it is called a *spin structure*. Since different models are characterised by different boundary conditions assigned to the worldsheet free fermions, the spin structure is the key component which defines the models constructed and, in general, is expressed in terms of a set of basis vectors V_i [135].

Consistent models are constrained by the modular invariance conditions, invariance of the worldsheet supercurrent, and correct space-time spin-statistics; all of these constraints will be satisfied so long as

$$\begin{aligned} m_j k_{ij} &= 0 && \mod (1) \\ k_{ij} + k_{ji} &= V_i \cdot V_j && \mod (1) \\ k_{ii} + k_{i0} + s_i &= \frac{1}{2} V_i \cdot V_i && \mod (1) \,, \end{aligned} \tag{4.1.7}$$

where the k_{ij} are otherwise arbitrary structure constants that completely specify the theory, m_i is the lowest common denominator amongst the components of V_i, and $s_i \equiv V_i^1$ is the spin-statistics associated with the vector V_i. The basis vectors span a finite additive group $G = \sum_k \alpha_k V_k$ where $\alpha_k \in \{0, \ldots, m-1\}$, each element of which describes the boundary conditions associated with a different individual sector of the theory. Within each sector $(\overline{\alpha V})$, the physical states are those which are level-matched and whose fermion-number operators ($N_{\overline{\alpha V}}$) satisfy the generalised GSO projections:

$$V_i \cdot N_{\overline{\alpha V}} = \sum_j k_{ij} \alpha_j + s_i - V_i \cdot \overline{\alpha V} \quad \mod (1) \qquad \forall\, i \,. \tag{4.1.8}$$

The worldsheet energies associated with such states are given by

$$M_{L,R}^2 = \sum_l \left\{ E_{\overline{\alpha V^l}} + \sum_{q=1}^{\infty} \left[(q - \overline{\alpha V^l})\overline{n}_q^l + (q + \overline{\alpha V^l} - 1)n_q^l \right] \right\} - \frac{(D-2)}{24} + \sum_{i=2}^{D} \sum_{q=1}^{\infty} q M_q^i \,, \tag{4.1.9}$$

where l sums over left or right worldsheet fermions, $n_q, \overline{n}_q$ are the occupation numbers for complex fermions, M_q are the occupation numbers for complex bosons, D is the number of uncompactified spacetime dimensions, and $E_{\overline{\alpha V^l}}$ is the vacuum-energy contribution of the l^{th} complex worldsheet fermion:

$$E_{\overline{\alpha V^l}} = \frac{1}{2}\left[(\overline{\alpha V^l})^2 - \frac{1}{12} \right] . \tag{4.1.10}$$

Level-matching then simply requires that $M_L^2 = M_R^2$. Explicitly, the fermions are labelled in the conventional manner:

- two complex space-time fermions, denoted by ψ^{34}, ψ^{56}, which correspond to the transverse modes of the ψ^μ, where $\mu = 1, \ldots, 6$;
- two complex internal fermions, denoted by χ^{34}, χ^{56}, which are present in the original ten-dimensional heterotic string model;
- eight real right-moving internal fermions, denoted by $y^{3,\ldots,6}, \omega^{3,\ldots,6}$, which are obtained from the fermionization of each compactified bosonic coordinate in the six-dimensional theory.

The left-moving worldsheet fermions consist of twenty complex degrees of freedom:

- sixteen complex left-moving fermions, denoted by $\overline{\psi}^{1,\ldots,5}, \overline{\eta}^{1,\ldots,3}, \overline{\phi}^{1,\ldots,8}$, which are present in the ten-dimensional heterotic theory;
- eight real left-moving internal fermions, denoted by $\overline{y}^{3,\ldots,6}, \overline{\omega}^{3,\ldots,6}$, corresponding to the internal right-moving fermions obtained from the fermionization procedure.

In terms of the fields listed above, the worldsheet supercurrent is defined as

$$T_F(z) = \psi^\mu(z)\,\partial_z X_\mu(z) + \sum_{I=3}^{6} \chi^I y^I \omega^I\,. \tag{4.1.11}$$

Moreover, the vector of $U(1)$ charges for each complex worldsheet fermion is given by

$$\mathbf{Q} = N_{\overline{\alpha V}} + \overline{\alpha V}\,, \tag{4.1.12}$$

where $\overline{\alpha V}$ is $0/-\frac{1}{2}$ for an **NS**/**R** boundary condition. This has only been a quick summary on the particulars of the free-fermionic construction. There are, however, numerous subtleties which come into play when dealing with necessarily real worldsheet fermions, especially if there is to be a subsequent coordinate dependent compactification, as in this work. For this reason, extreme care is required for the construction and analysis of these models, and one must adopt a consistent set of phase conventions in agreement with the GSO projections and real-fermionic modes. For this study, however, the conventions adopted are exactly those of Ref. [135].

4.2 Theoretical groundwork

In this section, a review of the aspects of those strings which will be important for this work is presented, and in particular those aspects related to the theoretical properties with which every heterotic string theory is endowed.

4.2.1 The partition function

The one-loop partition function $Z(\tau)$ forms a powerful tool when examining the stability properties of a given closed string. Generally, such a partition function is expressed as the trace over the left- and right-moving string Fock spaces, and for a string formulated in D spacetime dimensions, it takes the form

$$\begin{aligned} Z(\tau) &= \mathrm{Tr}\ (-1)^F\, \overline{q}^{H_R} q^{H_L} \\ &= {\tau_2}^{1-\frac{D}{2}} \sum_{m,n} a_{mn}\, \overline{q}^m q^n \,, \end{aligned} \tag{4.2.1}$$

where $q \equiv e^{2\pi i\tau}$. Here (H_R, H_L) are the right- and left-moving worldsheet Hamiltonians whose eigenvalues are the right- and left-moving worldsheet energies (E_R, E_L), and F denotes the spacetime fermion number. In addition, $\tau_1 \equiv \mathrm{Re}\,\tau$, $\tau_2 \equiv \mathrm{Im}\,\tau$, and a_{mn} counts the net number of spacetime bosonic minus fermionic string states with right- and left-moving worldsheet energies $(E_R, E_L) = (m, n)$.

The spacetime mass M of a given (m, n) state is given by $\alpha' M^2 \sim m + n$ where $M_{string} \equiv \frac{1}{\sqrt{\alpha'}}$ is the string scale; as a result states with $m + n < 0$ are tachyonic. Level-matched states with $m = n$ are physical and can survive the generalised GSO projections; these are the states that are usually described as being part of the string spectrum. By contrast, states with $m \neq n$ are unphysical or "off-shell"; such states cannot survive in the string spectrum but can nevertheless propagate within and hence contribute only to loop amplitudes. For heterotic strings, $m \geq -\frac{1}{2}$, $n \geq -1$ and the leading factors of τ_2 in Eq. (4.2.1) are obtained from the "traces" over the continuum of states corresponding to the uncompactified transverse spacetime dimensions.

Modular invariance requires that the partition function $Z(\tau)$ be invariant under the modular transformations in Eq. (4.1.4). Invariance under the T transformation therefore requires that any (m, n) string states have $m - n \in \mathbb{Z}$, with the physical states satisfying $m - n = 0$. If the heterotic string in question has a spectrum exhibiting spacetime SUSY, then $a_{mn} = 0$ for all (m, n) and hence $Z(\tau) = 0$ for every supersymmetric theory. On the other hand, string theories with $Z(\tau) \neq 0$ are therefore necessarily non-supersymmetric, which is one reason the partition function is a particularly powerful tool for the exploration of non-supersymmetric theories.

It is worth emphasising that modular invariance is a fundamental symmetry of *all closed perturbative strings*, regardless of the absence of spacetime SUSY. There is still a surviving myth in some quarters that modular invariance somehow requires spacetime SUSY. However, this is a misconception since there are a number of non-supersymmetric theories that are nevertheless modular invariant. Another misconception that still survives and deserves explicit refutation is that string theories that do not exhibit spacetime SUSY have tachyons surviving in their spectra. This assertion is not valid, at least at tree level; there exist many examples of non-supersymmetric string models whose tree-level spectra are entirely tachyon-free. Note that freedom from tachyonic states in this context merely requires that $a_{nn} = 0$ for all $n < 0$, *i.e.* that the number of bosonic tachyons match the number of fermionic tachyons at all tachyonic mass levels. However, fermionic tachyons with $m = n < 0$ are generally forbidden by Lorentz invariance, therefore the claim that $a_{nn} = 0$ for all $n < 0$ actually implies that there are no tachyonic states of any spin whatsoever. Bear in mind that once quantum effects are included, the vacua of non-supersymmetric theories which are tachyon-free at tree level can generally shift, and tachyons might be generated at higher loops. A brilliant example of a modular invariant, non-supersymmetric, tachyon-free model is the ten-dimensional $O(16) \otimes O(16)$ heterotic string theory which was originally constructed in Ref. [23]. Indeed, all of the string theories that shall be discussed in this paper are of this variety.

In the case that a D-dimensional string theory with partition function $Z^{(D)}$ is compactified on a d-dimensional volume V_d, then the resulting $(D-d)$-dimensional string theory has a partition function $Z^{(D-d)}$, which is related to $Z^{(D)}$ via

$$Z^{(D)} = \lim_{V_d \to \infty} \left[\frac{1}{\mathcal{M}^d V_d} Z^{(D-d)} \right] . \tag{4.2.2}$$

Here $\mathcal{M} \equiv (2\pi)^{-1} M_{string} = (2\pi \sqrt{\alpha'})^{-1}$ is the reduced string scale. Likewise, for theories with closed strings, the $V_d \to 0$ limit generally yields a D-dimensional string, as a result of T-duality. The relation is still the same as in Eq. (4.2.2) but with V_d replaced with a suitably identified *T-dual* volume $\tilde{V}_d$.

As a whole, there is a diversity in the kinds of physical and unphysical states which contribute to $Z(\tau)$ and there is a theorem pertaining to this feature of string theories. The theorem holds regardless of the specific class of non-supersymmetric string model under study and regardless of the particular GSO projections that might be imposed [30].

Theorem 4.2.1 *Every non-supersymmetric string model necessarily contains off-shell tachyonic states with* $(m, n) = (0, -1)$*, leading to* $a_{0,-1} \neq 0$.

To understand the origin of these states and their effect on the partition function, recall that every string model contains an **NS-NS** sector from which the gravity multiplet arises:

$$\text{graviton} \quad \subset \quad \tilde{\psi}^{\mu}_{-\frac{1}{2}} \, |0\rangle_R \otimes \alpha^{\nu}_{-1} \, |0\rangle_L \, . \tag{4.2.3}$$

Here $|0\rangle_{R,L}$ are the right- and left-moving vacua of the heterotic string: $\tilde{\psi}^{\mu}_{-\frac{1}{2}}$ represents the excitation of the right-moving worldsheet **NS** fermion $\tilde{\psi}^{\mu}$, and α^{ν}_{-1} represents the excitation of the left-moving coordinate boson X^{ν}. There is no self-consistent GSO projection which can possibly eliminate this gravity multiplet from the string spectrum, hence it always contains the graviton. Along with the graviton there always exists in the string spectrum a corresponding state for which the left-moving vacuum is *not* excited. This state is called *"proto-graviton"* and is identified as

$$\text{proto-graviton} \quad \rightarrow \quad \tilde{\psi}^{\mu}_{-\frac{1}{2}} \, |0\rangle_R \otimes |0\rangle_L \, . \tag{4.2.4}$$

The proto-graviton state has worldsheet energies $(E_R, E_L) = (m, n) = (0, -1)$, and is thus both off-shell and tachyonic. Normally, such states are considered to be irrelevant for phenomenology purposes, as they cannot appear as asymptotic states in any scattering. Furthermore, in a supersymmetric theory, there likewise exists in the spectrum a superpartner state, known as the *"proto-gravitino"* which automatically cancels any contribution to the partition function from the proto-graviton. As a result, the full partition function lacks a contribution $\sim q^{-1}$, and hence vanishes entirely. The "proto-gravitino" is ultimately related to the gravitino in exactly the same way as the proto-graviton is related to the graviton and is identified as

$$\text{proto-gravitino} \quad \rightarrow \quad \{\tilde{\psi}_0\}^{\mu} \, |0\rangle_R \otimes |0\rangle_L \, . \tag{4.2.5}$$

Here $\{\tilde{\psi}_0\}^{\alpha}$ schematically indicates the **R** zero-mode combinations which collectively give rise to the spacetime Lorentz spinor index μ. In the context of non-supersymmetric strings, however, *any GSO projection that eliminates the gravitino from the string spectrum, will also correspondingly eliminate the proto-gravitino state*. This is a direct consequence of the fact that GSO projections are 'blind' to the excitations of the bosonic coordinates. Once the proto-gravitino is projected from the spectrum, there is nothing to cancel the contribution from the proto-graviton states, and the resulting non-supersymmetric partition function will necessarily have $a_{0,-1} > 0$. As deduced from Eq. (4.2.4), the off-shell, tachyonic states transform as vectors under the transverse spacetime Lorentz symmetry $SO(D-2)$. Consequently, the first term in the partition function of a non-supersymmetric string theory in D uncompactified spacetime dimensions is always of the form

$$Z(\tau) = \frac{D-2}{q} + \dots \, . \tag{4.2.6}$$

This outcome provides a convenient way of verifying the overall normalisation of a given string partition function.

4.2.2 The one-loop cosmological constant

There is one known physical parameter which is not natural: the cosmological constant Λ. Undeniably, various experimental tests verify that our world is quantum mechanical and the value of the cosmological constant is found to be already at a very low energy scale, $\Lambda \sim (10^{-3}\, eV)^4$. Even if it is set to zero, there is no symmetry enhancement of any form. This indicates that the gravitational phenomena violate naturalness, something that no one can argue against since quantum gravity is not fully yet understood. On a theoretical level, the value of the cosmological constant is measured in QFT and is found to be many orders of magnitude greater than its actual value. This discrepancy between the theory and experiment is known as the *cosmological constant* problem [28].

If string theory is the framework for the unification of all fundamental forces in nature, then it must provide convincing explanations about the observed vanishing of the cosmological constant. Similarly to QFTs, in superstring theories a zero cosmological constant is attainable as a result of spacetime SUSY, at least in a flat background [137–140]. Spacetime SUSY, however, is a broken symmetry and hence string theory must be able to provide a mechanism for a vanishing or almost vanishing cosmological constant which does not depend on SUSY. So far, no such mechanism is known, therefore the purpose of this work is to open an investigation into how the cosmological constant problem could be addressed in the context of non-supersymmetric, tachyon-free string theories. To proceed on this course, it is essential to give the background details on the cosmological constant calculations in heterotic non-supersymmetric string theories.

Generally, the cosmological constant is the vacuum energy density and admits contributions from both the physical and unphysical states of the theory. The dominant contribution to this quantity comes at one-loop order because conformal invariance eliminates the tree-level contribution. So, for any string model in D uncompactified dimensions with partition function $Z(\tau)$, the corresponding D-dimensional one-loop vacuum energy density is evaluated as

$$\Lambda^{(D)} \equiv -\tfrac{1}{2}\,\mathcal{M}^D \int_{\mathcal{F}} \frac{d^2\tau}{{\tau_2}^2}\, Z(\tau)\,, \tag{4.2.7}$$

where $\mathcal{M}$ is the reduced string scale and

$$\mathcal{F} \equiv \{\tau : |\mathrm{Re}\,\tau| \le \tfrac{1}{2}, \mathrm{Im}\,\tau > 0,\ |\tau| \ge 1\} \tag{4.2.8}$$

is the fundamental domain of the modular group. For convenience, the fundamental domain $\mathcal{F}$ is regarded as being composed of two separate regions, an “upper” region with $\tau_2 \ge 1$ and a “lower” region with $\tau_2 < 1$. The upper region extends across the full width $-\frac{1}{2} \le \tau_1 \le +\frac{1}{2}$; in this region, the τ_1-integration then guarantees that only the states with $m = n$ survive as contributors to Λ. In addition, the unphysical states with $m - n \in \mathbb{Z} \neq 0$ contribute to Λ through integration over the curved lower region ($\tau_2 < 1$) within $\mathcal{F}$.

In the case that a D-dimensional string with partition function $Z^{(D)}$ is compactified on a d-dimensional volume V_d, resulting in a $(D-d)$-dimensional string with partition function $Z^{(D-d)}$, then $\Lambda^{(D-d)}$ will typically diverge as $V_d \to \infty$. To avoid this, the cosmological constant of the $(D-d)$-dimensional theory is redefined as $\tilde{\Lambda}^{(D-d)} \equiv \Lambda^{(D-d)}(V_d)^{-1}$. Even though $\tilde{\Lambda}^{(D-d)}$ describes the $(D-d)$-dimensional theory, it now has the mass dimensions appropriate for a D-dimensional vacuum energy density. Substituting this result into Eq. (4.2.2) it is found that

$$\Lambda^{(D)} = \lim_{V_d \to \infty} \tilde{\Lambda}^{(D-d)} . \tag{4.2.9}$$

The same relations also hold in the $V_d \to 0$ limit, provided that V_d is replaced with the appropriate T-dual volume $\tilde{V}_d$. In the following, the prefactor $\frac{1}{2}\mathcal{M}^D$ in Eq. (4.2.7) is disregarded and Λ remains a pure *real number*.

For reasons that will become clear in the next chapter, it is necessary to have a frame of reference regarding the relative sizes of the contributions to the cosmological constant that arise from individual (m, n) string states. A given state with $(E_R, E_L) = (m, n)$ contributes a term $\overline{q}^m q^n$ to the partition function, which in turn yields the following contribution to the one-loop cosmological constant:

$$I^{(D)}_{m,n} \equiv \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \, \tau_2^{1-D/2} \, \overline{q}^m q^n . \tag{4.2.10}$$

Taking into account *all* states that contribute in Eq. (4.2.10), it is a common supposition that massless physical states (*i.e.*, states which satisfy $m = n = 0$) make the dominant contributions to vacuum amplitudes. It can be demonstrated that $I_{nn} \sim e^{-4\pi n}$ for large n, confirming the trend that the contributions from heavy physical states are exponentially suppressed relative to those from lighter states. It can be also demonstrated that the contributions from states with $m \neq n$ are generally suppressed relative to those with $m = n$, even for fixed total energy/mass $m + n$.

A calculation of the contribution from relatively light states in a ten-dimensional and four-dimensional heterotic string theory is presented in Table 4.1. It is evident that the states that make the largest contributions to the cosmological constant are actually the *off-shell tachyonic states* with $(E_R, E_L) = (m, n) = (0, -1)$! These states are the proto-gravitons discussed above and remarkably the contributions from these states are actually bigger than those from the physical massless states by a

m	n	$I^{(10)}_{m,n}$	$I^{(4)}_{m,n}$
0	−1	−14.258	−12.192
1	−1	0.014	0.010
1/2	−1/2	−0.038	−0.032
0	0	0.257	0.549
2	−1	-2.569×10^{-5}	-1.803×10^{-5}
3/2	−1/2	4.682×10^{-5}	3.456×10^{-5}
1	0	-1.029×10^{-4}	-8.463×10^{-5}
1/2	1/2	3.021×10^{-4}	3.304×10^{-4}

Table 4.1: The contribution of the lightest states to the one-loop cosmological constant of a heterotic string theory constructed in ten and four dimensions.

factor of ~ 55 for $D = 10$ and ~ 22 for $D = 4$. Thus, these large contributions to Λ are necessarily present for any non-supersymmetric string model, and any attempt to cancel Λ must therefore find a way of cancelling these contributions as well.

4.3 Exploring the finiteness and stability

In this section, a review of the aspects related to the finiteness and stability properties of non-supersymmetric strings, is presented.

4.3.1 Misaligned Supersymmetry

As was mentioned in the previous section, supersymmetric string theories necessarily have vanishing one-loop partition functions, *i.e.* $a_{nn} = 0$ for all n, and hence $\Lambda = 0$. This cancellation is at the root of the finiteness properties which unbroken spacetime SUSY bestows upon the theories in which it is exhibited. Moreover, this cancellation makes spacetime SUSY an unparalleled candidate for solving the hierarchy problems associated with the Higgs mass m_H and the cosmological constant Λ.

Finiteness effects are commonly quantified through the calculation of *supertraces*, which are essentially statistics-weighted sums over the entire spectrum of the theory:

$$\text{Str}\,\mathcal{M}^{2\beta} \equiv \sum_{states\ i} (-1)^F (M_i)^{2\beta}\,. \tag{4.3.1}$$

In supersymmetric theories the direct pairing of degenerate bosonic and fermionic states implies the vanishing of all supertraces:

$$\text{Str}\,\mathcal{M}^{2\beta} = 0 \qquad \forall\,\beta \geq 0\,. \tag{4.3.2}$$

These supertraces are important because they relate directly to hierarchy issues by governing the quantum-mechanical sensitivities of light energy scales (such as m_H or Λ) to heavy mass scales (*e.g.*, a cut-off λ):

$$\delta m_H^2 \sim (\text{Str}\,\mathcal{M}^0)\lambda^2 + (\text{Str}\,\mathcal{M}^2)\,\log\lambda + \ldots \tag{4.3.3a}$$

$$\Lambda \sim (\text{Str}\,\mathcal{M}^0)\lambda^4 + (\text{Str}\,\mathcal{M}^2)\lambda^2 + (\text{Str}\,\mathcal{M}^4)\,\log\lambda + \ldots \tag{4.3.3b}$$

These relations hold supermultiplet by supermultiplet across the entire spectrum. In the case of the cosmological constant, all states in the theory are included, whereas in the case of the Higgs, only those states to which it is coupled are included. Thus, from Eq. (4.3.2) it is proved that supersymmetric theories solve both the hierarchy problem associated with the Higgs mass and cause Λ to vanish completely.

Nature complicates things as it does not exhibit unbroken SUSY. Given how well the properties of supersymmetric theories tackle the hierarchy issues, it is only logical to wonder if the non-supersymmetric strings could possibly possess any of these properties. It turns out that string theory maintains a degree of finiteness which stems directly from the nature of strings as extended objects. The preeminent agent that sits at the root of the extra finiteness properties, even when spacetime SUSY is completely absent, is none other than modular invariance.

At the level of a QFT without any trace of spacetime SUSY, it is well understood how modular invariance achieves finiteness: the expected field-theoretic one-loop divergences reside in the UV (*i.e.* $\tau \to 0$) region of the modular-group's fundamental domain. During the calculation of quantum effects, the emerging modular symmetries truncate the integrals and exclude this specific region, leaving the theory UV finite. Hence, modular symmetries act as automatic 'regulators' of the one-loop UV divergences. It turns out that in string theory, modular invariance does amazingly well due to the fact that it provides a powerful restriction on the degree to which SUSY can actually be broken. In principle, *in any tachyon-free closed string theory, spacetime SUSY may be broken but a residual so-called "misaligned supersymmetry" must always remain in the string spectrum* [20].

Misaligned SUSY is a general feature of *all* non-supersymmetric string models, and forms the basic mechanism by which the spectrum of such theories manages to configure itself at all mass levels so as to maintain finiteness. In supersymmetric theories, there is an equal number of bosonic and fermionic states at each mass level, hence all associated cancellations occur level by level across the entire spectrum. The marked contrast in non-supersymmetric theories is that level by level cancellations are not possible. The reason lies in the fact that if at a given mass level there is a surplus of spacetime bosonic states, then in the next higher mass level there will be *an even larger* surplus of fermionic states, which is then followed by an even larger surplus of bosonic states at an even higher energy level, and so forth. This pattern, as illustrated in Fig. 4.2, is therefore characterised by the exponential growth of bosonic states succeeded by fermionic states at alternate mass levels. All states tend to *oscillate* and progress upwards across the entire spectrum of the non-supersymmetric theory. It is deduced, therefore, that the cancellation of UV divergences is achieved via a combination of contributions from all the different levels across the *whole* string spectrum. This is the most prominent phenomenological signature of misaligned SUSY, which is essential for this study.

When a theory exhibits spacetime SUSY, the number of bosonic states matches *exactly* the number of fermionic states at every mass level. So, even though each number separately undergoes an exponential growth, their difference - a_{nn} - remains strictly zero and thus there are no oscillations in the string spectrum such as that in Fig. 4.2. By contrast, in a theory with no spacetime SUSY, the bosonic states are "misaligned" resulting in a discernible oscillatory behaviour.

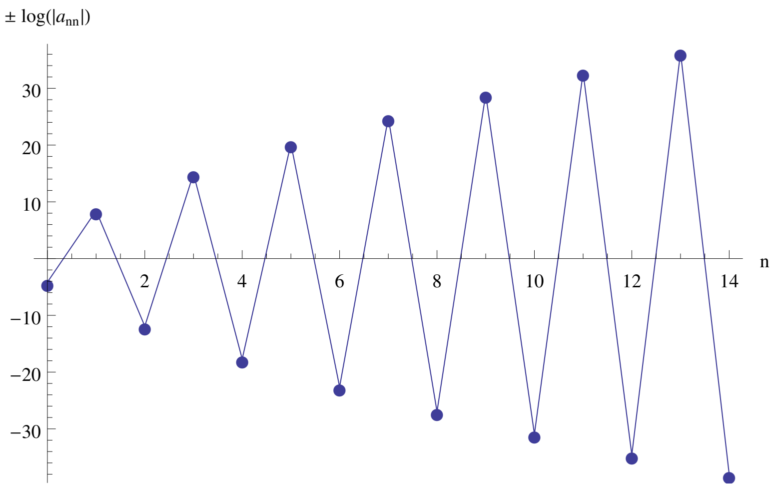


Figure 4.2: A plot of the boson/fermion oscillations which are the hallmark of a hidden "misaligned supersymmetry" existing in the spectrum of *all* tachyon-free non-supersymmetric closed strings. At each mass level n, the quantity plotted is $\pm\log(|a_{nn}|)$, where a_{nn} is the number of bosonic minus fermionic states at that level. The overall sign is chosen according to the sign of a_{nn}; thus in this convention the positive values indicate surpluses of bosonic states over fermionic states, and negative values indicate the reverse. The points are connected in order of increasing n so as to emphasise the alternating, oscillatory behaviour of the boson and fermion surpluses throughout the spectrum. These oscillations represent the mechanism through which non-supersymmetric string theories exhibit UV finiteness. The plot is adapted from Ref. [15].

The plot shown in Fig. 4.2 is an ideal case for a simple system. The actual semi-realistic string models admit quite complex oscillation patterns. Nevertheless, the general signature of misaligned SUSY remains intact. Specifically, the string spectrum is characterised by repeating patterns of oscillations between bosonic and fermionic degeneracies (a_{nn}) which lie in an exponentially growing envelope. The behaviour of the degeneracies as a function of the mass level n, is generally governed by the functional forms $\Phi(n) \sim |a_{nn}| \sim e^{c\sqrt{n}}$. Thus, a given bosonic surplus may have magnitude $\Phi(n_i)$, but this requires a corresponding fermionic surplus of magnitude $\Phi(n_i + \Delta n)$, which in turn requires a corresponding bosonic surplus of magnitude $\Phi(n_i + 2\Delta n)$, and so forth. To be more precise, different sectors of the theory correspond to different bosonic and fermionic envelope functions $\Phi_B^{(i)}(n)$ and $\Phi_F^{(i)}(n)$, as discussed in Ref. [20]. There are well-defined methods to generate analytically *exact* expressions for the functional forms [141] and it is found that these functional forms take the form of a leading "Hagedorn" exponential, followed by an infinite series of subleading exponential functions, followed ultimately by terms which are polynomial in n. This is a well-known feature of *all* string theories, supersymmetric or not, and leads directly to the string Hagedorn transition [142]. Given this, misaligned SUSY requires that the sum of the bosonic functional forms necessarily experiences a relative cancellation against the sum of the fermionic functional forms, *i.e.*

$$\frac{\sum_i \Phi_B^{(i)}(n) - \sum_i \Phi_F^{(i)}(n)}{\sum_i \Phi_B^{(i)}(n) + \sum_i \Phi_F^{(i)}(n)} \to 0 \qquad \text{as} \qquad n \to \infty\,. \tag{4.3.4}$$

The precise degree to which $\sum_i \Phi_B^{(i)}(n)$ cancels directly against $\sum_i \Phi_F^{(i)}(n)$ as a function of n depends on the off-shell tachyonic structure of the theory as well as its overall stability properties; this cancellation is even conjectured to be complete under certain circumstances. Even for large n, the net degeneracies a_{nn} never head towards zero; the spacings between energy levels remain fixed, and the oscillations between bosonic and fermionic surpluses continue unabated with ever-growing amplitudes. Alternating surpluses thus no longer cancel in any pairwise fashion, and it is only through a combination between the states at *all* mass levels across the entire string spectrum that finiteness is ultimately achieved. Further technical details regarding the idea of misaligned SUSY are extensively presented in Ref. [20].

In a sense, there is a fundamental limit on the extent to which spacetime SUSY can be broken in string theory; it can be broken only to the extent that a "misaligned" SUSY remains in the spectrum. Just as in the case of unbroken spacetime SUSY, the finiteness inherent in misaligned SUSY is also encoded in supertrace cancellations. Due to the infinite towers of states with exponentially growing degeneracies, the string supertrace is regulated as shown [21]:

$$\text{Str}\; M^{2\beta} \equiv \lim_{y\to 0} \sum_{states} (-1)^F\, M^{2\beta}\, e^{-y\,\alpha' M^2}\,. \tag{4.3.5}$$

Note that the regulator y, which respects modular invariance, leads to a convergent sum over states and is then removed once the sum is evaluated. The spectrum of any non-supersymmetric, tachyon-free closed string theory, in D uncompactified spacetime dimensions is then found to satisfy [21]:

$$\text{Str } M^0 = 0, \quad \text{Str } M^2 = 0, \quad \ldots \quad \text{Str } M^{D-4} = 0\,, \tag{4.3.6a}$$

$$\text{Str } M^{D-2} = 6\,(-4\pi)^{\frac{D}{2}} \left(\frac{D}{2}-1\right)! \,\frac{\Lambda}{M^2_{string}}\,. \tag{4.3.6b}$$

Here the convention used is the standard normalisation for closed strings, where the spacetime mass of any state with worldsheet energies (m, n) is determined by $M^2 = 2(m+n)M^2_{string}$.

The results in Eqs. (4.3.5) and (4.3.6) are independent of the construction method, the compactification manifold or the low energy phenomenology of the associated theory, and represent the collective behaviour of states at all mass levels simultaneously. A notable outcome of these supertrace relations is the fact that Str M^{D-2} is actually directly proportional to Λ. This is greatly encouraging because the issues of finiteness and cancellation of the cosmological constant are deeply intertwined, and modular invariance in conjuction with misaligned SUSY are powerful tools in the quest to achieve both. Furthermore, as it is discussed below, in a given string model the value of the cosmological constant is directly connected to its ultimate stability. Thus in string theory, the issues of hierarchy, finiteness, and stability are all connected to each other. Consequently, any mechanism that leads to a suppressed cosmological constant for a given string model will simultaneously help to stabilise it and enhance its finiteness properties, regardless of the absence of spacetime SUSY.

4.3.2 Dilaton tadpoles and higher-loop tachyons

Having explored the finiteness of non-supersymmetric strings it is time to move on the subject of self-consistency and examine whether such strings admit stable ground states. Recall that in supersymmetric strings there are flat directions at all orders of perturbation theory which correspond to massless moduli. So long as the flat directions remain unchanged then superstrings exhibit an equilibrium. This equilibrium is sensitive and is not *de facto* stable, as any attempt to lift these flat directions results in a runaway behaviour wherein moduli fields acquire VEVs that tend to run off to infinity. One of these moduli fields is the dilaton ϕ whose VEV determines the value of the string coupling thus such a runaway behaviour will yield phenomenologically unacceptable string couplings. For non-supersymmetric strings, the situation is considerably worse because a massive dilaton field implies a non-vanishing one-loop cosmological constant as illustrated in Fig. 1.1, and vice versa.

The existence of a non-zero dilaton tadpole is extremely dangerous as this entails that a dilaton can simply be absorbed into (or emitted from) the vacuum with no other consequences. Such a process can repeat itself *ad nauseum*, rendering the ground state of the relevant theory totally unstable. As explained in Chapter 1, the dilaton potential $V(\phi)$ contains a non-zero linear term proportional to ϕ itself which destabilises the vacuum, thereby producing a non-consistent theory. In the context of non-supersymmetric strings, the vacuum destabilisation is taken to another level altogether. For one thing, there is no assurance at all that there exists a "nearby" vacuum which continues to be non-supersymmetric but for which stability is restored. One could attempt to absorb the non-zero dilaton tadpoles via the Fischler-Susskind mechanism [17]. However, if such tadpoles remain unsuppressed then the resulting new background will be vastly different to the original one, thus invalidating the basic construction. For another thing, unsuppressed dilaton tadpoles could very well alter significantly the emerging spectra, especially when quantum effects are taken into account. The most dangerous alteration, which could have a strongly negative impact on the consistency of such non-supersymmetric strings, is the appearance of tachyonic states at higher orders, even if they are absent at tree-level.

Over the past thirty years, there has been considerable effort in constructing non-supersymmetric strings with (almost) vanishing, or suppressed cosmological constants [28–30, 32–34], without success. It has been conjectured however in Ref. [22] that if such theories are constructed, they could lead to an entirely new approach towards string phenomenology. The conclusion derived from this section is that the first and most critical issue when attempting to formulate a consistent non-supersymmetric string theory as an underlying platform for a non-supersymmetric string phenomenology is to ensure the stability of the strings. Since the experimental value of Λ is not absolutely set to zero, it is sufficient to achieve a very small value for it so that it matches the observational limit. Such a negligible value should be adequate for the suppression of the one-loop dilaton tadpole and thereby ensuring that all the stability properties exhibited by non-supersymmetric, tachyon-free theories remain unaltered.

Chapter 5

Exploring the class of interpolating heterotic string models

Nothing in life is to be feared, it is only to be understood. Now is the time to understand more, so that we may fear less.

Marie Curie

5.1 Motivation for interpolation

The critical issue in this study is to construct non-supersymmetric strings for which the corresponding one-loop cosmological constant Λ is almost vanishing. Before proceeding with the construction of any such models, it is important to understand what the mass scales are that come into play in a given theory. In general, string theories (whether open or closed) are defined by a single mass scale $M_{string} \equiv \frac{1}{\sqrt{\alpha'}}$, where α' is the string tension. There are, however, additional mass scales introduced due to the effect of the background geometry in which the strings are formulated and propagate.

A common example of an additional mass scale is the one associated with the compactification volume of a given theory. For strings with worldsheet SUSY the background spacetime has a critical dimension $D = 10$, which in order to yield a four-dimensional world, six of the spacetime dimensions have to be compactified. The process of compactification then introduces a new mass scale M_c into the problem, which is associated with the six-dimensional compactification volume $M_c \equiv (V_6)^{-1/6}$. Both the string and compactification scales play a crucial role in determining the masses of individual string states in a given string theory. On the one hand, the string *oscillator* states that reflect the internal twisting and stretching of a string depend solely on the former mass scale: $M_\ell^{osc} \sim \sqrt{\ell} M_{string}$. On the other hand, because the string sits within a compactified geometry, each string oscillator state is accompanied by an infinite tower of Kaluza Klein (KK) excitations. Relative to the mass of the zero-mode state, these excited KK modes have masses set by the latter mass scale: $M_m^{KK} \sim m M_c$. Finally, a closed string state can stretch and wrap around the compactified geometry, yielding *winding-mode* states whose masses depend on both M_{string} and M_c simultaneously: $M_n^{winding} \sim n M_{string}^2 M_c{}^{-1}$. All three kinds of states typically appear together within the spectra of closed strings and contribute to the overall value of the one-loop cosmological constant. Therefore, Λ is expect to be affected by the effects of both fundamental scales, namely M_{string} and M_c.

Naturally, it is assumed that $M_c \sim M_{string}$; this configuration of scales is "minimal" and does not require a dynamical mechanism by which a hierarchy of scales might be generated. Such models are characterised by the fact that there is not always a clear distinction between the oscillator, KK and/or winding states, and typically the cosmological constant is found to be of order $\Lambda \sim M_{string}$. Throughout the years, various proposals have been presented regarding the possibility of suppressing Λ through some other mechanism. For example, the proposals in Refs. [28–30, 32] all rely on different kinds of symmetry arguments for reducing the value of Λ within closed string models that satisfy $M_c \sim M_{string}$. However, there has been no success in constructing string models that exhibit the symmetries proposed in Refs. [28–30], and the mechanism proposed in Ref. [32] may actually fail at higher loops [34, 35].

An alternate approach to tackle this problem is to consider models in which M_{string} is fixed, but M_c (which characterises the compactification volume) is taken to be a free, adjustable variable. Considering a scenario in which the d-dimensional compactification manifold is a d-torus with different radii of compactification R_i; $i = 1, \ldots, d$, then the compactification volume V_d admits many different compactification scales $M_c^{(i)}$, each of which could be considered as a free parameter. As $V_d \to \infty$, there is a string model produced in d additional spacetime dimensions, denoted as M_1. T-duality ensures that as $V_d \to 0$, there is another string model produced in d additional spacetime dimensions, denoted as M_2. Thus, any model constructed with variable compactification volumes could be said to *interpolate* between these two higher-dimensional endpoints, models M_1 and M_2.

Such interpolating models form a unique class of non-supersymmetric models and provide the necessary circumstances under which it is possible for the cosmological constant to be *exponentially suppressed*, with $\Lambda \sim O(e^{-M_c/M_{string}})$:

- It is possible to dial V_d to a sufficiently large value in order to obtain a cosmological constant of whatever small size is desirable. This stems from the realisation that if the model M_1 is supersymmetric, while M_2 is non-supersymmetric, then spacetime SUSY is restored at $V_d \to \infty$ but is likely to be broken for all finite V_d. This idea resembles the scenario for large volume compactification, as suggested in Refs. [24,79]. Thus, large compactification volumes are relatively easy to be incorporated in the interpolating-model framework at $M_c \sim O(1)$ TeV.

- Like all non-supersymmetric models, the spectra of these interpolating models exhibit a *misaligned supersymmetry*, with boson/fermion oscillations. This ensures that they are finite and well behaved at the UV limit.

- The *scale* of the cosmological constant need not necessarily be tied to the effective scale of the SSSB. In fact, the interpolation framework offers the intriguing possibility of separating the effective scale of SSSB from the scale of the cosmological constant, thereby bestowing a certain enhanced stability on these models even if the effective scale of SSSB is relatively large. This can also be important phenomenologically because the magnitude of the first non-vanishing supertrace in Eq. (4.3.6) is set by Λ rather than by the expected M_c; this is only consistent because the supertrace relations are not satisfied supermultiplet by supermultiplet across the entire string spectrum.

Due to these properties, interpolating models are the centrepiece of this work.

5.2 The general structure of interpolation

The discussion in this section is centred on the structure of the simplest case of heterotic interpolating string models in which the compactification manifold is a circle with a $\mathbb{Z}_2$ twist. Such models were originally introduced in Ref. [24, 26, 27] and later in Ref. [79][1]. There are many different construction techniques that might be followed, depending on various factors such as the spacetime dimension of the original model, the number of spacetime dimensions to be compactified, and so forth. Fortunately, these constructions all share certain common features.

The first step in the construction of interpolating models is to choose a heterotic string theory formulated in D spacetime dimensions, with partition function $Z(\tau)$, as given in Eq. (4.2.1). This theory will ultimately serve as one endpoint of interpolation.

The next step is to compactify this theory on a circle of radius R. For any compactification radius R, the corresponding dimensionless radius is defined as $R \equiv \frac{r}{\sqrt{\alpha'}}$. Any field compactified on a circle with this radius then accrues integer momentum and winding modes around this circle, resulting in left- and right-moving spacetime momenta given by

$$p_L = \frac{1}{\sqrt{2\alpha'}}\left(\frac{\ell}{r} + \jmath r\right), \qquad p_R = \frac{1}{\sqrt{2\alpha'}}\left(\frac{\ell}{r} - \jmath r\right). \tag{5.2.1}$$

The quantities ℓ and $\jmath$ represent the momentum and winding quantum numbers of the field respectively. The contribution to the partition function from such modes

[1]Through the temperature/radius correspondence, such models also serve as the finite-temperature extensions of zero-temperature heterotic string models and thus appear frequently in studies of string thermodynamics [49].

is then given by

$$Z_{circ}(\tau, R) = \sqrt{\tau_2} \sum_{\ell,\, J \in \mathbb{Z}} q^{\frac{\alpha' p_L^2}{2}} \, \overline{q}^{\frac{\alpha' p_R^2}{2}} = \sqrt{\tau_2} \sum_{\ell,\, J \in \mathbb{Z}} q^{\frac{1}{4}\left(\frac{\ell}{r}+Jr\right)^2} \, \overline{q}^{\frac{1}{4}\left(\frac{\ell}{r}-Jr\right)^2} , \tag{5.2.2}$$

whose trace is sufficient for compactifications on a circle. Note that $Z_{circ} \to r$ as $r^{-1} \to 0$, while $Z_{circ} \to r^{-1}$ as $r^{-1} \to \infty$. For such an untwisted compactification, each field within the original D-dimensional string theory accrues the *same* set of momentum and winding modes. Therefore, the total partition function of the resulting $(D-1)$-dimensional string theory representing the untwisted compactification is simply given by

$$Z(r) = Z(\tau)\, Z_{circ}(r) . \tag{5.2.3}$$

It is actually deduced from Eq. (5.2.2) that in the limit of $r \to \infty$ or $r \to 0$ one recovers the original D-dimensional theory. Thus, due to the T-dual nature of this compactification, this process results in a $(D-1)$-dimensional string model which trivially interpolates between the original higher-dimensional string model as $R \to \infty$ and the T-dual of itself as $R \to 0$.

Finally, the purpose of this work is actually to focus on the $(D-1)$-dimensional string theories which represent the compactifications of the original D-dimensional string theory on *twisted* circles, *i.e.* on $\mathbb{Z}_2$ orbifolds, and illustrate the general structure of interpolating models. To this end, a twist is introduced into the $(D-1)$-dimensional model; the choice of which is constrained by a number of factors that are relevant to the self-consistency of the resulting interpolation. This twist is ultimately what allows the interpolation of the $(D-1)$-dimensional theory between two *different* endpoints and thus allows the breaking of spacetime SUSY within interpolation. Any such $(D-1)$-dimensional models have a partition function that takes the general form [24, 45–48]

$$\begin{aligned} Z_{string}(\tau, R) \;&=\; Z^{(1)}(\tau)\, \mathcal{E}_0(\tau, R) \;+\; Z^{(2)}(\tau)\, \mathcal{E}_{1/2}(\tau, R) \\ &\quad +\; Z^{(3)}(\tau)\, \mathcal{O}_0(\tau, R) \;+\; Z^{(4)}(\tau)\, \mathcal{O}_{1/2}(\tau, R) . \end{aligned} \tag{5.2.4}$$

Here the $\mathcal{E}_{0,1/2}$ and $\mathcal{O}_{0,1/2}$ functions indicate various restricted summations over

KK (ℓ) and winding modes ($\jmath$), as described in Sect. III.B of Ref. [15], and are defined as in Ref. [24]:

$$\begin{aligned}
\mathcal{E}_0 &= \{\ell \in \mathbb{Z},\ \jmath\ even\} \\
\mathcal{E}_{1/2} &= \left\{\ell \in \mathbb{Z} + \tfrac{1}{2},\ \jmath \text{ even}\right\} \\
\mathcal{O}_0 &= \{\ell \in \mathbb{Z},\ \jmath \text{ odd}\} \\
\mathcal{O}_{1/2} &= \left\{\ell \in \mathbb{Z} + \tfrac{1}{2},\ \jmath \text{ odd}\right\}.
\end{aligned} \tag{5.2.5}$$

The models are considered to be "interpolating" between two different endpoints. As was discussed in the previous section, these two endpoints are the D-dimensional models M_1 and M_2 as $R \to \infty$ and $R \to 0$, respectively. A $(D-1)$-dimensional interpolating model can thus be considered to be a twisted compactification of the D-dimensional model M_1; the possible twists correspond to the possible choices for self-consistent D-dimensional models M_2 at the other end of the interpolation. As a result, M_1 and M_2 must be related to each other through $\mathbb{Z}_2$ twists which are in fact self-consistent $\mathbb{Z}_2$ orbifolds of the D-dimensional string model. Given this, the partition functions of both M_1 and M_2 models are related to the partition function in Eq. (5.2.4); $Z^{(1)} + Z^{(2)}$ reproduces the partition function of M_1 and $Z^{(1)} + Z^{(3)}$ reproduces the partition function M_2.

From a different perspective, the fact that two D-dimensional models M_1 and M_2 are directly related to each other through a $\mathbb{Z}_2$ orbifold twist Q implies that these models serve as ideal candidates for the structure of a $(D-1)$-dimensional interpolating model. Specifically, one could construct a $(D-1)$-dimensional interpolating model by compactifying M_2 on a circle of radius R, and orbifold the resulting $(D-1)$-dimensional theory by the twist $\mathcal{T}Q$. In this case Q acts on the internal part of the string, while $\mathcal{T}$ acts on the compactified circle. In particular, $\mathcal{T}$ corresponds to the $\mathbb{Z}_2$ shift $y \to y + \pi R$ where y is the T-dual coordinate along the compactified dimension. States with even values of $\jmath$, such as those within $\mathcal{E}_{0,1/2}$ are invariant under $\mathcal{T}$, while those with odd values of $\jmath$, such as those within $\mathcal{O}_{0,1/2}$ pick up a minus sign. Together, the resulting orbifold procedure yields a partition function of the form in Eq. (5.2.4).

If M_1 has spacetime SUSY but M_2 does not, the relevant $\mathbb{Z}_2$ orbifold twist Q must at least include $(-1)^F$. However, Q normally includes additional twist factors which act on the purely internal gauge quantum numbers and provide different choices of how the breaking of spacetime SUSY occurs. Thus, specifying the separation of the M_1 partition function into $Z^{(1)}$ and $Z^{(2)}$ is tantamount to specifying these additional gauge twists, and thereby specifying a choice for the ultimate breaking of SUSY in M_1. If the M_2 is tachyonic, then the $R \to \infty$ limit of the interpolating model will be tachyon-free. As the radius shrinks towards zero, certain states which were previously massive for radii R exceeding some critical radius R^* will become massless at $R = R^*$ and tachyonic for all $R < R^*$, hence the associated interpolating models would certainly be tachyonic.

However, and indeed this is the case that is of utmost interest in this study, the M_2 could be non-supersymmetric and tachyon-free, thus yielding interpolating models that are tachyon-free for all radii $0 \le R \le \infty$. Two concrete examples of such interpolating models are provided with extensive details in Sect. III.B of Ref. [15]. For ease of analysis, the examples are the simplest heterotic interpolating models that can be constructed: two distinct nine-dimensional string models which interpolate between different ten-dimensional endpoints. For the first example, M_1 is the ten-dimensional supersymmetric $SO(32)$ string model and M_2 is the ten-dimensional non-supersymmetric $SO(16)\times SO(16)$ string model, whereas in the second example M_1 is replaced by the ten-dimensional supersymmetric $E_8 \otimes E_8$ string model while M_2 is the same as before. Having concrete examples allows the examination of the emerging mass spectra in detail, and determines the manner in which SUSY is broken in these models.

5.3 Mass spectra in (D-1)-dimensional theories

To understand the spectrum of such interpolating models at all energies, one calculates the net degeneracies of physical bosonic minus fermionic states as functions of the worldsheet energy level $E_R = E_L = n$. Specifically, for a given interpolating model, the degeneracies a_{nn} are extracted from the total partition function which is expanded as a double power series into the form given in Eq. (4.2.1). The entire spectrum of the theory is then displayed in a plot of $\pm\log(|a_{nn}|)$ versus n, where the overall sign is chosen so that it matches with the statistics of a_{nn}. These plots demonstrate the Bose-Fermi non-degeneracies of $(D-1)$ non-supersymmetric tachyon-free theories that fall in the class of interpolating models. For the nine-dimensional model that interpolates between the supersymmetric $E_8 \otimes E_8$ and non-supersymmetric $SO(16) \otimes SO(16)$, the plots are shown in Fig. 5.1, evaluated at several different values of the dimensionless inverse radius $a \equiv \sqrt{\alpha'}R^{-1}$ [15].

Several important features are immediately evident from Fig. 5.1. First and foremost, the spectrum exhibits an oscillatory behaviour which stems from surpluses of bosonic states alternating with surpluses of fermionic states as one proceeds upwards in n. Looking back to Section 4.3.1 of Chapter 4, this is the signature trademark of misaligned SUSY, a mechanism which applies within all modular invariant non-supersymmetric tachyon-free string theories and which is ultimately responsible for their UV finiteness. Even more interestingly, the densities of states begin to exhibit a distinct pattern as the radius increases to infinity. On the whole, string models in which all the compactification radii admit small, finite values, *i.e.* the radii are of the order of the string scale, are characterised by a spectrum which resembles the one illustrated in the upper left panel of Fig. 5.1. The densities of states are marked by oscillations between bosonic and fermionic surpluses which are mathematically described by an exponentially rising envelope function $|a_{nn}| \sim e^{c\sqrt{n}}$. Indeed, this exponential rise in the total state degeneracies is triggered by the exponential rise in the number of *oscillator* string states as opposed to KK or winding states[2]. However, as the values of the compactification radii increase, it is observed that the oscillator string states and their KK excitations begin to establish a hierarchy between them: The oscillator states have worldsheet energies which are quantised in units of n and continue to exhibit exponentially growing state degeneracies (even though the rate of growth becomes somewhat smaller as the radii values increase). By contrast, the KK excitations of these os-

[2]This ultimately leads to a Hagedorn transition.

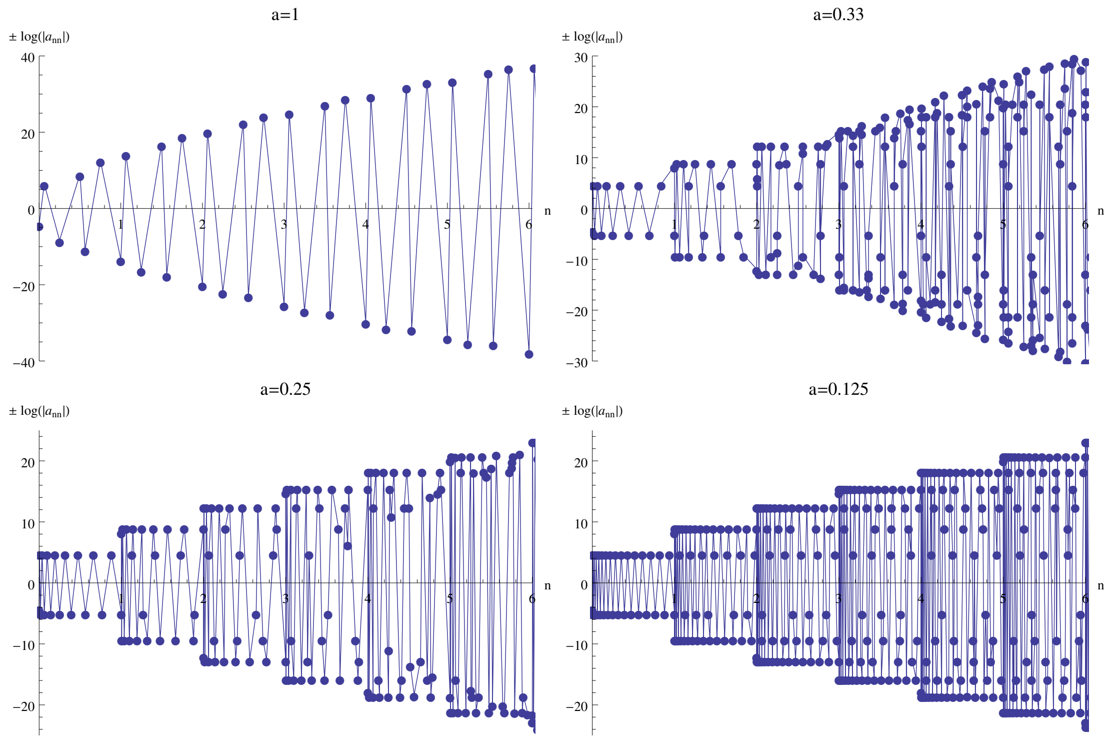


Figure 5.1: Degeneracies of physical states for a particular nine-dimensional interpolating model. The model is described in Ref. [15] and the results are shown for $a = 1$ (upper left), $a = 0.33$ (upper right), $a = 0.25$ (lower left), $a = 0.125$ (lower right). Within each plot, points are connected in order of increasing worldsheet energy n. It is evident that surpluses of bosonic states alternate with surpluses of fermionic states as the energy level increases; this behaviour is due to an underlying "misaligned supersymmetry" which exists within all modular invariant non-supersymmetric tachyon-free string theories and which is ultimately responsible for the UV finiteness of closed strings. For $R = \sqrt{\alpha'}$ (or $a = 1$), the oscillation between bosonic and fermionic surpluses occurs within the exponentially growing envelope function $|a_{nn}| \sim e^{c\sqrt{n}}$. As the value of R (or $a \to 0$) increases, a hierarchy begins to emerge between the oscillator states and their KK excitations; the oscillator states continue to experience exponentially growing densities of states as functions of n. All the while, their corresponding KK excitations are densely packed within each interval $(n, n+1)$ and, as expected, exhibit constant state degeneracies. This figure is adapted from Refs. [15, 16].

cillator states have worldsheet energies which are quantised in units of $R^{-1} \leq n$. So, string models in which all of the compactification radii admit very large finite values, are characterised by a spectrum which resembles the one illustrated on the lower right panel of Fig. 5.1. The densities of states increase dramatically, with each interval $(n, n+1)$ populated by the different KK excitations of the oscillator states at the energy level n. It is worth stressing the observation that within these intervals, the degeneracies of KK states continue the same oscillatory pattern as before but do so only within an envelope function of constant amplitude.

This spectral information clearly indicates that there is no soft SUSY breaking in such models. So long as the values of the compactification radii are finite, it is not possible to restore SUSY, no matter how large the values may be. Hence the spectrum is entirely non-supersymmetric with $a_{mn} \neq 0$ for all (m, n). Despite this feature, for large but *finite* R the low-lying spectrum resembles that of a higher-dimensional theory in which spacetime SUSY is broken (with towers of KK modes and their slightly displaced would-be superpartners). At the same time, the intermediate and heavy spectra remain purely non-supersymmetric and thereby produce important non-supersymmetric threshold effects. Eventually, as the values of compactification radii tend to *infinity* the KK states become infinitely dense. The bosonic and fermionic degeneracies fall into alignment so that level by level cancellations are possible and hence spacetime SUSY is restored. Consequently, the cosmological constant vanishes exactly.

In general, the spectra of non-supersymmetric strings do not have bosonic and fermionic states which could be identified as belonging to the same supermultiplet. However, interpolating models are privileged in this aspect due to the fact that they contain a free tunable parameter, the compactification radius, which allows these models to connect smoothly back to a supersymmetric limit. At this limit, states could be identified as belonging to the same supermultiplets. From this standpoint, the spectra of non-supersymmetric models belonging in the class of interpolation differ fundamentally from those of other non-supersymmetric models. Interpolating models are nevertheless endowed with *all* the features of misaligned SUSY, including the supertrace relations given in Eq. (4.3.6). These supertrace relations ultimately depend on Λ rather than M_c and rely on the combination of contributions from the KK, winding and oscillator modes all inextricably tied together through modular invariance. Thus, situations in which Λ is suppressed give rise to supertraces whose overall magnitudes are smaller than those supertraces which are evaluated supermultiplet by supermultiplet.

5.4 Suppression of the cosmological constant at one-loop

Clearly, as $R \to \infty$, it is established that $\Lambda \to 0$, reflecting the restoration of SUSY in this limit. This piece of information, in conjuction with the significant outcomes derived from the mass spectrum examination, provide compelling evidence: In order to successfully suppress the one-loop cosmological constant Λ associated with such interpolating models one has to require a large and finite value for the compactification radius R. The stage having thus been set, it is imperative to understand the behaviour of Λ for large R. This task requires the derivation of the leading and subleading corrections that emerge when R is large but finite. The following discussion presents the general results regarding the leading and subleading contributions to the one-loop Λ. Note that in all the following results the unit $-\frac{1}{2}\mathcal{M}^D$ included in Eq. (4.2.7) is omitted!

5.4.1 Contribution of physical states

First, since SUSY is restored in the $R \to \infty$ limit, then $Z^{(2)} = -Z^{(1)}$ is true at the level of their q-expansions. As a result, the general partition function of the interpolating model, given in Eq. (5.2.4), takes the form

$$\begin{aligned} Z_{string}(\tau, R) &= Z^{(1)}(\tau)\, [\mathcal{E}_0(\tau, R) - \mathcal{E}_{1/2}(\tau, R)] \\ &+ Z^{(3)}(\tau)\, \mathcal{O}_0(\tau, R) + Z^{(4)}(\tau)\, \mathcal{O}_{1/2}(\tau, R)\,. \end{aligned} \tag{5.4.1}$$

Hence, there are only three different sectors making non-supersymmetric contributions to the cosmological constant: $\mathcal{E}_0 - \mathcal{E}_{1/2}$, $\mathcal{O}_0$, and $\mathcal{O}_{1/2}$.

Starting from the sectors that have **vanishing winding modes**, *i.e.* $\jmath = 0$, the only contributions to the cosmological constant come from the $\mathcal{E}_0$ and $\mathcal{E}_{1/2}$ sectors, thus only the first term in Eq. (5.4.1) is taken into account. Assuming that the theory is devoid of on-shell tachyonic states, there are contributions both from massless and massive states.

• For the *massless states*, *i.e.* $m_i = 0$ the partition function is written as

$$Z_{string}(\tau, R) = \tau_2^{-4}\left[(N_b^{(0)} - N_f^{(0)})(\bar{q}^0 q^0)(\mathcal{E}_0 - \mathcal{E}_{1/2}) + \ldots\right], \tag{5.4.2}$$

where $N_b^{(0)}$ and $N_f^{(0)}$ represent the numbers of bosonic and fermionic string states, respectively, that remain massless in the theory *after SUSY breaking has already occurred*. For $\jmath = 0$ and large $R \equiv \frac{r}{\sqrt{\alpha'}}$, the quantity $\mathcal{E}_0 - \mathcal{E}_{1/2}$ is determined by

$$\begin{aligned}
\mathcal{E}_0 - \mathcal{E}_{1/2} &= \sqrt{\tau_2}\sum_{\ell\in\mathbb{Z}}\left[(q\bar{q})^{\ell^2 r^{-2}/4} - (q\bar{q})^{(\ell+1/2)^2 r^{-2}/4}\right] + \ldots \\
&= \sqrt{\tau_2}\sum_{\ell\in\mathbb{Z}}\left[e^{-\pi\tau_2\ell^2 r^{-2}} - e^{-\pi\tau_2(\ell+1/2)^2 r^{-2}}\right] + \ldots \\
&= \sqrt{\tau_2}\left[\vartheta_3(i\tau_2 r^{-2}) - \vartheta_2(i\tau_2 r^{-2})\right] + \ldots \\
&= r\left[\vartheta_3(ir^2{\tau_2}^{-1}) - \vartheta_4(ir^2{\tau_2}^{-1})\right] + \ldots \\
&= r\sum_{\ell\in\mathbb{Z}}\left[e^{-\pi\ell^2 r^2{\tau_2}^{-1}} - (-1)^\ell e^{-\pi\ell^2 r^2{\tau_2}^{-1}}\right] + \ldots \\
&= r\sum_{\ell=odd} e^{-\pi\ell^2 r^2{\tau_2}^{-1}} + \ldots,
\end{aligned} \tag{5.4.3}$$

where the modular transformations $\vartheta_{3,2}(\tau) = \frac{1}{\sqrt{-i\tau}}\vartheta_{3,4}(-\frac{1}{\tau})$ are used in passing from the third to the fourth line. The *generalised* Jacobi ϑ-functions are explicitly defined in Eq. (D.0.6) of Appendix D. These transformations effectively resum the infinite series so that the leading large-R behaviour can be reliably extracted. It is then deduced from Eq. (5.4.3) that the dominant contribution from the massless states is given by the $\ell = \pm 1$ terms, thus

$$\mathcal{E}_0 - \mathcal{E}_{1/2} \sim 4r\, e^{-\pi r^2{\tau_2}^{-1}} \qquad \text{as} \qquad r \to \infty\,. \tag{5.4.4}$$

In a general setup, where a string theory in $D = 10$ is compactified on a $\mathbb{Z}_2$ orbifold of radius R, the cosmological constant is evaluated as

$$\begin{aligned}
\Lambda &= 4r\,(N_f^{(0)} - N_b^{(0)})\int_{\mathcal{F}}\frac{d^2\tau}{{\tau_2}^6}\,e^{-\pi r^2{\tau_2}^{-1}} + \ldots \\
&\approx \frac{96}{\pi^5 r^9}\,(N_f^{(0)} - N_b^{(0)}) + \ldots,
\end{aligned} \tag{5.4.5}$$

and for a string theory in $D = 4$, the compactification goes through as above, resulting in:

$$\begin{aligned}\Lambda &= 4r\,(N_f^{(0)} - N_b^{(0)}) \int_{\mathcal{F}} \frac{d^2\tau}{{\tau_2}^3}\, e^{-\pi r^2 {\tau_2}^{-1}} + \ldots \\ &\approx \frac{4}{\pi^2 r^3}\,(N_f^{(0)} - N_b^{(0)}) + \ldots .\end{aligned} \tag{5.4.6}$$

In the final results above, the range of integration is restricted to the upper ($\tau_2 \geq 1$) portion of the modular group's fundamental domain. The above integral is solved analytically and the resulting subleading terms of order $O(e^{-\pi r^2})$ are disregarded. In general, for all spacetime dimensions D it is deduced that

$$\Lambda = \left[\frac{4}{\pi^{\frac{D}{2}}}\left(\frac{D}{2} - 1\right)!\,(N_f^{(0)} - N_b^{(0)})\right]\frac{1}{r^{D-1}} + \ldots , \tag{5.4.7}$$

with the understanding (relevant for odd D) that $\left(\frac{1}{2}\right)! = \Gamma\left(\frac{3}{2}\right) = \frac{1}{2}\sqrt{\pi}$, *etc.*

- For the *massive states*, *i.e.* $m_i \neq 0$ the partition function is written as

$$Z_{string}(\tau, R) = \tau_2^{-4}\left[(N_b^{(i)} - N_f^{(i)})(\overline{q}^{m_i} q^{m_i})(\mathcal{E}_0 - \mathcal{E}_{1/2}) + \ldots\right], \tag{5.4.8}$$

where $N_b^{(i)}$ and $N_f^{(i)}$ represent the numbers of bosonic and fermionic string states, respectively, that have a mass m_i in the theory *after SUSY breaking has already occurred*. For $\jmath = 0$ and large $R \equiv \frac{r}{\sqrt{\alpha'}}$, the quantity $\mathcal{E}_0 - \mathcal{E}_{1/2}$ is determined by an expression similar to the one in Eq. (5.4.3); the main difference being that for the massive states there is an additional exponential factor which depends on the masses of the states. The integral that describes the contributions to the cosmological constant from the individual states within the $Z^{(1)}$ sector of such interpolating models is given by

$$\tilde{I}_{m,n}\left(\frac{1}{r}\right) \equiv 4r \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2}\, \tau_2^{1-D/2}\, \overline{q}^m q^n\, e^{-\pi r^2 {\tau_2}^{-1}} . \tag{5.4.9}$$

Following the same procedure as in the case for the massless states, in general, the contribution to the cosmological constant from the massive states in the sectors with vanishing winding modes is found to be

$$\tilde{I}_{n,n} \approx 4\,(N_f^{(i)} - N_b^{(i)})\left(\frac{2m_i}{r}\right)^{(D-1)/2} e^{-4\pi\, m_i\, r} \quad \text{as} \quad r \to \infty\,. \tag{5.4.10}$$

It is evident that at large R values, these terms are exponentially suppressed. However, the largest contributions arise from light but physical massive states within $Z^{(1)}$, *i.e.* states with $m = n$ for small n. In general, this value of n is determined by examining the individual terms within $Z^{(1)}$; the result is generally model-dependent since it is sensitive to the nature of the orbifold twists involved in the construction of the model at hand.

Moving on to the sectors that have **non-vanishing winding modes**, *i.e.* $\jmath \neq 0$, the cosmological constant admits contributions from each one of $\mathcal{E}_0 - \mathcal{E}_{1/2}$, $\mathcal{O}_0$, and $\mathcal{O}_{1/2}$ sectors of the partition function. The physical states coming from the $\mathcal{E}_0 - \mathcal{E}_{1/2}$ part of the partition function are all exponentially suppressed at large R values. In fact, the exponential suppression of these states exceeds the exponential suppression of the physical massive states coming from sectors with vanishing winding modes. The reason for this can be clearly deduced from the general expression of the partition function, given in Eq. (5.2.2): the $\mathcal{E}_0 - \mathcal{E}_{1/2}$ quantity for these states carries an extra exponential factor which depends on the value of the winding mode. This additional exponential factor in conjuction with the exponential factor that depends on the mass m_i of such physical states, cause the resulting Λ terms to be heavily suppressed.

It is also observed that all physical states within the $\mathcal{O}_0$ and $\mathcal{O}_{1/2}$ sectors are extremely heavy as a result of non-vanishing winding modes $\jmath \neq 0$. Similarly, the contributions from such heavy states are also exponentially suppressed. The contributions of the physical states from the $\mathcal{O}_0$ and $\mathcal{O}_{1/2}$ sectors in the limit of large but finite R are evaluated through an algebraic procedure which parallels the analysis of $\mathcal{E}_0 - \mathcal{E}_{1/2}$ in Eq. (5.4.3). As before, the $\mathcal{O}_{0,1/2}$ functions in the $R \to \infty$

limit are now related to the *generalised* Jacobi ϑ-functions as follows:

$$\begin{aligned} O_0 &= 2\sqrt{\tau_2}\, e^{-\pi\tau_2 r^2}\, \vartheta_3(\tau_1, i\tau_2 r^2) + \ldots \\ O_{1/2} &= 2\sqrt{\tau_2}\, e^{-\pi\tau_2 r^2}\, \vartheta_2(\tau_1, i\tau_2 r^2) + \ldots . \end{aligned} \tag{5.4.11}$$

Using the modular transformations of ϑ-functions, as given in Eq. (D.0.7) of Appendix D, and keeping only the leading terms, it is found that

$$O_{0,1/2} \approx 2r\, e^{-\pi|\tau|^2 r^2 \tau_2^{-1}} \quad \text{as} \quad r \to \infty . \tag{5.4.12}$$

Thus, a given state with worldsheet energies (m, n) within the $O_{0,1/2}$ sectors contributes to the cosmological constant as follows:

$$\hat{I}_{m,n}\left(\frac{1}{r}\right) \equiv 2r \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2}\, \tau_2^{1-D/2}\, \overline{q}^m q^n\, e^{-\pi|\tau|^2 r^2 \tau_2^{-1}} \quad \text{as} \quad r \to \infty . \tag{5.4.13}$$

From this result follows that the physical states from $O_{0,1/2}$ with masses m_i contribute to the one-loop cosmological constant terms given by

$$\hat{I}_{n,n}\left(\frac{1}{r}\right) \approx \frac{2\sqrt{2}}{\pi}\, \frac{1}{r^2}\, (N_b^{(i)} - N_f^{(i)})\, e^{-4\pi m_i^2}\, e^{-\pi r^2} \quad \text{as} \quad r \to \infty . \tag{5.4.14}$$

Note that this result is independent of the spacetime dimensionality D on which a theory is formulated prior to the compactification on the $\mathbb{Z}_2$ orbifold.

5.4.2 Contribution of unphysical states

As pointed out in Section 4.2.1 of Chapter 4, the spectrum of every non-supersymmetric string model contains off-shell proto-graviton states with $(m, n) = (0, -1)$ and whose contributions to the partition function are completely uncancelled. In general, such states come from the $\mathcal{E}_0 - \mathcal{E}_{1/2}$ sector of the partition function and there is a plethora of other similar light (or even off-shell tachyonic) states in the spectra of such strings. The results shown in Table 4.1, indicate that such states often make the largest contributions to the one-loop cosmological constant. The integral that describes the contributions to the cosmological constant from such states is the same one as in Eq. (5.4.9).

In contrast to the massive physical states whose contributions are model-dependent, the contribution of proto-gravitons is model-independent. It turns out that as $R \to \infty$, these states contribute a correction term to the cosmological constant which scales approximately as

$$\tilde{I}_{0,-1}\left(\frac{1}{r}\right) \approx \frac{4\sqrt{2}}{\pi}\,\frac{1}{r^2}\,e^{2\pi}\,N_{proto}\,e^{-\pi r^2} + \ldots \quad \text{as} \quad r \to \infty\,, \tag{5.4.15}$$

where N_{proto} is the number of proto-graviton states in the model. In general, since the proto-graviton states generally transform as a vector of the transverse Lorentz group $SO(D-2)$, their number is expected to be $N_{proto} = D - 2$.

A comparison of the results from massless states and proto-graviton states for the nine-dimensional model interpolating between the supersymmetric $E_8 \otimes E_8$ and non-supersymmetric $SO(16)\otimes SO(16)$ is shown in the plot of Fig. 5.2 [15]. The plot is performed in terms of the dimensionless inverse radius $a \equiv \sqrt{\alpha'}R^{-1}$ and it is observed that the contributions from the massless states exceed the contributions from the proto-graviton states in the $R \to \infty$ limit. This demonstrates that the physical massless states indeed dominate for large and finite values of R.

Of all unphysical states, *i.e.* states for which $m \neq n$, it turns out that the contributions from the proto-gravitons are the largest. In fact, the result in Eq. (5.4.15) is only a special case of the more general result

$$\tilde{I}_{m,n}\left(\frac{1}{r}\right) \approx -\frac{4\sqrt{2}}{\pi}\,\frac{1}{r^2}\,e^{-2\pi(m+n)}\,e^{-\pi r^2} + \ldots \quad \text{as} \quad r \to \infty\,. \tag{5.4.16}$$

Quite remarkably, this result holds for all D as well as for all (m, n), as long as $m \neq n$; this result is even independent of $|m - n|$.

The final task is to determine the contributions of the unphysical states that emerge in the O_0 and $O_{1/2}$ sectors of the partition function. Similarly to the case of the massive states, the integral that governs the contribution of these states is the same as given in Eq. (5.4.13). Recalling that the states within $Z^{(4)}$ have worldsheet energies (m, n) with $m - n \in \mathbb{Z} + 1/2$, whereas those within $Z^{(3)}$ have $m - n \in \mathbb{Z}$, it turns

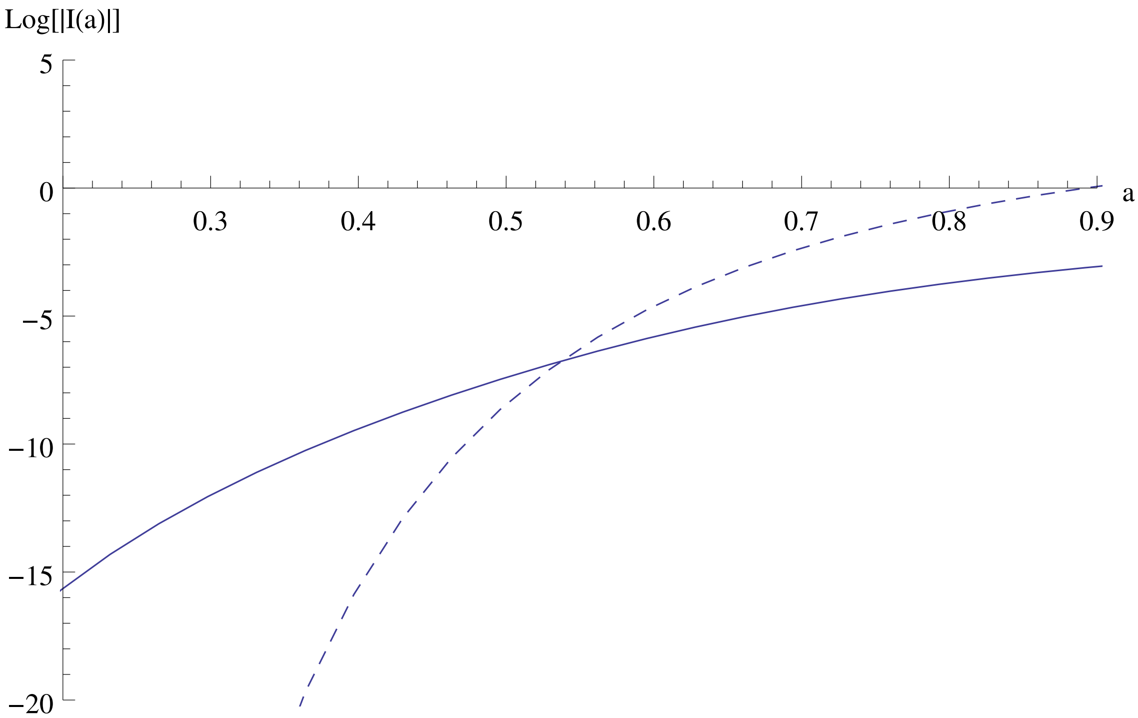


Figure 5.2: Vacuum-energy contributions from massless states (solid line) versus proto-gravitons (dashed line) for a particular nine-dimensional interpolating model. The model is described in Ref. [15] and the results are shown for $\log(|\tilde{I}^{(10)}_{0,0}(a)|)$ and $\log(|\tilde{I}^{(10)}_{0,-1}(a)|)$ plotted versus a. It is evident that the former exceeds the latter in the $R \to \infty$ (or $a \to 0$) limit. The figure is adapted from Ref. [15].

out that these states contribute a correction term of the form

$$\hat{I}_{m,n}\left(\frac{1}{r}\right) \approx \frac{2\sqrt{2}}{\pi}\,\frac{1}{r^2}\,e^{-2\pi(m+n)}\,e^{-\pi r^2} \quad \text{as} \quad r \to \infty\,. \tag{5.4.17}$$

Indeed, this result holds for all spacetime dimensions D as well as all energy configurations, regardless of whether $m = n$ or not.

This result is quite remarkable, since it exactly duplicates the result obtained in Eq. (5.4.16) for the $\mathcal{E}_0 - \mathcal{E}_{1/2}$ sector up to an overall sign and a factor of two! This duplication occurs *even though* the form of the integral in Eq. (5.4.9) is quite different from the form of the integral in Eq. (5.4.13), and *even though* the result in Eq. (5.4.16) is subjected to a restriction (namely $m \neq n$) which does not apply to the result in Eq. (5.4.17). Clearly, the mathematical elegance of these features resides in the power of the asymptotically large but finite radii limit. However, there are also potentially important phenomenological implications that stem from this deduction: the contribution from a state in the $\mathcal{E}_0 - \mathcal{E}_{1/2}$ sectors with energy configuration (m, n) can be completely cancelled by the contribution of a state in the O_0 or $O_{1/2}$ sectors with a completely different energy configuration (m', n'), so long as $m + n = m' + n'$. Even more remarkably, focusing on the special case with $m' = n' = \frac{1}{2}(m + n)$, it is deduced that the contribution of an *unphysical* state in the $\mathcal{E}_0 - \mathcal{E}_{1/2}$ sectors can be cancelled by the contribution of a *physical* state in the O_0-sector! These results provide a myriad of potential new ways of further suppressing the contributions to the cosmological constant.

A direct comparison of *all* the key results presented so far, leads to the derivation of some important conclusions, associated with the expected behaviour of the cosmological constant. The first conclusion originates from the comparison of the results in Eq. (5.4.7), (5.4.10),(5.4.15) and (5.4.17) and states that the *leading terms* in the overall Λ calculation are the ones from the *physical massless states* that come from the sectors with vanishing winding modes. Specifically, the contributions from the physical massless states dominate not only over the contributions from the physical massive states, but also over the contributions of the proto-graviton states, as demonstrated in Fig. 5.2! Undoubtedly, the leading term in the evaluation of the one-loop cosmological constant Λ of an interpolating model in $(D-1)$ spacetime dimensions is the term given in Eq. (5.4.7). The second conclusion comes with the realisation that all the other terms that contribute in the evaluation of Λ must be *subleading*. A cursory examination of Eq. (5.4.10), (5.4.15) and (5.4.17) reveals that the largest subleading terms are the ones obtained from the lightest physical states within $Z^{(1)}$. These contributions, which are given in Eq. (5.4.10) always scale as e^{-r}. Obviously, this factor is larger than the scale factor of the subleading terms given in Eq. (5.4.15) and (5.4.17) which in both cases scale as e^{-r^2}. Moreover, any other subleading terms, especially the ones associated with much heavier states, are all exponentially suppressed.

In summary, within such interpolating models, the contributions to the cosmological constant in the $a \to 0$ limit from a given state with worldsheet energies (m, n) in the different $\mathcal{E}/O$ sectors are listed in Table 5.1 [15].

To verify the expected behaviour of the one-loop cosmological constant for the

Sector	State	Λ terms
$\mathcal{E}_0 - \mathcal{E}_{1/2}$	$m = n = 0$	$-4\,\pi^{-D/2}\,(D/2-1)!\,a^{D-1}$
$\mathcal{E}_0 - \mathcal{E}_{1/2}$	$m = n = m_i \neq 0$	$4\,(2m_i a)^{(D-1)/2}\,e^{-4\pi\,m_i\,a^{-1}}$
$\mathcal{E}_0 - \mathcal{E}_{1/2}$	$m \neq n$	$-[4\sqrt{2}\,\pi^{-1}]\,e^{-2\pi(m+n)}a^2 e^{-\pi\,a^{-2}}$
$\mathcal{O}_{0,1/2}$	any (m,n)	$[2\sqrt{2}\,\pi^{-1}]\,e^{-2\pi(m+n)}a^2 e^{-\pi\,a^{-2}}$

Table 5.1: The contributions to the cosmological constant in the $a \to 0$ limit from a given state with worldsheet energies (m,n) in the different $\mathcal{E}/\mathcal{O}$ sectors. In this table, D represents the dimensionality of the theory *prior* to the compactification on the $\mathbb{Z}_2$ orbifold.

interpolating models, one calculates the Λ associated with the full partition function, which is given in Eq. (5.2.4), as a function of a. For the nine-dimensional model that interpolates between the supersymmetric $E_8 \otimes E_8$ and non-supersymmetric $SO(16) \otimes SO(16)$, the results are plotted as shown in Fig. 5.3 [15]. The plot is of the rescaled $(2a^{-1})\Lambda$ versus a (solid line) and is based on the numerical integration of the cosmological constant. The numerical integration is performed over the entire fundamental domain of the modular group and includes *all* terms in the partition function of the supersymmetric $E_8 \otimes E_8$ theory. Note that the factor of $(a/2)$ is the effective (T-dual) "volume" of compactification in the $a \to \infty$ limit; dividing by this factor allows the interpolating nine-dimensional cosmological constant to asymptote to a finite ten-dimensional limit as $a \to \infty$.

As shown in Fig. 5.3, the model successfully interpolates between $\Lambda = 0$ at $a = 0$ and $\Lambda \approx 725$ at $a \to \infty$, where $\Lambda \approx 725$ is the cosmological constant associated with the ten-dimensional $SO(16) \otimes SO(16)$ heterotic string. It should be noted that a similar plot appears in Ref. [26]. Moreover, it is verified that the leading term $\Lambda \sim \frac{6144}{\pi^5}\,a^9 \sim (20.08)\,a^9$ as $a \to 0$ indeed provides an excellent approximation to the cosmological constant for $a \ll 1$, holding to several significant digits throughout the relevant range. This verifies not only the overall radius-dependence (scaling power-law behaviour) predicted in Eq. (5.4.7) but also the numerical coefficient which precedes it.

There is also another conclusion extracted from all of these detailed remarks, which is by far the most significant. At the limit of large and finite radius of com-

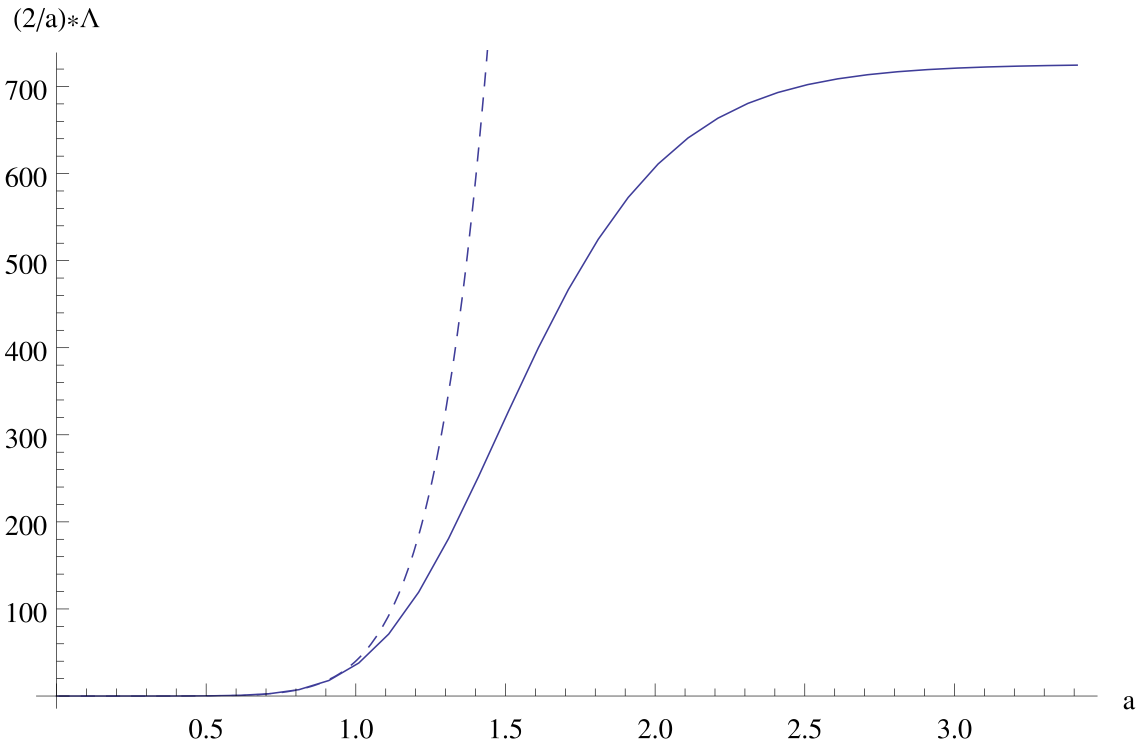


Figure 5.3: A plot of the one-loop cosmological constant for a particular nine-dimensional interpolating model. The model is described in Ref. [15] and the results are shown for $(2a^{-1})\Lambda$ plotted versus $a \equiv \sqrt{\alpha'}R^{-1}$ (solid line). It is evident that $(2a^{-1})\Lambda$ indeed interpolates between $\Lambda = 0$ at $R \to \infty$ and $\Lambda \approx 725$ as $R \to 0$, where $\Lambda \approx 725$ is the cosmological constant associated with the ten-dimensional $SO(16) \otimes SO(16)$ heterotic string. The dashed line shows the behaviour of the cosmological constant for large and finite R, which is governed by the term $\Lambda \sim \frac{6144}{\pi^5}a^9$. The figure is adapted from Ref. [15].

pactification, the contributions from the lightest states and proto-graviton states are also exponentially suppressed! The only states that yield a very large contribution and are therefore responsible for a non-vanishing cosmological constant, are the physical massless states. Their general contribution takes the form

$$\Lambda \sim (N_b^{(0)} - N_f^{(0)})\,\frac{1}{r^{D-1}} + \dots\,. \tag{5.4.18}$$

In a more general setup, where a D-dimensional supersymmetric theory is compactified down to d dimensions one would find

$$\Lambda_d \sim (N_b^{(0)} - N_f^{(0)})\,\frac{1}{r^{d}} + \dots\,. \tag{5.4.19}$$

For a compactification on a twisted two-torus $S^1 \otimes S^1$, where each circle S^1 has its own radius $R_i \equiv \frac{r_i}{\sqrt{\alpha'}}$ and is subjected to its own SUSY breaking $\mathbb{Z}_2$ orbifold twist, the one-loop cosmological constant of the interpolating model in $(D-2)$ spacetime dimensions is given by

$$\Lambda \sim \frac{4\, r_1\, r_2}{\pi^{D/2}} \left(\frac{D}{2} - 1\right)! \left[(N_b^{(0)} - N_f^{(0)}) \frac{1}{{r_1}^D} + (\tilde{N}_b^{(0)} - \tilde{N}_f^{(0)}) \frac{1}{{r_2}^D} \right] + \ldots \quad \text{as} \quad r_1, r_2 \to \infty\,. \tag{5.4.20}$$

Here $N_b^{(0)} - N_f^{(0)}$ and $\tilde{N}_b^{(0)} - \tilde{N}_f^{(0)}$ denote the net numbers of physical massless states which are invariant under the first and second twists, respectively. Note that the leading factor $\sim (a_1 a_2)^{-1}$ is the volume factor for the two-torus, which according to the discussion in Section 4.2.2 of Chapter 4 is expected to appear in this expression. Moreover, taking either r_1 or r_2 to be infinite then one would reproduce the D-dimensional supersymmetric theory.

A non-vanishing cosmological constant implies that the theories emerging from interpolation admit large dilaton tadpoles and are therefore unstable. Fortunately, there is a way to remedy this shortcoming; *if the number of massless bosonic states equals the number of massless fermionic states then the leading term will cancel*. In such case, the contributions from the subleading terms are of critical importance because they yield an exponentially suppressed and hence almost vanishing one-loop cosmological constant. It should be stressed that $N_b^{(0)} = N_f^{(0)}$ is a statement which is applied *only* for the massless states. By having an equal number of massless bosonic and fermionic d.o.f neither means that the massless spectrum is supersymmetric nor implies that all these d.o.f exist in a visible sector. Indeed, some of the massless d.o.f may carry quantum numbers that correspond to visible-sector states with $N_b^{(0)} \neq N_f^{(0)}$, provided that other massless d.o.f may carry quantum numbers that correspond to hidden-sector states so that all states fill out the matching condition $N_b^{(0)} = N_f^{(0)}$.

Chapter 6

General outline for constructing stable, semi-realistic interpolating string models

A ship in port is safe, but that is not what ships are for. Sail out to sea and do new things.

Grace Hopper

6.1 Recipe for the construction technique

At this point, the narrative of this thesis turns to the construction of phenomenologically viable semi-realistic interpolating models which incorporate all the features described in the preceding chapters. Ultimately, the goal is to find a non-supersymmetric SM-like theory that has an equal number of massless bosonic ($N_b^{(0)}$) and massless fermionic ($N_f^{(0)}$) d.o.f and hence an exponentially suppressed cosmological constant. In this context, recall that demanding $N_b^{(0)} = N_f^{(0)}$ does *not* imply that the theory maintains spacetime SUSY. It is not an absolute requirement that all of these massless string states come from the visible sector of the theory. Due to this outcome, there is a perfectly valid reason to expect that the observed low-energy world - for which there are obviously unequal numbers of bosonic and fermionic states - emerges from this class of non-supersymmetric tachyon-free theories exhibiting $N_b^{(0)} = N_f^{(0)}$ without demanding any additional visible states.

Models with $N_b^{(0)} = N_f^{(0)}$ are not easy to construct; an even more difficult task is to ensure that they simultaneously exhibit low-energy spectra resembling either the SM or one of its numerous extensions. As discussed in Chapter 5, one aspect of this class of models is that they have one or more adjustable radii (R_i); moreover, it is required that at $R_i \to \infty$ limits the models have SUSY restored, with vanishing cosmological constant. To accomplish this, it is reasonable to start with a supersymmetric semi-realistic model in higher dimensions, and then compactify on some sort of twisted manifold. This procedure will introduce the needed radii, thereby ensuring that the corresponding cosmological constant vanishes when the radii are taken to be infinite.

As a result, the construction of such models in a four-dimensional spacetime follows a rather deliberate, step-by-step approach which involves starting in six dimensions and then performing a so-called "CDC" down to four dimensions. CDCs are generalisations of ordinary Scherk-Schwarz compactifications [57] which were introduced and developed in Refs. [48, 50–52, 80]. The best course of action is to begin with six-dimensional M_1 models that have $\mathcal{N} = 1$ SUSY. Such models serve as a starting point for the following analysis and are most conveniently obtained by lifting already existing semi-realistic four-dimensional $\mathcal{N} = 1$ string models, for example those in Refs. [143–150], into six dimensions. A direct implication of this procedure is that the newly derived six-dimensional theories are already endowed with many desirable phenomenological features of their semi-realistic four-dimensional 'parent' theories.

Once suitable M_1 models are successfully constructed, the next step is to compactify two spacetime dimensions so as to yield a four-dimensional theory. There are actually various methods of compactification; one being on a two-torus resulting in a *non-chiral* four-dimensional $\mathcal{N} = 2$ theory, another being on a $\mathbb{Z}_2$ orbifold resulting in a *chiral* $\mathcal{N} = 1$ four-dimensional theory which under certain choices for GSO projections can produce an MSSM-like theory. Finally, one can perform a CDC on a $\mathbb{Z}_2$ orbifold using the techniques of Refs. [50–52], thereby obtaining a four-dimensional $\mathcal{N} = 0$ theory. Again, under certain choices for GSO projections one can produce a SM-like theory. In particular, it is useful to compare the partition function of the four-dimensional model that results from the compactification on a $\mathbb{Z}_2$ orbifold with the partition function of the four-dimensional $\mathcal{N} = 0$ model that results from a CDC on the *same* orbifold. This comparison will demonstrate directly how the breaking of SUSY manifests itself. Be aware that the four-dimensional models obtained from compactifications are expected to have different spin structures from the semi-realistic four-dimensional 'parent' model which serves as the construction basis of the six-dimensional theory.

The final step is to use the resulting $\mathcal{N} = 0$ model and introduce modifications so as to obtain $N_b^{(0)} = N_f^{(0)}$, as required to produce an exponentially suppressed cosmological constant. One way to get SM-like theories with $N_b^{(0)} = N_f^{(0)}$ is to alter the final CDC twist but retain the prior GSO symmetry breaking. However, if one alters the final CDC twist and also removes prior GSO projections then a number of different $N_b^{(0)} = N_f^{(0)}$ models can be obtained, *e.g.* a Pati-Salam-like model, a flipped-$SU(5)$ "unified" model, and an $SO(10)$ "unified" model. These examples are only a small number of possible models that one could construct using interpolation and gives a strong indication that the interpolation class is still in its infancy; in a sense these examples could be considered as drops of water in a vast ocean that still remains unexplored. A schematic illustration of the construction is depicted in Fig. 6.1.

Throughout this study, the formalism employed is the free-fermionic formalism of Refs. [132–135], which was described in Section 4.1 of Chapter 4, and the notation used is that of Refs. [134, 135]. All the d.o.f required for the cancellation of conformal anomalies are provided by *real* two-dimensional worldsheet fermionic fields which are *complexified*, *i.e.* in this context *two real* free fermions pair up to make a single complex two-dimensional Weyl-Majorana spinor. This technicality is essential because it provides an explicit way to understand the CDC mechanism and its impact on breaking SUSY. Due to this particular technicality though, it has proven to be extremely difficult to construct a consistent five-dimensional theory which is modular invariant and phenomenologically viable. The origin of

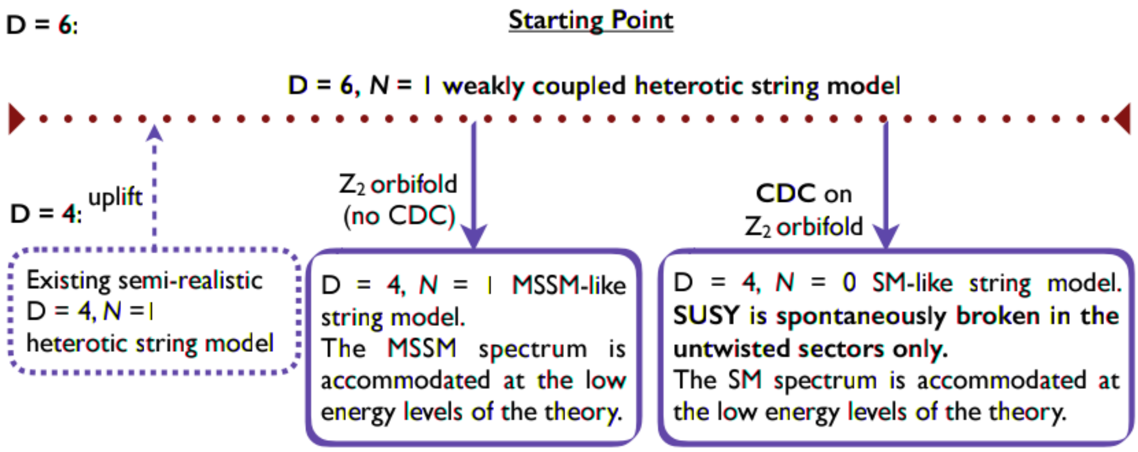


Figure 6.1: A schematic illustration of the procedure for constructing semi-realistic non-supersymmetric string models with $N_b^{(0)} = N_f^{(0)}$ in four dimensions, as discussed in the text.

this hurdle lies with the right-moving worldsheet free fermions. Recall that these are the internal free fermions obtained through fermionization and the spacetime d.o.f which define the spacetime spin-statistics of the theory. Recall also that in D-dimensional string theories, there are $D-2$ real spacetime d.o.f and hence the worldsheet fields that define the spacetime spin-statistics reside in the representations of the $SO(D-2)$ internal group. This means that in five dimensions there are three real worldsheet free fermions that define the spin-statistics and in turn, this implies that they cannot be complexified properly. One of these spacetime free fermionic d.o.f will always be left out to pair up with an internal free fermionic d.o.f. Such a pairing turns out to be catastrophic for the modular invariance, and thereby the consistency of the theory. Luckily, such an inconvenience is bypassed in six-dimensions because the number of real spacetime free fermionic d.o.f is four, thus yielding two complex free fermionic d.o.f that govern the spacetime spin-statistics of the theory. Therefore, it is advantageous to interpolate between M_1 and M_2 models in *six* dimensions rather than five. Moreover, the orbifold compactification from six to four dimensions can be treated using the "unified" formalism of Ref. [151], which is a generalisation of the work in Ref. [134, 135]. In particular, as shall be discussed, only the untwisted sectors feel the CDC, hence the presence of the orbifold does not change the physics of SUSY breaking.

6.2 A supersymmetric theory in six dimensions

Having determined the route that must be followed in the construction process, all is set for embarking upon this journey. The starting point is the construction of the $\mathcal{N} = 1$, $6D$ theory. Following the discussion in Section 4.1 of Chapter 4, each two-dimensional free fermion is assigned a set of boundary conditions which preserve modular invariance and SUSY (both worldsheet and spacetime). To insure the cancellation of conformal anomalies, it is required to have 40 left- and 16 right-moving *real* free fermions which are complexified such that there are 20 left- and 8 right-moving *complex* free fermionic d.o.f. Thus, the spin structure of the model is summarised by the set of basis vectors in Table 6.1. Along with these vectors is a matrix k_{ij} which specifies the phases involved in the corresponding GSO projections:

$$k_{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 0 & 0 & 0 & \frac{3}{4} \end{pmatrix}. \tag{6.2.1}$$

Note that different choices of k_{ij} may break SUSY by discrete torsion. This scenario is examined in Appendix E.

The vectors $\{V_0, V_1, V_2\}$ correspond to the so-called NAHE [143] vectors $\{\mathbf{1}, \mathbf{S}, \mathbf{b}_1\}$, lifted into six dimensions. In contrast, the additional vectors V_5, V_6, and V_7 are inspired by the four-dimensional MSSM-like theories listed in the Appendix of Ref. [150], which in turn are based on the earlier models of Refs. [144–148]. The vector V_1 is the SUSY generator of the model (with the superpartners of the states in sector $\overline{\alpha V}$ residing in $\overline{V_1 + \alpha V}$). The internal right-moving fields χ^{34}, χ^{56} carry the supersymmetric charges. Of course V_1 also projects out the tachyonic states from the spectrum thanks to the generalised GSO projections in Eq. (4.1.8). By themselves, the vectors V_0, V_1 generate an $\mathcal{N} = 4$ theory which is broken to $\mathcal{N} = 2$ by V_2. The **NS-NS** sector of the theory ($\overline{\alpha V} = \mathbf{0}$) gives rise to not only the gravity

Sector	ψ^{34}	ψ^{56}	χ^{34}	y^{34}	ω^{34}	χ^{56}	y^{56}	ω^{56}	$\bar{y}^{34}$	$\bar{\omega}^{34}$	$\bar{y}^{56}$	$\bar{\omega}^{56}$	$\bar{\psi}^1$	$\bar{\psi}^2$	$\bar{\psi}^3$	$\bar{\psi}^4$	$\bar{\psi}^5$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^1$	$\bar{\phi}^2$	$\bar{\phi}^3$	$\bar{\phi}^4$	$\bar{\phi}^5$	$\bar{\phi}^6$	$\bar{\phi}^7$	$\bar{\phi}^8$
V_0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_1	1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
V_2	1	1	0	1	0	0	1	0	1	0	1	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
V_5	0	0	0	0	0	0	1	1	0	1	0	0	1	1	1	0	0	0	0	0	1	1	1	1	0	0	1	1
V_6	0	0	0	0	0	0	0	0	1	1	0	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	0
V_7	0	0	0	1	1	0	0	0	1	0	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1	$\frac{1}{2}$

Table 6.1: Spin structure of the $\mathcal{N} = 1$, $6D$ model, where all entries are understood to be multiplied by a factor of $-\frac{1}{2}$. Thus the '1' entries denote **R** ground states, while '0' entries are **NS** and '$\frac{1}{2}$' entries denote phases of $-\frac{1}{4}$ for the corresponding complexified fermions. These conventions apply to all subsequent tables in which explicit spin structures are listed. As has become standard practice in string theory, the spacetime states listed on the top left are the right-moving fermions while those on the right are left-moving fermions.

multiplet but also to the massless scalar states required to build $\mathcal{N} = 2$ gauge multiplets, as well as hypermultiplets. The sector V_2 produces the sets of fermions in the spinorial representations of the parent $SO(16) \supset SO(10)$ 'visible sector' gauge group defined by the internal left-moving complex fermions $\bar{\psi}^1, \ldots, \bar{\psi}^5$. The superpartners arise in V_1 and $\overline{V_1 + V_2}$ sectors accordingly. Introducing $V_{5,6,7}$ breaks the gauge group to

$$[SU(3)]^2 \otimes [SU(2)]^2 \otimes [SO(4)] \otimes [U(1)]^8 . \tag{6.2.2}$$

The additional vectors do not overlap with V_1, and therefore spacetime SUSY is not broken further at this stage. However, these vectors are needed to break the horizontal symmetries embedded in the gauge group. These horizontal symmetries arise from fermions that are not complexified with a phase of $\frac{1}{2}$ in the V_7 sector. Since the horizontal symmetries are generation-dependent, their breaking reduces the number of *net* generations of matter fields obtained from the V_2 sector.

6.3 Orbifold compactification

Since an $\mathcal{N} = 1$, $6D$ theory is now specified, the next step in the model-construction procedure is to compactify this model to four dimensions, by applying one of the methods proposed in the previous section. If one compactifies the two extra dimensions on a T_2 torus, the $\mathcal{N} = 1$, $6D$ theory is converted into a $\mathcal{N} = 2$, $4D$ theory.

This toroidal compactification immediately signals two problems: The first one is that there is an extended SUSY that needs to be broken and the second one is that phenomenologically this theory is not viable as it is not chiral. Of course, these problems are solved if one compactifies the theory on a $\mathbb{Z}_2$ orbifold so as to get an $\mathcal{N} = 1$, $4D$ model.

Such compactifications are obtained from toroidal compactifications by identifying *fixed points* under some discrete subgroup of the internal rotations [127]. These points are basically transformations on the smooth toroidal manifold that remain as an *exact symmetry* of the compactified theory; this is the reason for the characterisation 'fixed'. Even though there are singularities associated with the fixed points, the consistency of the string theory remains unaffected due to modular invariance. This is achieved by requiring the presence of a new sector, defined as *twisted*, which corresponds to string states that are localised at the fixed points of the orbifold. The internal rotations of the two-dimensional compact space form an $SO(2) \equiv U(1)$ group and the condition for unbroken SUSY requires that the T_2 torus has a discrete subgroup of U_1 that leaves one of the two gravitinos invariant. Thus, the $\mathbb{Z}_2$ orbifold reduces the number of supersymmetries by half, and inverts the sign of the two internal coordinates $X^i \to -X^i$, $(i = 1, 2)$ and of their superpartners [127]. As a result, $\mathbb{Z}_2$ acts non-trivially on the left-moving free fermions which are essentially the gauge d.o.f and in addition to SUSY it can also break the gauge symmetries.

6.3.1 A chiral supersymmetric theory in four dimensions

It is common practice to translate the possible action of the orbifold on the worldsheet d.o.f by introducing *twists*. In mathematical terms, a twist is a collection of the phases that the free fermions acquire under a specific orbifold action. In the present case, there are actually two twists introduced, which correspond to the vectors b_3 and b_4. The resulting model then has the spin structure summarised in Table 6.2. Note that in writing the four-dimensional model in terms of a six-dimensional spin structure, it should be implicitly recognised that the remaining two dimensions are not fermionized: they are instead retained as bosons compact-

Sector	ψ^{34}	ψ^{56}	χ^{34}	y^{34}	ω^{34}	χ^{56}	y^{56}	ω^{56}	$\overline{y}^{34}$	$\overline{\omega}^{34}$	$\overline{y}^{56}$	$\overline{\omega}^{56}$	$\overline{\psi}^1$	$\overline{\psi}^2$	$\overline{\psi}^3$	$\overline{\psi}^4$	$\overline{\psi}^5$	$\overline{\eta}^1$	$\overline{\eta}^2$	$\overline{\eta}^3$	$\overline{\phi}^1$	$\overline{\phi}^2$	$\overline{\phi}^3$	$\overline{\phi}^4$	$\overline{\phi}^5$	$\overline{\phi}^6$	$\overline{\phi}^7$	$\overline{\phi}^8$
V_0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_1	1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
V_2	1	1	0	1	0	0	1	0	1	0	1	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
b_3	1	0	1	0	0	0	0	1	0	0	0	1	1	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0
b_4	1	0	0	0	1	1	0	0	0	1	0	0	1	1	1	1	1	0	0	1	0	0	0	0	0	0	0	0
V_5	0	0	0	0	0	0	1	1	0	1	0	0	1	1	1	0	0	0	0	0	1	1	1	1	0	0	1	1
V_6	0	0	0	0	0	0	0	0	1	1	0	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	0
V_7	0	0	0	1	1	0	0	0	1	0	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1	$\frac{1}{2}$

Table 6.2: Spin structure of the worldsheet fermions of the $\mathcal{N} = 1$, $4D$ model before applying the CDC. This spin structure is accompanied by two bosonic d.o.f compactified on a $\mathbb{Z}_2$ orbifold with twist action corresponding to the vectors $b_{3,4}$.

ified on a twisted two-torus with arbitrary radii, and thus their treatment is beyond the scope of the free fermionic formalism. However, as discussed in the Appendix of Ref. [41], there is an alternative way of specifying the resulting four dimensional string model. This permits a description of the resulting model in terms of a four-dimensional spin structure, and the free fermionic radii are required to be $R_i = \sqrt{2\alpha'}$. This model can then be extended back to arbitrary radii following the procedure outlined in Ref. [41].

In the remainder of the discussion of this model, the terms "twisted" and "untwisted" will be used to refer to the two compactified dimensions. The $\mathbb{Z}_2$ projection is

$$\widehat{g}\phi = \begin{cases} \phi\,\widehat{g} & \text{for } \phi \notin b_3 \text{ or } b_4 \\ -\phi\,\widehat{g} & \text{for } \phi \in b_3 \text{ or } b_4\,, \end{cases}$$

where $\widehat{g}$ is a generator of the $\mathbb{Z}_2$ orbifold. Furthermore, this particular choice of b_3 and b_4 is consistent with global invariance of the worldsheet supercurrent as defined in Eq. (4.1.11):

$$\widehat{g}\,T_F(z) = -T_F(z)\,\widehat{g}\,. \tag{6.3.1}$$

As mentioned above, the *right-moving* free fermionic d.o.f define the spacetime spin-statistics of the theory. In addition, they determine the spacetime supersymmetries that remain unbroken. Therefore, the b_3 and b_4 right-moving entries are assigned so as to break the extended spacetime supersymmetries after compactication, leaving only an $\mathcal{N}=1$ SUSY. As the vector

$$\begin{aligned} V_4 &= \overline{b_3 + b_4} \\ &= -\tfrac{1}{2}[\,0\,0\,1\,0\,1\,1\,0\,1\,|\,0\,1\,0\,1\,0\,0\,0\,0\,0\,0\,1\,1\,0\,0\,0\,0\,0\,0\,0\,0\,] \end{aligned} \tag{6.3.2}$$

itself provides an additional untwisted sector, an entirely equivalent route is to start with an $\mathcal{N}=1$, $6D$ fermionic theory that has V_4 as an additional vector, and from there to compactify on the $\mathbb{Z}_2$ orbifold with a single twist action (b_3, for example).

In the $\{V_0, V_1, V_2, b_3, V_4, V_5, V_6, V_7\}$ basis, the structure constants that define the generalised GSO projection phases are as follows:

$$k_{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{3}{4} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{4} \end{pmatrix}. \tag{6.3.3}$$

Considerable care has to be taken when assigning these constants. As described in Appendix E, an incorrect choice could amount to breaking SUSY explicitly or, conversely, disallow SSSB in the case of CDC. This scenario shall be examined in Section 7.3 of Chapter 7.

The *left-moving* free fermionic d.o.f are the ones which govern the gauge structure of the theory. Therefore, the b_3 and b_4 left-moving entries are assigned so as to initially break the $SO(16)$ gauge group to $SO(10) \otimes SO(6)$, ensuring that the matter fields carry the correct SM charges once V_5, V_6, and V_7 are added. The final gauge group of the theory is then found to be

$$SU(3) \otimes SU(2) \otimes U(1)_Y \otimes G'_{semi-hidden} \otimes G'_{hidden}\,, \tag{6.3.4}$$

where the convention for the weak hypercharge is given by

$$-\frac{1}{2}U(1)_Y \equiv \frac{1}{3}\left[U(1)_{\bar{\psi}^1} + U(1)_{\bar{\psi}^2} + U(1)_{\bar{\psi}^3}\right] + \frac{1}{2}\left[U(1)_{\bar{\psi}^4} + U(1)_{\bar{\psi}^5}\right] . \tag{6.3.5}$$

Note that string states which are charged under the SM gauge group are derived directly from the visible sector of the theory. However, there are additional string states that are charged both under the SM gauge group as well as other gauge groups. These string states are called semi-hidden and their associated gauge groups form the $G'_{semi-hidden}$. Moreover, there are string states that remain completely uncharged under the remaining gauge groups. These states are called hidden as they are derived directly from the hidden sector of the theory, hence the gauge groups that do not admit any charged string states form the G'_{hidden}.

6.3.2 Mass spectrum of the supersymmetric four-dimensional theory

This $\mathcal{N} = 1$, $4D$ model obviously has string states that are charged under the SM gauge group. For phenomenological purposes, and in order to verify that indeed the emerging theory at the low-energy limit resembles the MSSM, it is wise to consider the massless spectrum of this theory. At the level of an unbroken SUSY (and hence an unbroken electroweak symmetry), all the MSSM states are massless. Therefore, it should be natural for these states to be accommodated in the massless spectrum of this $\mathcal{N} = 1$, $4D$ theory.

The massless spectrum is obtained by applying the generalised GSO projections in Eq. (4.1.8) but one must also take into account the additional effects of the orbifolding, which can be found in Ref. [151]. Alternatively, they can be deduced from the form of the partition function presented in Eq. (6.3.15). One first evaluates the would-be projections on the states, *i.e.* one evaluates the projection coefficients C^{α}_{β} with b_3 and b_4 obeying the same rules as for the other basis vectors. Under the orbifolding, the oscillators in a given state may be odd, and thus contribute additional phase shifts that must be included in the projections. This can be deduced from a generic state with total winding number (in untwisted sectors)

$\jmath = \jmath_1 + \jmath_2$ and KK number $\ell = \ell_1 + \ell_2$ which transforms under the orbifold (with action $\beta \cdot V \equiv b_3, b_4$) as

$$
\begin{aligned}
& \prod_{c=1} X^{(a_c)}_{-\jmath_c} \prod_{e=1} \Psi^{(b_e)}_{-\ell_e} \prod_{d=1} \tilde{X}^{(a_d)}_{-\jmath_d} \prod_{f=1} \tilde{\Psi}^{(b_f)}_{-\ell_f} |\ell, \jmath\rangle \\
\longrightarrow (-1)^A \quad & \prod_{c=1} X^{(a_c)}_{-\jmath_c} \prod_{e=1} \Psi^{(b_e)}_{-\ell_e} \prod_{d=1} \tilde{X}^{(a_d)}_{-\jmath_d} \prod_{f=1} \tilde{\Psi}^{(b_f)}_{-\ell_f} |-\ell, -\jmath\rangle \, . \qquad (6.3.6)
\end{aligned}
$$

The overall phase A is calculated by introducing an overall minus sign for each excitation in the X_5 or X_6 direction and maintaining the other GSO phases. For the untwisted states, invariance under the orbifold action is equivalent to shifting the GSO projections of b_3, b_4 by the *additional* phase coming from the compact bosons, $\frac{1}{2}\sum_{a_c, a_d \in 5,6}$ (since $\frac{1}{2}\sum_{b_e, b_f \in b_{3,4}}$ is already included). For states with non-zero $\jmath$ or ℓ, one then has opposite projections for the even/odd wave-functions, so the remaining invariant combination is $\frac{1}{\sqrt{2}}\left(|\jmath, \ell\rangle + (-1)^A|-\jmath, -\ell\rangle\right)$, while any *zero-modes* that are odd under the orbifolding are projected out in the usual way. To understand this procedure it is better to discuss contributions from the untwisted and the twisted b_3 and b_4 sectors in turn.

• **Untwisted sectors without** $V_4 = \overline{b_3 + b_4}$: Here none of the projections are altered by the orbifold action on the bosonic oscillators except for that of the radion, and the vacuum energies obey

$$
E_{L,R} = \frac{1}{2}\sum_{l}\left[(\overline{\alpha V^l})^2 - \frac{1}{12}\right] - \frac{(D-2)}{24} - \frac{1}{12}\, . \qquad (6.3.7)
$$

Here $D = 4$, the sum is over complex fermions, and the factor of $-\frac{1}{12}$ accounts for the *two real compactified* bosons. The **NS-NS** sector yields the massless bosons for the gauge and gravity sector including the complex radion for dimensions (5,6). In addition, it yields three pairs of complex Higgs scalars and three pairs of singlet scalar states. The V_1 sector generates their superpartners. The resulting observable content from these sectors is summarised in Tables 6.3 and 6.4.

An attractive feature of this model is that there are no massless Higgs triplets, and the only massless visible-sector scalars transform in the $\left(\mathbf{2}, \pm\frac{1}{2}\right)$ representation of $SU(2) \otimes U(1)_Y$. Specifically the three pairs of Higgs doublets H_{U_i} and H_{D_i} that

Sector	States remaining after CDC	Spin	Particles
$\mathbf{0}$	$\psi^{34}_{-\frac{1}{2}}\,\|0\rangle_R \otimes X^{34}_{-1}\,\|0\rangle_L$	2 0	Graviton $g_{\mu\nu}$, Antisymmetric tensor $B_{[\mu\nu]}$, Dilaton ϕ
	$\psi^{56}_{-\frac{1}{2}}\,\|0\rangle_R \otimes X^{56}_{-1}\,\|0\rangle_L$	0	Complex radion Φ
	$\psi^{34}_{-\frac{1}{2}}\,\|0\rangle_R \otimes \Psi^i_{-\frac{1}{2}}\Psi^j_{-\frac{1}{2}}\,\|0\rangle_L$	1	Gauge bosons V_μ
	$\psi^{56}_{-\frac{1}{2}}\,\|0\rangle_R \otimes \Psi^i_{-\frac{1}{2}}\Psi^j_{-\frac{1}{2}}\,\|0\rangle_L$	0	Complex scalars H_{U_1}, H_{D_1}, Ξ_1, Ξ'_1
V_1	$\|\alpha\rangle_R \otimes \Psi^i_{-\frac{1}{2}}\Psi^j_{-\frac{1}{2}}\,\|0\rangle_L$	$\frac{1}{2}$	Weyl spinors $\tilde{H}_{U_2}$, $\tilde{H}_{D_2}$, $\tilde{\Xi}_2$, $\tilde{\Xi}'_2$
	$\|\alpha\rangle_R \otimes \Psi^i_{-\frac{1}{2}}\Psi^j_{-\frac{1}{2}}\,\|0\rangle_L$	$\frac{1}{2}$	Weyl spinors $\tilde{H}_{U_3}$, $\tilde{H}_{D_3}$, $\tilde{\Xi}_3$, $\tilde{\Xi}'_3$

Table 6.3: The $\mathbb{Z}_2$-untwisted visible-sector states of the $\mathcal{N}=1$, $4D$ model. All of these states will remain massless after the CDC is imposed. The Ψ_i refer to generic left-moving d.o.f, with indices $i,j=1,\ldots,20$.

Sector	States projected after CDC	Spin	Particles
V_1	$\|\alpha\rangle_R \otimes X^{34}_{-1}\,\|0\rangle_L$	$\frac{3}{2}$ $\frac{1}{2}$	Gravitino ψ_μ, Dilatino $\widetilde{\phi}$
	$\|\alpha\rangle_R \otimes X^{56}_{-1}\,\|0\rangle_L$	$\frac{1}{2}$	Radino $\widetilde{\Phi}$
	$\|\alpha\rangle_R \otimes \Psi^i_{-\frac{1}{2}}\Psi^j_{-\frac{1}{2}}\,\|0\rangle_L$	$\frac{1}{2}$	Gauginos λ_μ
	$\|\alpha\rangle_R \otimes \Psi^i_{-\frac{1}{2}}\Psi^j_{-\frac{1}{2}}\,\|0\rangle_L$	$\frac{1}{2}$	Weyl spinors $\tilde{H}_{U_1}$, $\tilde{H}_{D_1}$, $\tilde{\Xi}_1$, $\tilde{\Xi}'_1$
$\mathbf{0}$	$\chi^{34}_{-\frac{1}{2}}\,\|0\rangle_R \otimes \Psi^i_{-\frac{1}{2}}\Psi^j_{-\frac{1}{2}}\,\|0\rangle_L$	0	Complex scalars H_{U_2}, H_{D_2}, Ξ_2, Ξ'_2
	$\chi^{56}_{-\frac{1}{2}}\,\|0\rangle_R \otimes \Psi^i_{-\frac{1}{2}}\Psi^j_{-\frac{1}{2}}\,\|0\rangle_L$	0	Complex scalars H_{U_3}, H_{D_3}, Ξ_3, Ξ'_3

Table 6.4: Additional $\mathbb{Z}_2$-untwisted visible-sector states of the $\mathcal{N}=1$, $4D$ model. By contrast, these states will no longer remain in the massless spectrum after the CDC is imposed, and will instead obtain masses $\frac{1}{2}\sqrt{R_1^{-2}+R_2^{-2}}$.

survive the GSO and orbifold projections are

$$\begin{aligned}
[H_{U_1}]_{1,0,0,0,0,0,0,0}\,,\ \ [H_{D_1}]_{-1,0,0,0,0,0,0,0} &= \psi^{56}_{-\frac{1}{2}}\,|0\rangle_R \otimes \overline{\psi}^{4,5}_{-\frac{1}{2}}\,\overline{\eta}^{1}_{-\frac{1}{2}}\,|0\rangle_L \\
[H_{U_2}]_{0,1,0,0,0,0,0,0}\,,\ \ [H_{D_2}]_{0,-1,0,0,0,0,0,0} &= \chi^{34}_{-\frac{1}{2}}\,|0\rangle_R \otimes \overline{\psi}^{4,5}_{-\frac{1}{2}}\,\overline{\eta}^{2}_{-\frac{1}{2}}\,|0\rangle_L \\
[H_{U_3}]_{0,0,1,0,0,0,0,0}\,,\ \ [H_{D_3}]_{0,0,-1,0,0,0,0,0} &= \chi^{56}_{-\frac{1}{2}}\,|0\rangle_R \otimes \overline{\psi}^{4,5}_{-\frac{1}{2}}\,\overline{\eta}^{3}_{-\frac{1}{2}}\,|0\rangle_L\,.
\end{aligned} \tag{6.3.8}$$

In addition to these, the three pairs of singlet scalar states Ξ_i and Ξ'_i that survive the GSO and orbifold projections are given by

$$
\begin{aligned}
[\Xi_1]_{0,1,-1,0,0,0,0}\,,\ \ [\Xi_1']_{0,-1,1,0,0,0,0} &= \psi^{56}_{-\frac{1}{2}}\,|0\rangle_R \otimes \overline{\eta}^{2}_{-\frac{1}{2}}\,\overline{\eta}^{3}_{-\frac{1}{2}}\,|0\rangle_L \\
[\Xi_2]_{1,0,-1,0,0,0,0}\,,\ \ [\Xi_2']_{-1,0,1,0,0,0,0} &= \chi^{34}_{-\frac{1}{2}}\,|0\rangle_R \otimes \overline{\eta}^{1}_{-\frac{1}{2}}\,\overline{\eta}^{3}_{-\frac{1}{2}}\,|0\rangle_L \\
[\Xi_3]_{1,-1,0,0,0,0,0}\,,\ \ [\Xi_3']_{-1,1,0,0,0,0,0} &= \chi^{56}_{-\frac{1}{2}}\,|0\rangle_R \otimes \overline{\eta}^{1}_{-\frac{1}{2}}\,\overline{\eta}^{2}_{-\frac{1}{2}}\,|0\rangle_L\,.
\end{aligned}
\tag{6.3.9}
$$

The generalised GSO projections pick out the relevant components, *i.e.* $b^{4,5}_{-\frac{1}{2}}$ or $d^{4,5}_{-\frac{1}{2}}$ (in the notation of Ref. [135]) of the $\overline{\psi}^{4,5}_{-\frac{1}{2}}$ operators for the electroweak doublets, and of the $\overline{\eta}^{1,2,3}$ for the singlet states. The presence of these scalar doublets and their superpartners in the massless spectrum is correlated with the existence of $U(1)$ horizontal symmetries embedded in the larger broken gauge group. The subscripts on the states above are the charges under these horizontal $U(1)$ symmetries; more details on these are given in Section 9.2.2 of Chapter 9.

At the level of four-dimensional theories, the V_2 sector gives rise to 16 generations of massless fermionic states, which transform in the **16** of the $SO(10)$ with scalar superpartners in the $\overline{V_2 + V_1}$ sector. However, half of the generations are projected out by the $\mathbb{Z}_2$ twists, while V_5, V_6, and V_7 overlap non-trivially with V_2, b_3, and b_4, constraining the total number of generations to two from the V_2 sector. The states derived from the V_2 sector are listed in Table 6.5 and their superpartners in Table 6.6; they are the usual decomposition of a **16** representation of $SO(10)$ under $SU(3) \otimes SU(2) \otimes U(1)_Y$. At this point, it should be noted that it is possible to use another choice of the V_5, V_6, and V_7 vectors, along with real instead of complexified free fermions, in order to further break the horizontal symmetries and get one generation of matter fields from the V_2 sector. However, the primary goal is to construct a viable phenomenological SM-like theory with an exponentially suppressed cosmological constant, so the number of chiral generations is not of critical importance.

• **Untwisted sectors with** $V_4 = \overline{b_3 + b_4}$: As mentioned above, this combination of the two orbifold twist actions is effectively just another untwisted sector. Checking the entirety of all sectors containing the combination $\overline{b_3 + b_4}$, it is found that

Sector	States remaining after CDC	Spin	Particles
V_2	$\lvert\alpha\rangle_R \otimes \lvert\hat{\alpha}\rangle_L$	$\frac{1}{2}$	e_R
	$\lvert\alpha\rangle_R \otimes \overline{\psi}_0^4\overline{\psi}_0^5 \lvert\hat{\alpha}\rangle_L$	$\frac{1}{2}$	ν_R
	$\lvert\alpha\rangle_R \otimes \overline{\psi}_0^i\overline{\psi}_0^a \lvert\hat{\alpha}\rangle_L$	$\frac{1}{2}$	Q_L
	$\lvert\alpha\rangle_R \otimes \overline{\psi}_0^i\overline{\psi}_0^j \lvert\hat{\alpha}\rangle_L$	$\frac{1}{2}$	d_R
	$\lvert\alpha\rangle_R \otimes \overline{\psi}_0^i\overline{\psi}_0^j\overline{\psi}_0^4\overline{\psi}_0^5 \lvert\hat{\alpha}\rangle_L$	$\frac{1}{2}$	u_R
	$\lvert\alpha\rangle_R \otimes \overline{\psi}_0^1\overline{\psi}_0^2\overline{\psi}_0^3\overline{\psi}_0^a \lvert\hat{\alpha}\rangle_L$	$\frac{1}{2}$	L_L

Table 6.5: The $\mathbb{Z}_2$-untwisted chiral multiplets of the $\mathcal{N} = 1$, $4D$ model, where $i, j \in SU(3)$ and $a \in SU(2)$. Just as with the states in Table 6.3, all of these states will remain massless after the CDC is imposed. The $|\alpha\rangle_R$ represent right-moving **R** ground states which are spacetime spinors, while $|\hat{\alpha}\rangle_L$ represents the left-moving **R** excitations that do not overlap with the SM gauge group. The multiplets are essentially the decomposition of the **16** of $SO(10)$. The same decomposition applies for the generations derived from the b_3 and b_4 twisted-sectors. However, the twisted-sector chiral matter fields remain unaffected by the CDC effects and therefore maintain their supersymmetric status.

Sector	States projected after CDC	Spin	Particles
$\overline{V_1+V_2}$	$\lvert\alpha\rangle'_R \otimes \lvert\hat{\alpha}\rangle_L$	0	$\tilde{e}_R$
	$\lvert\alpha\rangle'_R \otimes \overline{\psi}_0^4\overline{\psi}_0^5 \lvert\hat{\alpha}\rangle_L$	0	$\tilde{\nu}_R$
	$\lvert\alpha\rangle'_R \otimes \overline{\psi}_0^i\overline{\psi}_0^a \lvert\hat{\alpha}\rangle_L$	0	$\tilde{Q}_L$
	$\lvert\alpha\rangle'_R \otimes \overline{\psi}_0^i\overline{\psi}_0^j \lvert\hat{\alpha}\rangle_L$	0	$\tilde{d}_R$
	$\lvert\alpha\rangle'_R \otimes \overline{\psi}_0^i\overline{\psi}_0^j\overline{\psi}_0^4\overline{\psi}_0^5 \lvert\hat{\alpha}\rangle_L$	0	$\tilde{u}_R$
	$\lvert\alpha\rangle'_R \otimes \overline{\psi}_0^1\overline{\psi}_0^2\overline{\psi}_0^3\overline{\psi}_0^a \lvert\hat{\alpha}\rangle_L$	0	$\tilde{L}_L$

Table 6.6: The $\mathbb{Z}_2$-untwisted superpartners of the chiral multiplets in the $\mathcal{N} = 1$, $4D$ model where $i, j \in SU(3)$ and $a \in SU(2)$. Just as with the states in Table 6.4, these states will also obtain masses $\frac{1}{2}\sqrt{R_1^{-2} + R_2^{-2}}$ by the CDC. The $|\alpha\rangle'_R$ represent right-moving **R** ground states that are not spacetime spinors.

the only massless states are either singlets or additional Higgs-like doublets, albeit with charges that may prohibit their direct coupling to the matter fields in Yukawa couplings.

- **Twisted sectors**: In the twisted sectors, in addition to the orbifold itself acting on the compact bosonic oscillators, the vacuum energies are given by

$$E_{L,R} = \frac{1}{2}\sum_{l}\left[(\overline{\alpha V^l})^2 - \frac{1}{12}\right] - \frac{(D-2)}{24} + \frac{1}{24}\,, \tag{6.3.10}$$

where now $\frac{1}{24}$ accounts for the twisted complex boson. Similar to the untwisted V_2 sector, the twisted b_3 and b_4 sectors each give rise to additional **16** generations of chiral massless fields, each of which are projected down to a single generation for each fixed point of the $\mathbb{Z}_2$ orbifold. As there are four fixed points, each one of the b_3 and b_4 sectors gives rise to an additional four generations. The states derived from b_3 and b_4 are like those in Tables 6.5 and 6.6. Of course, there are many other linear combinations of twisted sectors $\overline{b_3 + \alpha V}$ and $\overline{b_4 + \alpha V}$ that contribute extra hidden states and singlets to the massless spectrum of the theory. However, these twisted sectors states are ultimately of minor phenomenological importance.

6.3.3 Partition function of the MSSM-like theory

Naturally, the quantum effects and the stability properties of the $\mathcal{N}=1$, $4D$ theory discussed above can be extracted from the partition function. In this case, the construction of the partition function relies on the results and definitions from Appendix D. In the untwisted sector, the modular invariant partition function for the two compact bosonic d.o.f in terms of the KK numbers $\vec{\ell} = \{\ell_1, \ell_2\}$, winding numbers $\vec{\jmath} = \{\jmath_1, \jmath_2\}$, and the radii r_1, r_2 is given by

$$Z_{\mathrm{B}}\begin{bmatrix} 0 \\ 0 \end{bmatrix}(\tau) = \sum_{\vec{\ell},\vec{\jmath}} Z_{\vec{\ell},\vec{\jmath}}; \tag{6.3.11}$$

$$Z_{\vec{\ell},\vec{\jmath}} = \frac{\mathcal{M}^2 r_1 r_2}{\tau_2 \overline{\eta}^2 \eta^2} \sum_{\vec{\ell},\vec{\jmath}} \exp\left\{-\frac{\pi}{\tau_2}\left[r_1^2|\ell_1 - \jmath_1\tau|^2 + r_2^2|\ell_2 - \jmath_2\tau|^2\right]\right\}. \tag{6.3.12}$$

In Eq. (6.3.11), the notation $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ indicates untwisted boundary conditions in both the spacelike and timelike toroidal directions. In contrast, the contribution from the twisted sectors is given by

$$Z_{\mathbf{B}}\left[\begin{array}{c}\overline{(\alpha_3+\alpha_4)/2}\\ \overline{(\beta_3+\beta_4)/2}\end{array}\right](\tau)=2|\eta(\tau)|^2\left|\vartheta\left[\begin{array}{c}1/2-\overline{(\alpha_3+\alpha_4)/2}\\ 1/2-\overline{(\beta_3+\beta_4)/2}\end{array}\right](\tau)\right|^{-2}, \tag{6.3.13}$$

where $\overline{\alpha_3+\alpha_4}$ and $\overline{\beta_3+\beta_4}$ indicate the $\mathbb{Z}_2$ twists on the complex boson as given in Ref. [151]. In Eq. (6.3.13) it is assumed that either $\overline{\alpha_3+\alpha_4}$ or $\overline{\beta_3+\beta_4}$ is odd. The modular transformations of $Z_{\mathbf{B}}$ are as follows:

$$\begin{aligned}
&Z_{\mathbf{B}}\left[\begin{array}{c}1/2\\ 1/2\end{array}\right](\tau+1)=Z_{\mathbf{B}}\left[\begin{array}{c}1/2\\ 0\end{array}\right](\tau), &&Z_{\mathbf{B}}\left[\begin{array}{c}1/2\\ 1/2\end{array}\right]\left(-\frac{1}{\tau}\right)=Z_{\mathbf{B}}\left[\begin{array}{c}1/2\\ 1/2\end{array}\right](\tau)\\
&Z_{\mathbf{B}}\left[\begin{array}{c}1/2\\ 0\end{array}\right](\tau+1)=Z_{\mathbf{B}}\left[\begin{array}{c}1/2\\ 1/2\end{array}\right](\tau), &&Z_{\mathbf{B}}\left[\begin{array}{c}1/2\\ 0\end{array}\right]\left(-\frac{1}{\tau}\right)=Z_{\mathbf{B}}\left[\begin{array}{c}0\\ 1/2\end{array}\right](\tau)\\
&Z_{\mathbf{B}}\left[\begin{array}{c}0\\ 1/2\end{array}\right](\tau+1)=Z_{\mathbf{B}}\left[\begin{array}{c}0\\ 1/2\end{array}\right](\tau), &&Z_{\mathbf{B}}\left[\begin{array}{c}0\\ 1/2\end{array}\right]\left(-\frac{1}{\tau}\right)=Z_{\mathbf{B}}\left[\begin{array}{c}1/2\\ 0\end{array}\right](\tau).
\end{aligned} \tag{6.3.14}$$

Note that the modular transformations of the $Z_{\mathbf{B}}(\tau)$ bosonic factors do not introduce any additional phases. Moreover, the untwisted $Z_{\mathbf{B}}\left[\begin{array}{c}0\\ 0\end{array}\right](\tau)$ partition function is modular invariant by itself; in general its divergent contribution to the total partition function is cancelled by vanishing contributions from worldsheet fermions so that it yields finite results. Putting all the pieces together, the complete one-loop partition function $Z(\tau)$ for the $\mathcal{N}=1$, $4D$ model is found to be

$$\begin{aligned}
Z(\tau)=&\frac{\mathcal{M}^2}{\tau_2|\eta(\tau)\overline{\eta}(\overline{\tau})|^2}\frac{1}{[\eta(\tau)]^8\,[\overline{\eta}(\overline{\tau})]^{20}}\sum_{\{\alpha,\beta\}}C^{\alpha}_{\beta}\,Z_{\mathbf{B}}\left[\begin{array}{c}\overline{(\alpha_3+\alpha_4)/2}\\ \overline{(\beta_3+\beta_4)/2}\end{array}\right](\tau)\\
&\times\prod_{i_R}\vartheta\left[\begin{array}{c}\overline{\alpha\mathbf{V}_i}\\ -\beta\mathbf{V}_i\end{array}\right](\tau)\prod_{i_L}\overline{\vartheta}\left[\begin{array}{c}\overline{\alpha\mathbf{V}_i}\\ -\beta\mathbf{V}_i\end{array}\right](\overline{\tau}).
\end{aligned} \tag{6.3.15}$$

The generalised GSO-projection coefficients are determined according to the conventions in Ref. [135] where the factor of $e^{2\pi i\beta V\cdot\overline{\alpha V}}$ from the partition function in Eq. (D.0.11) is absorbed into the following definition:

$$C^{\alpha}_{\beta}=\exp\left[2\pi i\left(\alpha s+\beta s+\beta_i\,k_{ij}\,\alpha_j\right)\right]. \tag{6.3.16}$$

Chapter 7

Coordinate Dependent Compactification

A scientist is happy, not in resting on his attainments, but in the steady acquisition of fresh knowledge.

Max Planck

7.1 A summary of the general properties of CDC'd models

As already mentioned, the CDC can be viewed as the string analogue of generalised Scherk-Schwarz compactifications from higher dimensions. As such, all the models constructed through CDC obey a specific SUSY breaking mechanism and share certain properties. In particular, the breaking of SUSY in this class of models is *spontaneous*: the models have an identifiable order parameter for the breaking, namely the compactification scale, which is referred to generically as R^{-1} where R in this context is the generic radius of the compact $(5, 6)$ dimensions. Therefore, the CDC lifts the mass of some of the states, including the mass of the gravitino, while the KK modes are split non-supersymmetrically at a scale $\sim R^{-1}$. Likewise, the cosmological constant *generically* goes like R^{-4}.

From this standpoint, the spectrum of the resulting theory is unerringly non-supersymmetric at all energy scales. The winding modes of the theory have masses proportional to R and experience gross shifts, which only increase with R. Indeed, there is no value of large and finite R beyond which the spectrum appears to be effectively supersymmetric. However, for $R \to \infty$ these theories effectively become *higher-dimensional* and spacetime SUSY is restored. Therefore, one should not confuse the theories obtained via CDC with theories which are supersymmetric at high energy scales and are subsequently subjected to a SUSY breaking mechanism at low energy scales.

It does makes sense though, at least to a certain extent, to consider the emerging four-dimensional theory as an effective spontaneously broken supersymmetric field theory, at the lowest orders of perturbation theory. Despite the threshold contribution to the non-supersymmetric EFT from heavy string modes, the theory does not lack UV finiteness so long as it is modular invariant due to the misaligned SUSY, which nevertheless remains in the spectrum of any CDC'd model. The overall outcome, then, is a theory in which the SUSY-breaking terms can be dialed to any value, even to the string scale itself, with non-supersymmetric UV divergences. In this way, CDC allows the parametric deformation of a theory away from one with supersymmetric content towards another one that is entirely non-supersymmetric. This property is on par with the interpolation and is therefore an integral and significant feature of the models constructed in this work.

7.2 A concise description of the method

In general, the CDC is a deformation which is designed to incorporate the *super-Higgs* phenomena of the Scherk-Schwarz procedure but in more general string configurations [48]. From the spacetime perspective, as described in Section 2.2.2 of Chapter 2, the super-Higgs mechanism involves the auxiliary field of some supermultiplet acquiring a non-vanishing VEV while the gravitino becomes massive by absorbing a Goldstino. From the worldsheet perspective however, the occurrence of super-Higgs mechanism requires the deformation of the worldsheet Lagrangian $\mathcal{L}_w$ by a non-Becchi-Rouet-Stora-Tyutin (BRST) invariant operator that must preserve only a *discrete* subgroup of a $U(1)$ symmetry [48]. The super-Higgs effect manifests itself when this discrete symmetry is spontaneously broken and auxiliary fields on the worldsheet develop a VEV.

In practice this means deforming an already existing model by adding a local generator $\mathbf{Q}$ of the parent $U(1)$ worldsheet symmetry, which at least partly involves the R-symmetry (in order to ensure that gravitinos and the graviton have the correct R charges). For the breaking to be spontaneous, the worldsheet supercurrent defined in Eq. (4.1.11) has to be invariant under the discrete symmetry, but it must not commute with the local generator $\mathbf{Q}$:

$$[T_F(z), \mathbf{Q}(z)] \neq 0\,. \tag{7.2.1}$$

Before proceeding to apply CDC to the six-dimensional supersymmetric theory defined by the spin structure in Table 6.1, it is necessary to specify the complexification of the internal right-moving fermions. This is given by

$$\begin{aligned} \chi_c^{(1)} &\equiv \chi_{34} = \frac{1}{\sqrt{2}}(\chi^3 + i\chi^4) & \chi_c^{(2)} &\equiv \chi_{56} = \frac{1}{\sqrt{2}}(\chi^5 + i\chi^6) \\ \omega_c^{(1)} &\equiv \omega_{34} = \frac{1}{\sqrt{2}}(\omega^3 + i\omega^4) & \omega_c^{(2)} &\equiv \omega_{56} = \frac{1}{\sqrt{2}}(\omega^5 + i\omega^6)\,. \end{aligned} \tag{7.2.2}$$

The y fields will be largely irrelevant for this discussion; therefore their complexification is of no importance. This complexification admits two *discrete* worldsheet symmetries, $\mathfrak{I}_1$ and $\mathfrak{I}_2$, which are subgroups of the internal $SO(4)$ group of the compactification from ten to six dimensions. Each of these symmetries is explicitly defined by the eight transformations

$$\begin{aligned} \chi^3 &\to -\chi^3 & \omega^3 &\to -\omega^3 \\ \chi^4 &\to -\chi^4 & \omega^4 &\to -\omega^4 \\ \chi^5 &\to -\epsilon\chi^5 & \omega^5 &\to -\epsilon\omega^5 \\ \chi^6 &\to -\epsilon\chi^6 & \omega^6 &\to -\epsilon\omega^6\,, \end{aligned} \tag{7.2.3}$$

where the y fields remain invariant under each symmetry and where $\epsilon = +1$ for $\mathfrak{J}_1$ and $\epsilon = -1$ for $\mathfrak{J}_2$. Since the y fields do not acquire a phase, the CDC can be expressed in terms of the $U(1)$ charges of the complex states $f_c \equiv \{\chi_c^{(i)}, \omega_c^{(i)}\}$:

$$\mathfrak{J}_{1,2}: \qquad f_c \to e^{2\pi i\, \mathbf{e}^{(1,2)}} f_c\,, \tag{7.2.4}$$

where $\mathbf{e}_i^{(1)}$ and $\mathbf{e}_i^{(2)}$ take the values

$$\mathbf{e}_i^{(1)},\ \mathbf{e}_i^{(2)} \;=\; \begin{cases} \frac{1}{2} & \text{for } \chi_c^{(1)}, \omega_c^{(1)} \\ \frac{1}{2}\epsilon & \text{for } \chi_c^{(2)}, \omega_c^{(2)} \\ 0 & \text{otherwise}\,. \end{cases} \tag{7.2.5}$$

Henceforth the choice applicable to this study is $\epsilon = +1$. The advantage of this complexification is of course that the operator associated with the $\mathfrak{J}_1$ symmetry can easily be written in the basis of the original $\mathcal{N} = 1$, $6D$ model of Table 6.1:

$$\widehat{J_1} = e^{2\pi i\, \mathbf{e}\cdot\mathbf{Q}}\,, \tag{7.2.6}$$

where

$$\begin{aligned} \mathbf{e}\cdot\mathbf{Q} \;&=\; \frac{1}{2}\frac{1}{2\pi i}\int dz \left(\overline{\chi}_c^{(i)}(z)\chi_c^{(i)}(z) + \overline{\omega}_c^{(i)}(z)\omega_c^{(i)}(z)\right) \\ &=\; \frac{1}{2}\left(Q_{\chi^{34}} + Q_{\chi^{56}} + Q_{\omega^{34}} + Q_{\omega^{56}}\right)\,. \end{aligned} \tag{7.2.7}$$

It is then convenient to work with $\mathbf{e}$ charges in which the right-moving d.o.f are defined in this basis:

$$\mathbf{e} = \tfrac{1}{2}[\,0\,0\,1\,0\,1\,1\,0\,1\,|\,0\,0\,0\,0\;0\,0\,0\,0\,0\;0\,0\,0\;0\,0\,0\,0\,0\,0\,0\,0\,]\,. \tag{7.2.8}$$

For later reference note that $|\mathbf{e}^2| = 1$ is necessary for modular invariance.

At the current stage of this work, the main aim is to examine the effects that arise when the $\mathcal{N} = 1$, $6D$ model of Table 6.1 undergoes a CDC. Following the discussion in Section 6.3 of Chapter 6, it is known that there can be a CDC on a two-dimensional torus or on a $\mathbb{Z}_2$ orbifold. Even though the first option does not produce phenomenologically viable theories, it is still necessary to develop a general approach for the models constructed via CDC.

7.2.1 *Non-chiral* non-supersymmetric four-dimensional theory

For convenience, the spin structure of the toroidally compactified $\mathcal{N} = 2$ model is specified only by the untwisted vectors $V_{0,1,2,4,5,6,7}$. This model, serves only as a stepping stone towards the final result, which is discussed in the next section.

In order to realise the effects of CDC on the resulting spectrum, it is appropriate to study the CDC-deformed one-loop partition function. For any supersymmetric six-dimensional theory that undergoes a CDC down to four dimensions, the one-loop partition function in the "charge-lattice" formalism is given by the following *general* form:

$$Z(\tau) = \mathrm{Tr} \sum_{\ell_{1,2}, \jmath_{1,2}} g\, q^{[L'_0]} \overline{q}^{[\overline{L'}_0]}\,, \tag{7.2.9}$$

where the primes indicate that these expressions are CDC deformations of the traditional supersymmetric Virasoro operators. The g is the generalised GSO fermion-number projection operator, which is independent of the values of the $\mathbf{e}$ charges. Following the conventions of Eq. (5.2.2), the Virasoro operators for the left- and right-moving sectors of the tachyon-free, non-supersymmetric model are expressed as

$$[L'_0] = \frac{\alpha' p_L^2}{2} + \text{osc.}\,, \qquad [\overline{L'}_0] = \frac{\alpha' p_R^2}{2} + \text{osc.} \tag{7.2.10}$$

The explicit form of the operators in Eq. (7.2.10) is obtained through the same procedure as described in Ref. [50–52], but with two additional bosonic coordinates (X^5, X^6) compactified with radii $R_1 = \frac{r_1}{\sqrt{\alpha'}}$ and $R_2 = \frac{r_2}{\sqrt{\alpha'}}$. Defining the respective KK and winding numbers to be $\ell_{1,2}$ and $\jmath_{1,2}$, the *general* forms of the Virasoro operators are expressed as

$$L'_0 = \frac{1}{2}\left[\mathbf{Q}_L - \mathbf{e}_L(\jmath_1 + \jmath_2)\right]^2 + \frac{1}{4}\left[\frac{\ell_1 + \mathbf{e}\cdot\mathbf{Q} - \frac{1}{2}(\jmath_1 + \jmath_2)\mathbf{e}^2}{r_1} + \jmath_1 r_1\right]^2$$
$$+ \frac{1}{4}\left[\frac{\ell_2 + \mathbf{e}\cdot\mathbf{Q} - \frac{1}{2}(\jmath_1 + \jmath_2)\mathbf{e}^2}{r_2} + \jmath_2 r_2\right]^2 - 1 + \text{other osc. contributions}\,,$$

$$\overline{L'}_0 = \frac{1}{2}\left[\mathbf{Q}_R - \mathbf{e}_R(\jmath_1 + \jmath_2)\right]^2 + \frac{1}{4}\left[\frac{\ell_1 + \mathbf{e}\cdot\mathbf{Q} - \frac{1}{2}(\jmath_1 + \jmath_2)\mathbf{e}^2}{r_1} - \jmath_1 r_1\right]^2$$
$$+ \frac{1}{4}\left[\frac{\ell_2 + \mathbf{e}\cdot\mathbf{Q} - \frac{1}{2}(\jmath_1 + \jmath_2)\mathbf{e}^2}{r_2} - \jmath_2 r_2\right]^2 - \frac{1}{2} + \text{other osc. contributions}\,, \quad (7.2.11)$$

where L_0 and $\overline{L}_0$ are the Virasoro operators of the original supersymmetric model in four dimensions, *i.e.* the Virasoro operators with $\mathbf{e} = \mathbf{0}$. It follows that

$$L'_0 + \overline{L'}_0 = L_0 + \overline{L}_0 + \frac{1}{2}\left[\mathbf{e}\cdot\mathbf{Q} - \frac{(\jmath_1 + \jmath_2)}{2}\mathbf{e}^2\right]^2\left(\frac{1}{{r_1}^2} + \frac{1}{{r_2}^2}\right) - (\jmath_1 + \jmath_2)\left(\mathbf{e}_L\cdot\mathbf{Q}_L + \mathbf{e}_R\cdot\mathbf{Q}_R\right)$$
$$+ \frac{1}{2}(\jmath_1 + \jmath_2)^2\left(\mathbf{e}_L^2 + \mathbf{e}_R^2\right) + \left(\frac{\ell_1}{r_1^2} + \frac{\ell_2}{r_2^2}\right)\left[\mathbf{e}\cdot\mathbf{Q} - \frac{(\jmath_1 + \jmath_2)}{2}\mathbf{e}^2\right],$$
$$L'_0 - \overline{L'}_0 = L_0 - \overline{L}_0\,, \quad (7.2.12)$$

where $\mathbf{e}_{R,L}$ and $\mathbf{Q}_{R,L}$ refer to just the right- or left-moving elements of these vectors and where $\mathbf{e}\cdot\mathbf{Q}$ denotes a Lorentzian dot product. Given this form of the Virasoro operators it is deduced that at the massless level of the resulting theory, *i.e.* for $\ell = \jmath = 0$, the CDC shifts the masses of those states with non-zero charges overlapping $\mathbf{e}$. The mass that the string states accrue is then given by

$$\alpha'\mathbf{m}^2 = |\mathbf{e}\cdot\mathbf{Q}|^2\left(\frac{1}{{r_1}^2} + \frac{1}{{r_2}^2}\right). \quad (7.2.13)$$

From these expressions it is easy to read off the effect of the CDC on the particle spectrum shown in Tables 6.3 through 6.6. The end result is that the states in Tables 6.3 and 6.5 survive the GSO projections and remain in the massless spectrum, while the states in Tables 6.4 and 6.6 gain masses and are eliminated. These states are just a subset of the total spectrum, which is altered dramatically after the CDC.

It is straightforward to understand why some states remain massless whereas the mass of some others is lifted. First, in the **NS**-**NS** sector, *all* the charges $\mathbf{Q}_i$ of the free fermions that overlap the $\mathbf{e}_i$ entries are zero. Likewise, all KK and winding masses are also unshifted. However, in the V_1 sector there are charges overlapping $\mathbf{e}$, and these can be $\pm\frac{1}{2}$, depending on the chirality of the string state. In order to see which states remain massless, one needs to examine the generalised GSO projections on the gravitinos. These projections include

$$\begin{aligned} V_0\cdot N+\frac{1}{4}\left(1-\gamma_{\psi_{34}}\gamma_{\psi_{56}}\gamma_{\chi_{34}}\gamma_{\chi_{56}}\right) &= k_{01}+\frac{1}{2}-V_0\cdot V_1 \quad \mathrm{mod}\ (1) \\ \frac{1}{4}\left(1-\gamma_{\psi_{34}}\gamma_{\psi_{56}}\gamma_{\chi_{34}}\gamma_{\chi_{56}}\right) &= k_{11}+\frac{1}{2}-V_1\cdot V_1 \quad \mathrm{mod}\ (1) \\ V_4\cdot N+\frac{1}{4}\left(1-\gamma_{\chi_{34}}\gamma_{\chi_{56}}\right) &= k_{41}-V_4\cdot V_1 \quad \mathrm{mod}\ (1)\,, \end{aligned} \tag{7.2.14}$$

where N corresponds to the number operator associated with the **NS** d.o.f and Γ corresponds to the number operator associated with the **R** d.o.f. There are also the general constraints

$$\begin{aligned} V_4\cdot V_1 = k_{14}+k_{41} = \frac{1}{2} \quad \mathrm{mod}\ (1) \\ V_0\cdot V_1 = k_{01}+k_{10} = 0 \quad \mathrm{mod}\ (1)\,, \end{aligned} \tag{7.2.15}$$

to be taken into account. Thus, the V_4 projection removes those gravitinos of the $\mathcal{N}=4$ theory for which

$$\tfrac{1}{4}\left(1-\gamma_{\chi_{34}}\gamma_{\chi_{56}}\right) = k_{14} \quad \mathrm{mod}\ (1)\,, \tag{7.2.16}$$

leaving behind an $\mathcal{N}=2$ theory. This outcome, in conjuction with the result in Eq. (7.2.13), reveals that states with chiralities such that $\mathbf{e}\cdot\mathbf{Q}=\pm\frac{1}{2}$ acquire masses of $\frac{1}{2}\sqrt{R_1^{-2}+R_2^{-2}}$, while states with $\mathbf{e}\cdot\mathbf{Q}=0$ mod (1) remain massless. Thus, the $\gamma_{\chi_{34}}\gamma_{\chi_{56}}=1$ states become heavy while the $\gamma_{\chi_{34}}\gamma_{\chi_{56}}=-1$ states are unaffected. It is therefore deduced that the $\mathcal{N}=2$ spacetime SUSY is broken completely by the CDC for states with $\gamma_{\chi_{34}}\gamma_{\chi_{56}}=1$. Clearly the same splitting applies to all fermions in the V_2 sector, as per Table 6.5. The mechanism whereby fermions gain masses while scalars (a.k.a. Higgses) remain massless is essentially the same as the one described in Ref. [80] for the Higgs/Higgsino, namely that the fermion masses are supersymmetric "μ-terms" while the scalars have soft terms that precisely cancel this contribution to their squared masses.

However, it is possible that the theory obtained via CDC could still be $\mathcal{N} = 2$ supersymmetric if *there is no* V_4 sector. After a cursory examination of Eq. (7.2.16), it is realised that if $k_{14} = 0$, then indeed there are no massless gravitinos remaining in the spectrum, whereupon the theory is non-supersymmetric. By contrast, the choice of $k_{14} = \frac{1}{2}$ leaves the $\mathcal{N} = 2$ symmetry of the original theory intact. In this case it is a different SUSY which is preserved and the conflict between the two supersymmetries is a matter of choice of the structure constants k_{ij}. Note, that the choice of structure constants is crucial so as to avoid the "discrete torsion" scenario, described in Appendix E.

7.2.2 *Chiral* non-supersymmetric four-dimensional theory

Given the previous model, attention is now turned on the orbifolding with action b_3 and b_4 so as to achieve chirality. It turns out that the CDC does not change the spectrum of the twisted sectors, which remain (globally) supersymmetric [48]. Therefore the following discussion is solely focused on what happens to the spectrum derived from the untwisted sectors of the $\mathcal{N} = 1$, $4D$ theory presented in Table 6.2. As noted in Ref. [50], an orbifold in the $X_{5,6}$ dimensions reverses the sign of the KK and winding modes. Following the discussion surrounding Eq. (6.3.6), as well as from the expressions of the Virasoro operators in Eq. (7.2.11), it is deduced that the orbifolding does not form a sensible projection on states with degenerate masses unless an odd element of the orbifolding acts on the charges as $\mathbf{e} \cdot \mathbf{Q} \to -\mathbf{e} \cdot \mathbf{Q}$. More succinctly, a sufficient condition for ensuring that the CDC'd theory is a four-dimensional $\mathcal{N} = 0$ theory, where all symmetries are spontaneously broken by $\mathbf{e}$, is that the operator $\widehat{J_1}$ must obey the condition [50]

$$\{\mathcal{L}, \widehat{g}\} = 0\,, \tag{7.2.17}$$

where $\mathcal{L} = \mathbf{e} \cdot \mathbf{Q}$ and $\widehat{g}$ corresponds to the possible odd actions $b_{3,4}$ under the $\mathbb{Z}_2$ orbifold in Eq. (6.3.1).

Orbifold actions that obey Eq. (7.2.17) then act on the fields appearing in $\mathbf{e}$ as a generalised conjugation, $\widehat{g}\chi_c^{(i)} = \pm\overline{\chi}_c^{(i)}$ and $\widehat{g}\omega_c^{(i)} = \pm\overline{\omega}_c^{(i)}$. This can be achieved if one takes the original b_3 and b_4 and applies them in a rotated complexification where they could overlap with the CDC vector $\mathbf{e}$, with

$$\begin{aligned} b_3 : \qquad & (\chi_{35}, \omega_{46}) \to (-\chi_{35}, -\omega_{46}) \\ b_4 : \qquad & (\chi_{46}, \omega_{35}) \to (-\chi_{46}, -\omega_{35}) , \end{aligned} \tag{7.2.18}$$

so that for example $b_3 : \chi_{34} \to -\overline{\chi}_{34}$. In order to clarify the connection with the $\mathcal{N} = 1$, $4D$ model, a complex basis is used in which the $\mathbf{e}$ vectors are diagonal. Since it is also possible to write the results in Eq. (7.2.18) using real-fermion boundary conditions, a convenient notation is introduced in which the complex-fermion boundary conditions are expressed using the real ones. In this notation, the boundary condition $\chi \to \overline{\chi}$ is represented with '$\overline{0}$', while $\chi \to -\overline{\chi}$ is represented with '$\overline{1}$'; to be more precise: $0 \equiv (00)_r$ and $1 \equiv (11)_r$ are the same as before, while $\overline{0} \equiv (01)_r$ and $\overline{1} \equiv (10)_r$.

The CDC'd model is then given by the boundary conditions in Table 7.1 with the same set of structure constants as in Eq. (6.3.3). Additional adjustments that have been made are due to the rotation of the orbifold basis; namely that certain entries within the vectors $V_{5,7}$ have also acquired bars in order to keep them aligned with the orbifold actions. This distinction is irrelevant for the other vectors in the spin structure because it is explicit that the barred and unbarred vectors are the same. Moreover, $V_4 = \overline{b_3 + b_4}$ is also unchanged, and therefore the action of the $\mathbf{e}$ shift on the untwisted massless spectrum is precisely as described in the $\mathcal{N} = 2$ theory with the appropriate change of basis.

Clearly, the theory which remains unaffected by the CDC effects is identical to the original theory, but with a different complexification, *e.g.* $\chi^{36} = \chi^3 + i\chi^6$ and $\chi^{45} = \chi^4 + i\chi^5$. Hence, the orbifolding and the CDC are effectively operating in different complexifications and it is therefore worth elucidating how this works

Sector	ψ^{34}	ψ^{56}	χ^{34}	y^{34}	ω^{34}	χ^{56}	y^{56}	ω^{56}	$\overline{y}^{34}$	$\overline{\omega}^{34}$	$\overline{y}^{56}$	$\overline{\omega}^{56}$	$\overline{\psi}^1$	$\overline{\psi}^2$	$\overline{\psi}^3$	$\overline{\psi}^4$	$\overline{\psi}^5$	$\overline{\eta}^1$	$\overline{\eta}^2$	$\overline{\eta}^3$	$\overline{\phi}^1$	$\overline{\phi}^2$	$\overline{\phi}^3$	$\overline{\phi}^4$	$\overline{\phi}^5$	$\overline{\phi}^6$	$\overline{\phi}^7$	$\overline{\phi}^8$
V_0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_1	1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
V_2	1	1	0	1	0	0	1	0	1	0	1	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
b_3	1	0	$\overline{1}$	0	$\overline{0}$	$\overline{0}$	0	$\overline{1}$	0	0	0	1	1	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0
b_4	1	0	$\overline{0}$	0	$\overline{1}$	$\overline{1}$	0	$\overline{0}$	0	1	0	0	1	1	1	1	1	0	0	1	0	0	0	0	0	0	0	0
V_5	0	0	0	$\overline{0}$	$\overline{0}$	0	$\overline{1}$	$\overline{1}$	0	1	0	0	1	1	1	0	0	0	0	0	1	1	1	1	0	0	1	1
V_6	0	0	0	0	0	0	0	0	1	1	0	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	0
V_7	0	0	0	$\overline{1}$	$\overline{1}$	0	$\overline{0}$	$\overline{0}$	1	0	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1	$\frac{1}{2}$
$\mathbf{e}$	0	0	1	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 7.1: Spin structure of the worldsheet fermions of the $\mathcal{N}=0$, $4D$ model after applying the CDC. The overlined entries in these vectors are explicitly defined in the text. This spin structure is accompanied by two bosonic d.o.f compactified on a $\mathbb{Z}_2$ orbifold with twist action corresponding to the vectors $b_{3,4}$. As always, every entry in this table is understood to be multiplied by $-\frac{1}{2}$.

for a particular state. Based on the notation of Ref. [135], with b_n and d_n denoting the positive and negative coefficients in a normal-mode expansion of these fields, and with boundary condition v, the b_3 projection then generally takes the form

$$b_3 \cdot \mathbf{Q} = \frac{1}{2} \sum_n \left(b^\dagger_{n+v-\frac{1}{2}} b_{n+v-\frac{1}{2}} - d^\dagger_{n+\frac{1}{2}-v} b_{n+\frac{1}{2}-v} \right) + \dots \mod (1)\,. \tag{7.2.19}$$

Suppose there is a massless state $\chi^{36}_{-\frac{1}{2}}\,|0\rangle_R \equiv (b^\dagger_{\chi^{36},\frac{1}{2}} \oplus d^\dagger_{\chi^{36},\frac{1}{2}})\,|0\rangle_R$ allowed by the b_3 projection. Such a projection cannot distinguish $b^\dagger$ from $d^\dagger$. In the meantime, $\mathbf{e}\cdot\mathbf{Q}$ shifts the spectrum of states; in terms of the coefficients of real fermions this shift is written as, *e.g.* $b_{\chi^{36},n} = \frac{1}{\sqrt{2}}(\chi^3_n + i\chi^6_n)$ and $d_{\chi^{36},n} = \frac{1}{\sqrt{2}}(\chi^3_n - i\chi^6_n)$. Every other state that remains unshifted must necessarily satisfy

$$\begin{aligned} \mathbf{e}\cdot\mathbf{Q} &= \frac{1}{2} \sum_{l=34,56} \left(b^\dagger_{\chi^l,\frac{1}{2}} b_{\chi^l,\frac{1}{2}} + b^\dagger_{\omega^l,\frac{1}{2}} b_{\omega^l,\frac{1}{2}} - d^\dagger_{\chi^l,\frac{1}{2}} d_{\chi^l,\frac{1}{2}} - d^\dagger_{\omega^l,\frac{1}{2}} d_{\omega^l,\frac{1}{2}} \right) + \dots = 0 \mod (1) \\ &= \frac{i}{2} \sum_{l=3,5} \left(\chi^{l\dagger}_{\frac{1}{2}} \chi^{l+1}_{\frac{1}{2}} - \chi^{l+1\dagger}_{\frac{1}{2}} \chi^{l}_{\frac{1}{2}} + \omega^{l\dagger}_{\frac{1}{2}} \omega^{l+1}_{\frac{1}{2}} - \omega^{l+1\dagger}_{\frac{1}{2}} \omega^{l}_{\frac{1}{2}} \right) = 0 \mod (1)\,. \end{aligned} \tag{7.2.20}$$

In the case of $\chi^{36}_{-\frac{1}{2}}\,|0\rangle_R$, the condition in Eq. (7.2.20) is not fulfilled, therefore this state would gain degenerate masses from the CDC of order $\sim R^{-1}$. There is an alternative way to see this, if one considers the same states written in the original complex basis as, *e.g.* $\chi^{34}_{-\frac{1}{2}}\,|0\rangle_R \equiv (b^\dagger_{\chi^{34},\frac{1}{2}} \oplus d^\dagger_{\chi^{34},\frac{1}{2}})\,|0\rangle_R$. Even though such states would also become massive, the b_3 projection leaves only the conjugation-invariant linear combination $\frac{1}{\sqrt{2}}(b^{\dagger}_{\chi^{34},\frac{1}{2}} + d^{\dagger}_{\chi^{34},\frac{1}{2}})\,|0\rangle_R = \chi^{3}_{-\frac{1}{2}}\,|0\rangle_R$. In a similar fashion the b_3 projection on the state $\chi^{56}_{-\frac{1}{2}}\,|0\rangle_R$ leaves behind only $\chi^{6}_{-\frac{1}{2}}\,|0\rangle_R$. Thus, either way, the states $\chi^{36}_{-\frac{1}{2}}\,|0\rangle_R$ remain in the spectrum with mass $\sim R^{-1}$. This finding should not come as a surprise because the orbifold action could be considered as a conjugation action in the original basis. This means that it is blind to states that remain neutral under the corresponding CDC charges. For example, such states are $b^{\dagger}_{\chi^{34},\frac{1}{2}} d^{\dagger}_{\chi^{34},\frac{1}{2}}\,|0\rangle_R$.

In this discussion real fermions are employed only as they appear in the complexification of the shifted charge lattice. However, as suggested above, the entire formalism for this class of models can actually be recast in a more straightforward manner using real fermions from the beginning of the construction procedure. Using real free fermions, the previous discussion becomes self-evident and it is also clarified how the CDC interacts with the $V_{5,7}$ vectors.

In order to write a model in terms of real fermions it is convenient to use the technique of Ref. [135]. Since the relevant phase in the GSO projection is either 0 or $-\frac{1}{2}$, the overall relative sign is of no particular importance, and hence the sign of the $d^{\dagger}d$ entries in the charge operators could be reversed. Employing this change, the theory is no longer defined in terms of a charge lattice, but the charge operator appearing in the GSO projections is replaced by the sum of number operators associated with the real fermions in addition to a vacuum "charge":

$$\begin{aligned}
\mathbf{e}\cdot\mathbf{Q}_r \;&\equiv\; \frac{1}{2}\sum_{l=34,56}\Big(b^{\dagger}_{\chi^l,\frac{1}{2}}b_{\chi^l,\frac{1}{2}} + b^{\dagger}_{\omega^l,\frac{1}{2}}b_{\omega^l,\frac{1}{2}} + d^{\dagger}_{\chi^l,\frac{1}{2}}d_{\chi^l,\frac{1}{2}} + d^{\dagger}_{\omega^l,\frac{1}{2}}d_{\omega^l,\frac{1}{2}}\Big) + \ldots = 0 \mod (1)\\
&=\; \frac{1}{2}\sum_{l=3,6,4,5}\Big(\chi^{l\dagger}_{\frac{1}{2}}\chi^{l}_{\frac{1}{2}} + \omega^{l\dagger}_{\frac{1}{2}}\omega^{l}_{\frac{1}{2}}\Big) + \ldots = 0 \mod (1)\,. \qquad (7.2.21)
\end{aligned}$$

Revisiting the action of the CDC on the Virasoro operators, it is necessary to define the real number operator at level n^l as $N_{n^l+\overline{\alpha V}^l-\frac{1}{2}} = b^{\dagger}_{n^l+\overline{\alpha V}-\frac{1}{2}} b_{n^l+\overline{\alpha V}-\frac{1}{2}}$. With this alteration, the Virasoro operators before the CDC are expressed as

$$L_0 = \sum_{l}\sum_{n^l=1}\Big(n^l + \overline{\alpha V}^l - \frac{1}{2}\Big)N_{n^l+\overline{\alpha V}^l-\frac{1}{2}} + \frac{1}{2}(\overline{\alpha V}^l)^2 + \ldots\,, \qquad (7.2.22)$$

and similar for $\overline{L}_0$. Here the quadratic piece is the relevant vacuum-energy contribution, and the dots indicate terms that do not depend on the fermion boundary condition. Of course, it is crucial that modular invariance remains intact. This is achieved by the CDC inducing a correct shift in the Virasoro operators of Eq. (7.2.11). A shift in the vacuum "charge" like $\overline{\alpha V} \to \overline{\alpha V} - \mathbf{e}(\jmath_1 + \jmath_2)$, generates the following shift in the Virasoro operators:

$$L_0 \to L_0' = L_0 - (\jmath_1 + \jmath_2)\,\mathbf{e} \cdot \left(N_{n^l + \overline{\alpha V}^l - \frac{1}{2}} + \overline{\alpha V}\right) + \frac{1}{2}\mathbf{e}_L \cdot \mathbf{e}_L (\jmath_1 + \jmath_2)^2 \,, \tag{7.2.23}$$

and similar for $\overline{L}_0 \to \overline{L}_0'$. Therefore, in the real-fermion formalism, $\mathbf{Q}$ in the CDC is replaced with

$$\mathbf{Q}_r \;\equiv\; N_{n^l + \overline{\alpha V}^l - \frac{1}{2}} + \overline{\alpha V}^l \tag{7.2.24}$$

and real Lorentz products are performed accordingly. Note that in practice the massless spectrum remaining after CDC could be determined by simply adding the general constraint $\mathbf{e} \cdot \mathbf{Q} = 0 \bmod (1)$, with real-fermion contributions incorporated, as described above, to the GSO projections. Indeed, it is in this notation that the spectra of theories including vectors with real fermions are more easily analysed.

Putting all the pieces together in the context of the model at hand, it is concluded that the states fulfilling the condition $\mathbf{e} \cdot \mathbf{Q} \neq 0 \bmod (1)$ are lifted. In this particular example, two pairs of Higgs fields and two of the singlet scalar states accrue masses while the scalars

$$\begin{aligned} H_{U_1}, H_{D_1} &= \psi^{56}_{-\frac{1}{2}}\,|0\rangle_R \otimes \overline{\psi}^{4,5}_{-\frac{1}{2}}\,\overline{\eta}^1_{-\frac{1}{2}}\,|0\rangle_L \\ \Xi_1,\ \Xi_1' &= \psi^{56}_{-\frac{1}{2}}\,|0\rangle_R \otimes \overline{\eta}^2_{-\frac{1}{2}}\,\overline{\eta}^3_{-\frac{1}{2}}\,|0\rangle_L \,, \end{aligned} \tag{7.2.25}$$

remain massless. Likewise, there remain two pairs of massless Higgsinos and Weyl spinors in the spectrum:

$$\begin{aligned} \tilde{H}_{U_2}, \tilde{H}_{D_2} &= \{\chi^{34}_0\}\,|0\rangle_R \otimes \overline{\psi}^{4,5}_{-\frac{1}{2}}\,\overline{\eta}^2_{-\frac{1}{2}}\,|0\rangle_L \\ \tilde{H}_{U_3}, \tilde{H}_{D_3} &= \{\chi^{56}_0\}\,|0\rangle_R \otimes \overline{\psi}^{4,5}_{-\frac{1}{2}}\,\overline{\eta}^3_{-\frac{1}{2}}\,|0\rangle_L \\ \tilde{\Xi}_2,\ \tilde{\Xi}_2' &= \{\chi^{34}_0\}\,|0\rangle_R \otimes \overline{\eta}^1_{-\frac{1}{2}}\,\overline{\eta}^3_{-\frac{1}{2}}\,|0\rangle_L \\ \tilde{\Xi}_3,\ \tilde{\Xi}_3' &= \{\chi^{56}_0\}\,|0\rangle_R \otimes \overline{\eta}^1_{-\frac{1}{2}}\,\overline{\eta}^2_{-\frac{1}{2}}\,|0\rangle_L \,. \end{aligned} \tag{7.2.26}$$

The Higgsinos can be coupled to the singlet scalars through Yukawa couplings of the form

$$\tilde{H}_{D_2}\tilde{H}_{U_3}\Xi_1 + \tilde{H}_{U_2}\tilde{H}_{D_3}\Xi_1' \,. \tag{7.2.27}$$

These and the other Yukawa couplings are discussed in Section 9.2.2 of Chapter 9. The phenomenological importance of Eq. (7.2.27) is evident from the fact that if the singlet scalars accrue a VEV, the Higgsinos will effectively become massive. So the low-energy theory will be free of Higgs superpartners. The fermionic matter fields in the V_2 sector are somewhat similar to those in the **NS-NS** sector in the sense that they have no charges overlapping with $\mathbf{e}$ and thus their masses are unshifted. However their superpartners in the $\overline{V_1 + V_2}$ sector behave precisely as for the V_1 sector, and all receive the same masses as the gravitinos if $k_{14} = 0$, as per Table 6.6.

From a phenomenological perspective, this splitting is very appealing: one pair of Higgs scalars and the chiral matter fields, which basically constitute the SM particle content, *all* remain massless. At the same time, *all their superpartners* become massive, with a mass of the order of $\sim R^{-1}$. This indicates that the emerging theory is a chiral, $\mathcal{N} = 0$, SM-like theory in four dimensions. Note, however, that the twisted sectors are unaffected, so only the untwisted generations are truly split, while the net twisted generations remain (quasi-) supersymmetric.

7.3 Partition function of the SM-like theory

Thus far, two models of interest have been presented: an MSSM-like model in Section 6.3 of Chapter 6 and an SM-like model in the previous section. In order to further elucidate the structure of the CDC'd model, examine the quantum effects and also study the cosmological constant behaviour, it is necessary to employ a powerful tool; the partition function.

Before examining the partition function of this model, recall that the partition function of the $\mathcal{N} = 1$, $4D$ theory is given by Eq. (6.3.15). Since the twisted sectors of this theory remain supersymmetric, even after the CDC, their contributions to the partition function are nil. Therefore, it will be useful for later purposes to focus on the explicit form of the contributions to the partition function of the $\mathcal{N}=1$, $4D$ theory from the *untwisted* sectors. In general, as with any $\mathbb{Z}_2$-orbifolded theory, the contributions to the total partition function are of the form

$$Z = \frac{1}{2}\left(Z\begin{bmatrix} 0 \\ 0 \end{bmatrix} + Z\begin{bmatrix} g \\ 0 \end{bmatrix} + Z\begin{bmatrix} 0 \\ g \end{bmatrix} + Z\begin{bmatrix} g \\ g \end{bmatrix} \right), \tag{7.3.1}$$

where '0' and 'g' indicate the sum over all untwisted and twisted sectors, respectively. As mentioned above and as discussed in Ref. [51], contributions with a twist on either cycle are independent of the vector $\mathbf{e}$. This is obvious when there is a twist on the a-cycle, but less so for the term $Z\begin{bmatrix} 0 \\ g \end{bmatrix}$. However, the reason the latter also does not depend on $\mathbf{e}$ is that, as described above, the orbifold reverses charges, windings and/or KK modes, and therefore precisely half of these states are projected out, leaving the invariant combination $(|\jmath,\ell,Q\rangle + |-\jmath,-\ell,-Q\rangle)$. Since there is an overall factor of $\frac{1}{2}$ in the projection, all states with non-zero $\jmath,\ell$ or with a Q that conjugates under the orbifolding are already counted by the untwisted $\frac{1}{2}Z\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ contribution. However, these are the only states that have $\mathbf{e}$-dependence in their Hamiltonian, and therefore $Z\begin{bmatrix} 0 \\ g \end{bmatrix}$ simply provides extra contributions from the orbifold projections on the rest of the spectrum. Consequently, the partition function of the $\mathcal{N}=0$, $4D$ theory can be written as

$$Z(\mathbf{e}) = Z(0) + \frac{1}{2}\left(Z\begin{bmatrix} 0 \\ 0 \end{bmatrix}(\mathbf{e}) - Z\begin{bmatrix} 0 \\ 0 \end{bmatrix}(0) \right) = \frac{1}{2}Z\begin{bmatrix} 0 \\ 0 \end{bmatrix}(\mathbf{e}), \tag{7.3.2}$$

Note that since the supersymmetric partition function results in a vanishing cosmological constant Λ, this result is useful for determining the Λ associated with the CDC'd theory. In this case, any effect on Λ would *only* be from the untwisted sectors of the CDC'd theory. Due to this particular feature, the effects on Λ can also be deduced if one works entirely within the toroidally compactified theory as long as all the untwisted-sector contributions are taken into account, including those containing the combination $V_4 = \overline{b_3 + b_4}$.

In order to understand clearly how the spontaneous SUSY breaking manifests itself in the spectrum of the CDC'd theory, it is more convenient to separate out the action of V_1 from the partition function, as this is what governs the supersymmetric cancellations. In addition to $V_{5,6,7}$, a convenient basis corresponds to the following linear combination of the vectors defined in Table 6.2:

$$\begin{aligned}
V_0' = \overline{V_0 + V_1} &= -\tfrac{1}{2}[\,0\,0\,0\,1\,1\,0\,1\,1\,|\,1\,\ldots\,1] \\
V_1 &= -\tfrac{1}{2}[\,1\,1\,1\,0\,0\,1\,0\,0\,|\,\ldots\,] \\
V_2' = \overline{V_2 + V_0 + b_3 + b_4} &= -\tfrac{1}{2}[\,0\,0\,0\,0\,0\,0\,0\,0\,|\,\ldots\,] \\
V_4 = \overline{b_4 + b_3} &= -\tfrac{1}{2}[\,0\,0\,1\,0\,1\,1\,0\,1\,|\,\ldots\,]\,.
\end{aligned} \tag{7.3.3}$$

In this basis, only V_4 overlaps with V_1; consequently terms that cancel due to supersymmetry will largely factor out. In addition the vectors V_i can be divided into two sets: $\{V_1, V_4\}$ and $\{V_a\}$ where $a \notin \{1,4\}$. Without loss of generality one can choose $k_{1a} = 0$, so that the partition function takes the form

$$\begin{aligned}
Z(\tau) = \frac{\mathcal{M}^2}{\tau_2\,\eta^{10}\,\overline{\eta}^{22}} \sum_{\substack{\alpha_{1,4}\\ \beta_{1,4}}} Z_{\mathbf{B}}\begin{bmatrix} 0 \\ 0 \end{bmatrix}(\tau)\, C^{\alpha}_{\beta} \prod_{\substack{i_R \in \\ \{1,2,3,6\}}} \vartheta\begin{bmatrix} \alpha\mathbf{V}_i \\ -\beta\mathbf{V}_i \end{bmatrix}(\tau) \\
\times \sum_{\substack{\alpha_{0',2',5,6,7}\\ \beta_{0',2',5,6,7}}} \prod_{\substack{i_R \notin \\ \{1,2,3,6\}}} \vartheta\begin{bmatrix} \alpha\mathbf{V}_i \\ -\beta\mathbf{V}_i \end{bmatrix}(\tau) \prod_{i_L} \overline{\vartheta}\begin{bmatrix} \alpha\mathbf{V}_i \\ -\beta\mathbf{V}_i \end{bmatrix}(\overline{\tau})\,.
\end{aligned} \tag{7.3.4}$$

The factors C^{α}_{β} can also be split. Letting $a \equiv 0', 2', 5, 6, 7$, the partition function coefficients are written as

$$\begin{aligned}
C^{\alpha}_{\beta} &= \exp\left[2\pi i\left(\alpha s + \beta s + \beta_i k_{ij} \alpha_j\right)\right] \\
&= e^{\pi i(\alpha_1+\beta_1)} \exp\left[2\pi i\left(\beta_a k_{ab}\alpha_b + \beta_4 k_{4b}\alpha_b + \beta_a k_{a4}\alpha_4 + \beta_1 k_{14}\alpha_4 + \beta_4 k_{41}\alpha_1\right)\right] \\
&= e^{\pi i(\alpha_1+\beta_1)} \exp\left[2\pi i\left(\beta_4 k_{4b}\alpha_b + \beta_a k_{a4}\alpha_4 + \beta_1 k_{14}\alpha_4 + \beta_4 k_{41}\alpha_1\right)\right] \hat{C}^{\alpha}_{\beta}\,.
\end{aligned} \tag{7.3.5}$$

One can then identify contributions involving different V_4 contributions to the spin structure:

$$Z(\tau) = \frac{\mathcal{M}^2}{\tau_2\,\eta^{10}\,\overline{\eta}^{22}} Z_{\mathbf{B}}\begin{bmatrix} 0 \\ 0 \end{bmatrix} \sum_{\alpha_4,\beta_4} \Omega\begin{bmatrix} \alpha_4 \\ \beta_4 \end{bmatrix}. \tag{7.3.6}$$

Using the double-index shorthand for the ϑ-functions as defined in Appendix D, the Ω functions take the form

$$\Omega\left[\begin{array}{c}0\\0\end{array}\right]=\left[\vartheta_{00}^4-\vartheta_{01}^4-\vartheta_{10}^4+\vartheta_{11}^4\right]\times\sum_{\substack{\alpha_{0',2',5,6,7}\\ \beta_{0',2',5,6,7}}}\hat{C}^{\alpha}_{\beta}\prod_{\substack{i_R\notin\\ \{1,2,3,6\}}}\vartheta\left[\begin{array}{c}\overline{\alpha\mathbf{V}_i}\\ -\beta\mathbf{V}_i\end{array}\right]\prod_{j_L}\overline{\vartheta}\left[\begin{array}{c}\overline{\alpha\mathbf{V}_j}\\ -\beta\mathbf{V}_j\end{array}\right],$$

$$\Omega\left[\begin{array}{c}1\\0\end{array}\right]=\left[\vartheta_{00}^2\,\vartheta_{10}^2-(-1)^{2k_{14}}\,\vartheta_{01}^2\,\vartheta_{11}^2-\vartheta_{10}^2\,\vartheta_{00}^2+(-1)^{2k_{14}}\,\vartheta_{11}^2\,\vartheta_{01}^2\right]$$
$$\times\sum_{\substack{\alpha_{0',2',5,6,7}\\ \beta_{0',2',5,6,7}}}\hat{C}^{\alpha}_{\beta}e^{2\pi i\beta_a k_{a4}}\prod_{\substack{i_R\notin\\ \{1,2,3,6\}}}\vartheta\left[\begin{array}{c}\overline{\alpha\mathbf{V}_i}\\ -\beta\mathbf{V}_i\end{array}\right]\prod_{j_L}\overline{\vartheta}\left[\begin{array}{c}\overline{\alpha\mathbf{V}_j}\\ -\beta\mathbf{V}_j\end{array}\right],$$

$$\Omega\left[\begin{array}{c}0\\1\end{array}\right]=\left[\vartheta_{00}^2\,\vartheta_{01}^2-\vartheta_{01}^2\,\vartheta_{00}^2-(-1)^{2k_{41}}\,\vartheta_{10}^2\,\vartheta_{11}^2+(-1)^{2k_{41}}\,\vartheta_{11}^2\,\vartheta_{10}^2\right]$$
$$\times\sum_{\substack{\alpha_{0',2',5,6,7}\\ \beta_{0',2',5,6,7}}}\hat{C}^{\alpha}_{\beta}e^{2\pi i\,k_{4a}\,\alpha_a}\prod_{\substack{i_R\notin\\ \{1,2,3,6\}}}\vartheta\left[\begin{array}{c}\overline{\alpha\mathbf{V}_i}\\ -\beta\mathbf{V}_i\end{array}\right]\prod_{j_L}\overline{\vartheta}\left[\begin{array}{c}\overline{\alpha\mathbf{V}_j}\\ -\beta\mathbf{V}_j\end{array}\right],$$

$$\Omega\left[\begin{array}{c}1\\1\end{array}\right]=\left[\vartheta_{00}^2\,\vartheta_{11}^2-(-1)^{2k_{14}}\,\vartheta_{01}^2\,\vartheta_{10}^2-(-1)^{2k_{41}}\,\vartheta_{10}^2\,\vartheta_{01}^2+(-1)^{2(k_{14}+k_{41})}\,\vartheta_{11}^2\,\vartheta_{00}^2\right]$$
$$\times\sum_{\substack{\alpha_{0',2',5,6,7}\\ \beta_{0',2',5,6,7}}}\hat{C}^{\alpha}_{\beta}e^{\left[2\pi i\,(k_{4a}\alpha_a+\beta_a k_{a4}+k_{44})\right]}\prod_{\substack{i_R\notin\\ \{1,2,3,6\}}}\vartheta\left[\begin{array}{c}\overline{\alpha\mathbf{V}_i}\\ -\beta\mathbf{V}_i\end{array}\right]\prod_{j_L}\overline{\vartheta}\left[\begin{array}{c}\overline{\alpha\mathbf{V}_j}\\ -\beta\mathbf{V}_j\end{array}\right].\qquad(7.3.7)$$

Since this is the partition function of the supersymmetric theory it is known that these contributions undergo an exact cancellation. The first term vanishes as a result of the first identity listed in Eq. (D.0.3), and the remaining ones vanish by inspection as a consequence of Eq. (7.2.15).

The procedure followed then, in order to obtain the partition function of the non-supersymmetric model, after CDC, is the same as described in Refs. [50–52]. For convenience the winding and KK numbers are defined as $\jmath=(\jmath_1+\jmath_2)\bmod(1)$ and $\ell=(\ell_1+\ell_2)\bmod(1)$ respectively. It is also useful to define $\bar{\jmath}=1-\jmath$ and $\bar{\ell}=1-\ell$.

The CDC'd partition function is then given by

$$Z'(\tau) \;=\; \frac{\mathcal{M}^2}{\tau_2\,\eta^{10}\,\overline{\eta}^{22}}\sum_{\vec{\ell},\vec{\jmath}} Z_{\vec{\ell},\vec{\jmath}}\sum_{\alpha_4,\beta_4}\Omega_{\ell,\jmath}\begin{bmatrix}\alpha_4\\ \beta_4\end{bmatrix}, \tag{7.3.8}$$

where

$$\begin{aligned}\Omega_{\ell,\jmath}\begin{bmatrix}\alpha_4\\ \beta_4\end{bmatrix} \;&=\; \prod_{\substack{i_R\in\\ \{1,2,3,6\}}}\vartheta\begin{bmatrix}\overline{\alpha\mathbf{V}}_i-\jmath\mathbf{e}_i\\ -\beta\mathbf{V}_i+\ell\mathbf{e}_i\end{bmatrix}\\ &\times\sum_{\substack{\alpha_{0',2',5,6,7}\\ \beta_{0',2',5,6,7}}}\tilde{C}^{\alpha}_{\beta}\prod_{\substack{i_R\notin\\ \{1,2,3,6\}}}\vartheta\begin{bmatrix}\overline{\alpha\mathbf{V}}_i-\jmath\mathbf{e}_i\\ -\beta\mathbf{V}_i+\ell\mathbf{e}_i\end{bmatrix}\prod_{j_L}\overline{\vartheta}\begin{bmatrix}\overline{\alpha\mathbf{V}}_j-\jmath\mathbf{e}_j\\ -\beta\mathbf{V}_j+\ell\mathbf{e}_j\end{bmatrix}.\end{aligned} \tag{7.3.9}$$

In this expression, the coefficients of the partition function are given by

$$\tilde{C}^{\alpha}_{\beta} = \exp\left[-2\pi i\left(\jmath\mathbf{e}\cdot\beta V-\frac{1}{2}\jmath\,\ell\,\mathbf{e}^2\right)\right]C^{\alpha}_{\beta}\,, \tag{7.3.10}$$

where C^{α}_{β} are the coefficients of the untwisted partition function before CDC, as given in Eq. (7.3.5).

Recall that up until now, all the d.o.f of the CDC vector $\mathbf{e}$ are assigned in a specific basis which is defined in Eq. (7.2.8). From this it is deduced that the partition function of the CDC'd model is the same as the old one, except that the $\mathbf{e}$ shifts the $i_R = 3,5,6,8$ arguments by a half unit when $\jmath$ or ℓ is odd, and there is a phase $e^{-2\pi i\,\jmath\mathbf{e}\cdot\beta V}$. This phase is only sensitive to V_1 and V_4 as these are the only *untwisted* vectors that overlap with $\mathbf{e}$ in the right-moving part. This phase is trivial when $\beta_1+\beta_4$ is even, and gives a factor $(-1)^{\jmath}$ when $\beta_1+\beta_4$ is odd. In total, then, it is obtained that

$$\begin{aligned}\Omega_{\ell,\jmath}\begin{bmatrix}0\\ 0\end{bmatrix} &= (-1)^{\jmath\,\ell}\left[\vartheta^2_{00}\,\vartheta^2_{\jmath\,\ell}-(-1)^{\jmath}\,\vartheta^2_{01}\,\vartheta^2_{\jmath\,\overline{\ell}}-\vartheta^2_{10}\,\vartheta^2_{\overline{\jmath}\,\ell}+(-1)^{\jmath}\vartheta^2_{11}\,\vartheta^2_{\overline{\jmath}\,\overline{\ell}}\right]\\ &\times\sum_{\substack{\alpha_{0',2',5,6,7}\\ \beta_{0',2',5,6,7}}}\hat{C}^{\alpha}_{\beta}\prod_{i_R\in\{4,7\}}\vartheta\begin{bmatrix}\overline{\alpha\mathbf{V}}_i\\ -\beta\mathbf{V}_i\end{bmatrix}\prod_{i_R\in\{5,8\}}\vartheta\begin{bmatrix}\overline{\alpha\mathbf{V}}_i-\jmath\mathbf{e}_i\\ -\beta\mathbf{V}_i+\ell\mathbf{e}_i\end{bmatrix}\prod_{j_L}\overline{\vartheta}\begin{bmatrix}\overline{\alpha\mathbf{V}}_j\\ -\beta\mathbf{V}_j\end{bmatrix},\end{aligned} \tag{7.3.11a}$$

$$\Omega_{\ell,\jmath}\begin{bmatrix}1\\0\end{bmatrix}=(-1)^{\jmath\ell}\Big[\vartheta_{00}^2\,\vartheta_{\bar{\jmath}\ell}^2-(-1)^{2k_{14}+\jmath}\,\vartheta_{01}^2\,\vartheta_{\bar{\jmath}\bar{\ell}}^2-\vartheta_{10}^2\,\vartheta_{\jmath\ell}^2+(-1)^{2k_{14}+\jmath}\,\vartheta_{11}^2\,\vartheta_{\jmath\bar{\ell}}^2\Big]$$
$$\times\sum_{\substack{\alpha_{0',2',5,6,7}\\ \beta_{0',2',5,6,7}}}\hat{C}^{\alpha}_{\beta}e^{2\pi i\beta_a k_{a4}}\prod_{i_R\in\{4,7\}}\vartheta\begin{bmatrix}\overline{\alpha\mathbf{V}_i}\\-\beta\mathbf{V}_i\end{bmatrix}\prod_{i_R\in\{5,8\}}\vartheta\begin{bmatrix}\overline{\alpha\mathbf{V}_i}-\jmath\mathbf{e}_i\\-\beta\mathbf{V}_i+\ell\mathbf{e}_i\end{bmatrix}\prod_{j_L}\bar{\vartheta}\begin{bmatrix}\overline{\alpha\mathbf{V}_j}\\-\beta\mathbf{V}_j\end{bmatrix},\tag{7.3.11b}$$

$$\Omega_{\ell,\jmath}\begin{bmatrix}0\\1\end{bmatrix}=(-1)^{\jmath\ell}\Big[(-1)^{\jmath}\,\vartheta_{00}^2\,\vartheta_{\jmath\bar{\ell}}^2-\vartheta_{01}^2\,\vartheta_{\jmath\ell}^2-(-1)^{2k_{41}+\jmath}\,\vartheta_{10}^2\,\vartheta_{\bar{\jmath}\bar{\ell}}^2+(-1)^{2k_{41}}\,\vartheta_{11}^2\,\vartheta_{\bar{\jmath}\ell}^2\Big]$$
$$\times\sum_{\substack{\alpha_{0',2',5,6,7}\\ \beta_{0',2',5,6,7}}}\hat{C}^{\alpha}_{\beta}e^{2\pi i\,k_{4a}\alpha_a}\prod_{i_R\in\{4,7\}}\vartheta\begin{bmatrix}\overline{\alpha\mathbf{V}_i}\\-\beta\mathbf{V}_i\end{bmatrix}\prod_{i_R\in\{5,8\}}\vartheta\begin{bmatrix}\overline{\alpha\mathbf{V}_i}-\jmath\mathbf{e}_i\\-\beta\mathbf{V}_i+\ell\mathbf{e}_i\end{bmatrix}\prod_{j_L}\bar{\vartheta}\begin{bmatrix}\overline{\alpha\mathbf{V}_j}\\-\beta\mathbf{V}_j\end{bmatrix},\tag{7.3.11c}$$

$$\Omega_{\ell,\jmath}\begin{bmatrix}1\\1\end{bmatrix}=(-1)^{\jmath\ell}\Big[(-1)^{\jmath}\,\vartheta_{00}^2\,\vartheta_{\bar{\jmath}\bar{\ell}}^2-(-1)^{2k_{14}}\,\vartheta_{01}^2\,\vartheta_{\bar{\jmath}\ell}^2-(-1)^{2k_{41}+\jmath}\,\vartheta_{10}^2\,\vartheta_{\jmath\bar{\ell}}^2+(-1)^{2(k_{14}+k_{41})}\,\vartheta_{11}^2\,\vartheta_{\jmath\ell}^2\Big]$$
$$\times\sum_{\substack{\alpha_{0',2',5,6,7}\\ \beta_{0',2',5,6,7}}}\hat{C}^{\alpha}_{\beta}e^{2\pi i\,(k_{4a}\alpha_a+\beta_a k_{a4}+k_{44})}\prod_{i_R\in\{4,7\}}\vartheta\begin{bmatrix}\overline{\alpha\mathbf{V}_i}\\-\beta\mathbf{V}_i\end{bmatrix}\prod_{i_R\in\{5,8\}}\vartheta\begin{bmatrix}\overline{\alpha\mathbf{V}_i}-\jmath\mathbf{e}_i\\-\beta\mathbf{V}_i+\ell\mathbf{e}_i\end{bmatrix}\prod_{j_L}\bar{\vartheta}\begin{bmatrix}\overline{\alpha\mathbf{V}_j}\\-\beta\mathbf{V}_j\end{bmatrix}.\tag{7.3.11d}$$

Previously in this chapter, it was claimed that if a CDC is performed on a torus, it is possible to maintain the $\mathcal{N}=2$ SUSY of the emerging theory. This scenario takes place if one chooses to assign different values to the structure constants. Specifically, for $k_{14}=\frac{1}{2}$, the generalised GSO projections allow the two gravitinos to survive in the massless spectrum and hence the theory remains supersymmetric. This claim can now be tested for the theory obtained when a CDC is performed on the $\mathbb{Z}_2$ orbifold so as to check whether the theory still has $\mathcal{N}=1$ SUSY when $k_{14}=\frac{1}{2}$. In order to do this, it is sufficient to study the expressions in Eq. (7.3.11). By inspection as well as through the first identity in Eq. (D.0.3), it is deduced that all the prefactors cancel for any $\jmath$ and ℓ when $k_{14}=\frac{1}{2}$, and that they do not when $k_{14}=0$.

A corollary is that the theory without V_4 is inevitably still supersymmetric despite the CDC: there is still a shift in the spectrum due to CDC but this shift is simply tantamount to shifting the R-charges of the states on the KK tower. This remark is supported by the fact that the expression in Eq. (7.3.11a) *vanishes completely*, regardless of ℓ, $\jmath$ and k_{14}.

7.4 Cosmological constant of the SM-like theory

As always, it is important to examine the one-loop cosmological constant Λ for the SM-like theory so as to study its stability properties. The analytical evaluation of Λ is based on the full expression of the partition function in Eq. (7.3.2). As discussed above, the CDC affects only the untwisted sectors of the theory, hence all the twisted sectors of the theory are still supersymmetric and make no net contribution to Λ. Moreover, since the model without V_4 is supersymmetric even after CDC, there are no contributions from the expression in Eq. (7.3.11a). As a result, in order to ensure that the theory is non-supersymmetric, the choice applicable to this study is $k_{14} = 0$. While the ℓ indices correspond to resummed KK modes, the sectors with $\jmath \neq 0$ correspond to winding-mode contributions. As can be seen from Eq. (6.3.12), and as anticipated by the discussion in Section 5.4 of Chapter 5, the winding-modes contributions are heavily suppressed by a factor of at least $\mathcal{O}(e^{-\pi r^2})$ for generic radius r. Therefore such terms are neglected in the following evaluation.

It is found that the only contributions to the partition function arise for *odd* ℓ when the winding number $\jmath$ is *zero* or *even*:

$$
\begin{aligned}
\Omega_{\ell,0}\begin{bmatrix} 1 \\ 0 \end{bmatrix} &= -\left[\vartheta_{00}^2\,\vartheta_{11}^2 - \vartheta_{01}^2\,\vartheta_{10}^2 - \vartheta_{10}^2\,\vartheta_{01}^2 + \vartheta_{11}^2\,\vartheta_{00}^2\right] + \ldots = 2\vartheta_{10}^2\,\vartheta_{01}^2 + \ldots \\
\Omega_{\ell,0}\begin{bmatrix} 0 \\ 1 \end{bmatrix} &= -\left[\vartheta_{00}^4 - \vartheta_{01}^4 + \vartheta_{10}^4\right] + \ldots \\
\Omega_{\ell,0}\begin{bmatrix} 1 \\ 1 \end{bmatrix} &= -\left[\vartheta_{00}^2\,\vartheta_{10}^2 - \vartheta_{01}^2\,\vartheta_{11}^2 + \vartheta_{10}^2\,\vartheta_{00}^2 - \vartheta_{11}^2\,\vartheta_{01}^2\right] + \cdots = -2\vartheta_{00}^2\,\vartheta_{10}^2 + \ldots
\end{aligned}
\tag{7.4.1}
$$

Although it is possible to evaluate the entire integral methodically, in order to examine the effect of the CDC for the $\jmath = 0$ contributions, it is simpler to go back to the original expression for the partition function in Eq. (7.3.8). When there are no winding-mode excitations, there are no shifts on the a-cycle and the only change is on the b-cycle due to the phase $2\pi \mathbf{e} \cdot \mathbf{Q}$. Also, from Eq. (7.3.10), the C^{α}_{β} remains unshifted, so the GSO projections are the same as for the supersymmetric theory. For the evaluation procedure it is also useful to define the vectors

$$\vec{\ell} \equiv (r_1 \ell_1, r_2 \ell_2), \qquad \vec{\jmath} \equiv (r_1 \jmath_1, r_2 \jmath_2). \tag{7.4.2}$$

Using the trace formula in Eq. (D.0.11) the non-vanishing contributions are written as

$$\begin{aligned} Z'_{\jmath=0}(\tau) &= \frac{\mathcal{M}^2}{\tau_2 \eta^{10} \overline{\eta}^{22}} \sum_{\vec{\ell}\,=\,\mathrm{odd}} Z_{\vec{\ell},0} \sum_{\alpha_4,\beta_4} e^{2\pi i \mathbf{e}\cdot\mathbf{Q}}\, \Omega\begin{bmatrix} \alpha_4 \\ \beta_4 \end{bmatrix} \\ &= \frac{\mathcal{M}^4 r_1 r_2}{\tau_2^2 \eta^{12} \overline{\eta}^{24}} \sum_{\vec{\ell}\,=\,\mathrm{odd}} e^{-\frac{\pi}{\tau_2}|\vec{\ell}|^2} e^{2\pi i \mathbf{e}\cdot\mathbf{Q}} \sum_{\alpha_4,\beta_4} \Omega\begin{bmatrix} \alpha_4 \\ \beta_4 \end{bmatrix}, \end{aligned} \tag{7.4.3}$$

where the Ω's are the expressions for the supersymmetric non-CDC theory. Note that $e^{2\pi i \mathbf{e}\cdot\mathbf{Q}}$ is an operator that does not depend on α, β so in the sum it becomes a simple overall factor. As a result, the net effect of the CDC on the Poisson-resummed partition function can be interpreted as one in which the sign of the contributions with $\mathbf{e} \cdot \mathbf{Q} = \frac{1}{2}$ in the supersymmetric theory, is reversed so that there are no longer exact cancellations.

This is especially straightforward in the large-τ_2 region which dominates the one-loop integral, where the τ_1-integral projects onto the physical spectrum and allows the counting of the physical states in the theory. Every fermion that is lifted by the CDC counts $+2$ and every boson -2. Conversely the partition-function contribution is proportional to the states remaining unshifted in the spectrum after CDC, namely $2(N_b^{(i)} - N_f^{(i)})$, at some degenerate mass level i. After the τ_1-integral has fixed the level-matching condition, it is found that

$$\Lambda \;=\; r_1 r_2 \mathcal{M}^4 \int_{\frac{1}{\mu^2}\approx 1}^{\infty} \frac{d\tau_2}{\tau_2^4} \sum_{\substack{\vec{\ell}=\mathrm{odd} \\ \text{level } i}} (N_f^{(i)} - N_b^{(i)}) e^{-\frac{\pi}{\tau_2}|\vec{\ell}|^2} e^{-\pi\tau_2 \alpha' m_i^2}, \tag{7.4.4}$$

where m_i^2 is the physical mass of the state i, and $N_b^{(i)}$ and $N_f^{(i)}$ are the numbers of *unshifted* bosons and fermions at the i'th mass level. The lower limit μ^{-2} reflects the fact that the τ_1 dependence of the UV end of the fundamental domain is not taken into account. Note that this expression is completely general for any model in six dimensions that after a CDC yields a non-supersymmetric four-dimensional theory.

Writing $\Lambda = \sum_{level\,i} \Lambda_i$, it is obvious that there are then two types of contributions, depending on whether the states are massless or massive. Assuming that r_1 is the smaller radius, the net contribution from massless states is determined by

$$\begin{aligned}
\Lambda_0 &= \frac{2r_1 r_2 \mathcal{M}^4}{\pi^3}(N_f^{(0)} - N_b^{(0)}) \sum_{\vec{\ell}=\text{odd}} |\vec{\ell}|^{-6}\left[1 - O(e^{-\pi|\vec{\ell}|^2\mu^2})\right] \\
&= \frac{4r_1 r_2 \mathcal{M}^4}{\pi^3}(N_f^{(0)} - N_b^{(0)})\,(2r_1)^{-6}\,\zeta\left(6, \frac{1}{2}\right) + \ldots \\
&= r_1 r_2 \mathcal{M}^4\,(N_f^{(0)} - N_b^{(0)})\,\frac{\pi^3}{240 r_1^6} + \ldots\,, \qquad (7.4.5)
\end{aligned}$$

where $\zeta(a,b)$ is the Hurwitz zeta-function and where a factor of two arises from $\ell = \pm 1$. Decompactifying to five dimensions by taking the $r_2 \to \infty$ limit and factoring out the infinite volume r_2 reproduces the single compactified-dimension result of Eq. (5.4.7) for $D = 6$.

The net contribution from massive states is determined by a saddle-point approximation with the saddle at $\tau_2 = \frac{1}{\sqrt{\alpha'}m_i}|\vec{\ell}|$ (which is valid for $\sqrt{\alpha'}m_i \ll 1$) and is found to be

$$\Lambda_{i\neq 0} = r_1 r_2 \mathcal{M}^4 (N_f^{(i)} - N_b^{(i)}) \sum_{\vec{\ell}=\text{odd}} |\vec{\ell}|^{-7/2}(\sqrt{\alpha'}m_i)^{5/2} e^{-2\pi\sqrt{\alpha'}m_i|\vec{\ell}|}\left[1 - O\left(\frac{1}{2\pi|\vec{\ell}|\sqrt{\alpha'}m_i}\right)\right]. \qquad (7.4.6)$$

Again the $r_2 \to \infty$ decompactification limit yields the $5D$ correction already anticipated in Eq. (5.4.10). Note that the subleading terms neglected in the saddle-point approximation are larger than the $\jmath \neq 0$ contributions, so it would not be appropriate to consider the latter at this order of approximation.

In summary, the leading and subleading terms on the one-loop cosmological constant of the non-supersymmetric theory emerged via CDC are the same as those found in Section 5.4 of Chapter 5, for the cosmological constant of the interpolating models. This conclusion is of utmost importance because it gives strong indications that CDC is a suitable method for constructing interpolating models which are all non-supersymmetric, UV finite due to the inherent misaligned SUSY and which could also be stable. As remarked in the discussion of interpolating models, stability could be achieved in a model with equal numbers of massless bosons and fermions. Indeed, if the SM-like theory presented here had $N_b^{(0)} = N_f^{(0)}$, then in the large but finite r_1, r_2 limit its one-loop cosmological constant would have been exponentially suppressed.

Chapter 8

Semi-realistic string models with exponentially suppressed cosmological constants

If the facts don't fit the theory,
change the facts.

Albert Einstein

8.1 Additional requirements imposed on the construction

Given the important preparatory model-building work in the previous Chapter as well as in Chapters 5 and 6, everything is ready for the centrepiece of this paper which is the construction of non-supersymmetric string models with exponentially suppressed cosmological constants. As alluded to above, only those models with almost vanishing cosmological constants will have suppressed one-loop dilaton tadpoles, as required for a theory to be stable in the absence of spacetime SUSY. Naturally, these models will have a variety of semi-realistic features and they could be identified as being either SM-like, Pati-Salam-like, or resembling a unified extension thereof, such as flipped $SU(5)$ or $SO(10)$. Despite these features, it should be noted that these models are not fully realistic and contain a number of direct phenomenological flaws. However, the purpose of this work is not to construct the absolute candidate model governing the mechanism of our world, but rather to demonstrate that it is indeed possible to construct *phenomenologically viable* theories, which are completely non-supersymmetric and possess an acceptable degree of stability. Moreover, the construction techniques that are mainly centred on a CDC method suggest that such models are only the tip of the iceberg, considering all the possible theories that could be constructed. This can then hopefully pave the way for more refined model-building and future enhanced studies on their phenomenological properties.

Even though it might come as a surprise, it is in fact easier to construct $N_b^{(0)} = N_f^{(0)}$ unified theories than $N_b^{(0)} = N_f^{(0)}$ SM-like theories for models which are realised through the free-fermionic construction followed by a CDC. The term “unified” in this context implies that the gauge group is (semi-)simple and contains the SM gauge group as a subgroup[1]. Such unified models are constructed by stripping away the $V_{5,6,7}$ vectors from the previous constructions. However, in order to construct a model with the desired leading-order cancellation of the cosmological constant, two important adjustments must be included.

The first is that unlike before, the CDC vector $\mathbf{e}$ can act *both* on the right-moving part, where it adjusts the structure of the SUSY breaking, and simultaneously on the left-moving side, where it adjusts the structure of the gauge group. As noted in Section 7.2 of Chapter 7, the only constraint on the form of this vector is that one should retain $\mathbf{e} \cdot \mathbf{e} = 1$ in order to have modular invariance. Keeping the vectors $V_{1,\ldots,4}$ as before, where the new basis now is related to that in Table 6.2 with $V_1 \to \overline{V_1 + V_0}$ and $V_2 \to \overline{V_2 + V_0}$, it is found that the CDC vector given by

$$\mathbf{e} = \tfrac{1}{2}[\,0\,0\,1\,0\,1\,1\,0\,1\,|\,1\,0\,1\,1\;0\,0\,0\,0\,0\;0\,0\,0\;0\,0\,0\,1\,1\,1\,1\,1\,] \tag{8.1.1}$$

generates $N_b^{(0)} = N_f^{(0)}$ in the toroidally compactified $\mathcal{N} = 2 \to \mathcal{N} = 0$ theory.

[1] Note that at this level of analysis, it is not a particular concern whether or not the Higgses required for breaking the GUT symmetry are present.

The second adjustment arises from the fact that, as per Eq. (7.3.2), only the untwisted sectors yield non-vanishing contributions to the partition function of the spontaneously broken theory. Therefore, it is acceptable to start by finding an $\mathcal{N} = 2 \to \mathcal{N} = 0$ theory with $N_b^{(0)} = N_f^{(0)}$, such as the one above. Any orbifold twisting that is added in order to generate a chiral theory is then guaranteed (with a suitable adjustment of structure constants) to preserve the cancellation of $N_b^{(0)} = N_f^{(0)}$ because it halves the number of bosonic and fermionic d.o.f that contribute with an $\mathbf{e}$ dependence. At this point, one must be cautious because although the orbifolding twisting halves the massless d.o.f, this might not always be true for other theories. The only constraint then on the orbifold action is that in at least one sector it should overlap with precisely one-half of the CDC elements when written in the real formalism (with the possibility that more general twisted sectors may arise through the introduction of additional untwisted boundary-condition vectors). This additional constraint on the orbifolding means that it is somewhat more difficult to find twisted matter.

Of course, one must always recall that the choice of structure constants k_{ij} throughout this process is pivotal. There is always the danger lurking in the background of breaking SUSY by discrete torsion before even applying the CDC, or leaving the post CDC model with $\mathcal{N} = 1$ SUSY. In each case the presence or absence of spacetime SUSY can also be seen directly at the level of the partition function. Along the way, a significant number of other constraints are applied: As always, the boundary-condition vectors and k_{ij} structure constants must satisfy the constraints imposed by modular invariance. Likewise, the additional modular invariance constraints for real fermions must also be satisfied. For the construction to be in agreement with interpolation, it is required that upon removing the CDC, SUSY is restored. Likewise, it is required that there exists an alternative choice of certain k_{ij}'s which can also restore SUSY. Finally, it is required that in at least one twisted sector, the boundary conditions, including the orbifold vector b_3, must overlap with precisely one-half of the entries of the CDC vector $\mathbf{e}$. Moreover, in

Sector	ψ^{34}	ψ^{56}	χ^{34}	y^{34}	ω^{34}	χ^{56}	y^{56}	ω^{56}	$\bar{y}^{34}$	$\bar{\omega}^{34}$	$\bar{y}^{56}$	$\bar{\omega}^{56}$	$\bar{\psi}^1$	$\bar{\psi}^2$	$\bar{\psi}^3$	$\bar{\psi}^4$	$\bar{\psi}^5$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^1$	$\bar{\phi}^2$	$\bar{\phi}^3$	$\bar{\phi}^4$	$\bar{\phi}^5$	$\bar{\phi}^6$	$\bar{\phi}^7$	$\bar{\phi}^8$
V_0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_1	0	0	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_2	0	0	1	0	1	1	0	1	0	1	0	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
b_3	1	0	$\bar{1}$	0	$\bar{0}$	$\bar{0}$	0	$\bar{1}$	0	0	0	1	1	1	1	1	1	0	1	0	1	0	0	1	1	1	0	0
V_4	0	0	1	0	1	1	0	1	0	1	0	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
e	0	0	1	0	1	1	0	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1

Table 8.1: Spin structure of the worldsheet fermions of the $\mathcal{N} = 0$, $4D$ $SO(10)$ theory. This spin structure is accompanied by two bosonic d.o.f compactified on a $\mathbb{Z}_2$ orbifold with twist action corresponding to the vector b_3. As always, the entries of the table are understood to be multiplied by $-\frac{1}{2}$.

this context overlaps of ‘$-\frac{1}{4}$’ phases[2] with the CDC vector are not allowed; this ensures that there is a basis in which the orbifold acts as a charge conjugation on the CDC charges (plus possible untwisted phases, depending on the sector). These constraints apply to all the models presented below.

8.2 “Unified” SO(10) and flipped SU(5) theories

An example of an $\mathcal{N} = 1 \to \mathcal{N} = 0$ $SO(10)$ unified model with $N_b^{(0)} = N_f^{(0)}$ is presented in Table 8.1. The associated structure constants are given by

$$k_{ij} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}, \tag{8.2.1}$$

and the overall gauge group of the theory is

$$G = [SO(4)]^4 \otimes [U(1)]^4 \otimes \underbrace{SO(10)}_{\text{contains SM}} \otimes SO(6)\,. \tag{8.2.2}$$

This model has fundamental **10**’s as well as eight complex **16**’s in the untwisted

[2]These phases correspond to the ‘$\frac{1}{2}$’ entries in the spin structures presented throughout this thesis.

sector, all quasi-supersymmetric and with 328 massless complex d.o.f in total. As usual, the untwisted matter multiplets appear in the $\overline{V_0 + V_2}$ sector. By re-introducing a V_7 vector, one can construct a flipped-$SU(5)$ model with $N_b^{(0)} = N_f^{(0)}$. An example is presented in Table 8.2 and its structure constants are given by

$$k_{ij} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & \frac{3}{4} \\ \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix} . \tag{8.2.3}$$

The overall gauge group of this theory is then given by

$$G = [SO(4)]^3 \otimes [U(1)]^6 \otimes \underbrace{U(5)}_{\text{contains SM}} \otimes SO(6) . \tag{8.2.4}$$

This model has twisted massless matter as well as untwisted matter with 152 massless complex d.o.f in total. It contains not only four complete chiral quasi-supersymmetric untwisted generations of matter but also massless twisted generations of fermions arising in the $\overline{V_4 + b_3}$ sector, along with superpartners in the $\overline{V_0 + V_1 + V_4 + b_3}$ sector. All these states are in the fermionic representation of the parent $SO(10)$, so it is natural to associate the above $U(5)$ gauge-group factor with the $SU(5) \otimes U(1)_X$ gauge group of the flipped $SU(5)$ unification scenario. The model also has the vector-like $\mathbf{5} + \overline{\mathbf{5}}$ Higgs representations required for electroweak symmetry breaking, but no GUT Higgses.

As usual, the $\overline{\alpha V} = 0$ sector gives rise to the gravity multiplet and adjoint gauge bosons that are not removed by the particular choices of $\mathbf{e}$ shown above. Furthermore, there is no massless gravitino or dilatino, and likewise there are no corresponding massless gauginos in the $\overline{V_0 + V_1}$ sector. The fact that these models have $N_b^{(0)} = N_f^{(0)}$ at the level of the spectrum, can be confirmed by the original and large-radius Poisson-resummed partition functions. This can also be seen in the evaluation of the untwisted partition function, which contains no constant term.

Sector	ψ^{34}	ψ^{56}	χ^{34}	y^{34}	ω^{34}	χ^{56}	y^{56}	ω^{56}	$\overline{y}^{34}$	$\overline{\omega}^{34}$	$\overline{y}^{56}$	$\overline{\omega}^{56}$	$\overline{\psi}^1$	$\overline{\psi}^2$	$\overline{\psi}^3$	$\overline{\psi}^4$	$\overline{\psi}^5$	$\overline{\eta}^1$	$\overline{\eta}^2$	$\overline{\eta}^3$	$\overline{\phi}^1$	$\overline{\phi}^2$	$\overline{\phi}^3$	$\overline{\phi}^4$	$\overline{\phi}^5$	$\overline{\phi}^6$	$\overline{\phi}^7$	$\overline{\phi}^8$
V_0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_1	0	0	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_2	0	0	1	0	1	1	0	1	0	1	0	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
b_3	1	0	$\overline{1}$	0	$\overline{0}$	$\overline{0}$	0	$\overline{1}$	0	0	0	1	1	1	1	1	1	0	1	0	0	0	0	0	1	1	1	1
V_4	0	0	1	0	1	1	0	1	0	1	0	1	0	0	0	0	0	0	1	1	0	0	0	0	1	1	1	1
V_7	0	0	0	$\overline{1}$	$\overline{1}$	0	$\overline{0}$	$\overline{0}$	1	1	1	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1	1	0	0	1	0	0
e	0	0	1	0	1	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	1	1	1	0	1	1	0	1

Table 8.2: Spin structure of the worldsheet fermions of the $\mathcal{N} = 0$, $4D$ $SU(5)$ theory.

8.3 SM-like theories

In a similar way, it is also possible to construct SM-like models with $N_b^{(0)} = N_f^{(0)}$. Recall that the $\mathcal{N} = 0$, $4D$ theory described in Section 7.2 of Chapter 7, has a single complex Higgs pair that remains massless, as well as matter fields, two from the untwisted V_2 sector and extra ones from the twisted sectors. Given this, it is possible to construct models with similar mass spectra and with $N_b^{(0)} = N_f^{(0)}$; one such model is presented in Table 8.3. In this example, there are $N_b^{(0)} = N_f^{(0)} = 136$ complex massless bosons and fermions. The structure constants of this theory are given by

$$k_{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{3}{4} \\ \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{pmatrix}, \tag{8.3.1}$$

while the full gauge group is given by

$$G = [SO(4)]^3 \otimes [U(1)]^9 \otimes \underbrace{U(3) \otimes U(2)}_{\text{contains SM}}. \tag{8.3.2}$$

In here the $U(1)_Y$ is the same as the one defined in Eq. (6.3.5). The resulting mass spectrum from the $\overline{\alpha V} = \mathbf{0}$ and $\overline{V_0 + V_1}$ sectors is the same as that summarised in Tables 6.3 and 6.4, including extra “Higgses”. The model has two

Sector	ψ^{34}	ψ^{56}	χ^{34}	y^{34}	ω^{34}	χ^{56}	y^{56}	ω^{56}	$\overline{y}^{34}$	$\overline{\omega}^{34}$	$\overline{y}^{56}$	$\overline{\omega}^{56}$	$\overline{\psi}^1$	$\overline{\psi}^2$	$\overline{\psi}^3$	$\overline{\psi}^4$	$\overline{\psi}^5$	$\overline{\eta}^1$	$\overline{\eta}^2$	$\overline{\eta}^3$	$\overline{\phi}^1$	$\overline{\phi}^2$	$\overline{\phi}^3$	$\overline{\phi}^4$	$\overline{\phi}^5$	$\overline{\phi}^6$	$\overline{\phi}^7$	$\overline{\phi}^8$
V_0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_1	0	0	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_2	0	0	1	0	1	1	0	1	0	1	0	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
b_3	1	0	$\overline{1}$	0	$\overline{0}$	$\overline{0}$	0	$\overline{1}$	0	0	0	1	1	1	1	1	1	0	1	0	0	0	1	1	1	0	0	1
V_4	0	0	1	0	1	1	0	1	0	1	0	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
V_5	0	0	0	$\overline{0}$	$\overline{0}$	0	$\overline{1}$	$\overline{1}$	0	1	0	1	1	1	1	0	0	0	1	0	0	0	1	0	0	1	1	1
V_7	0	0	0	$\overline{1}$	$\overline{1}$	0	$\overline{0}$	$\overline{0}$	0	1	0	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	1	1	0	0	0	0
e	0	0	1	0	1	1	0	1	0	0	0	1	0	0	0	0	0	0	0	0	1	1	1	0	1	1	1	1

Table 8.3: Spin structure of the worldsheet fermions of the $\mathcal{N} = 0$, $4D$ SM-like theory.

entire supersymmetric chiral generations arising in the $\overline{V_0 + V_2}$ sector, with the spectrum shown in Table 6.5. There also appears to be a third untwisted generation in the $\overline{V_0 + V_1 + V_4 + V_7}$ and $\overline{V_0 + V_1 + V_4 + 3V_7}$ sectors, while the twisted b_3 and $\overline{b_3 + V_4}$ sectors provide mainly singlets with additional Higgs/Higgsinos. This model could be considered to be quite remarkable. It has chiral generations of SM-like matter and is clearly non-supersymmetric, but it also has equal numbers of massless bosons and fermions and hence an exponentially small one-loop cosmological constant.

8.4 A Pati-Salam-like theory

An alternative route to achieving $N_b^{(0)} = N_f^{(0)}$ is to remove the final breaking to unitary gauge groups which is driven by V_7. As was mentioned earlier, this vector makes the task of building a consistent modular invariant model significantly more difficult due to the $-\frac{1}{4}$ phases. As a result, constructions that do not involve a V_7 sector are significantly less constrained than those with V_7. The enlarged theory obtained from the SM-like theory presented above but without the additional V_6 and V_7 vectors is Pati-Salam-like with 208 massless complex d.o.f in total. The spin structure of this theory is defined in Table 8.4, the associated structure constants

Sector	ψ^{34}	ψ^{56}	χ^{34}	y^{34}	ω^{34}	χ^{56}	y^{56}	ω^{56}	$\bar{y}^{34}$	$\bar{\omega}^{34}$	$\bar{y}^{56}$	$\bar{\omega}^{56}$	$\bar{\psi}^1$	$\bar{\psi}^2$	$\bar{\psi}^3$	$\bar{\psi}^4$	$\bar{\psi}^5$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^1$	$\bar{\phi}^2$	$\bar{\phi}^3$	$\bar{\phi}^4$	$\bar{\phi}^5$	$\bar{\phi}^6$	$\bar{\phi}^7$	$\bar{\phi}^8$
V_0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_1	0	0	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_2	0	0	1	0	1	1	0	1	0	1	0	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
b_3	1	0	$\bar{1}$	0	$\bar{0}$	$\bar{0}$	0	$\bar{1}$	0	0	0	1	1	1	1	1	1	0	0	1	1	0	0	0	0	1	1	1
V_4	0	0	1	0	1	1	0	1	0	1	0	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
V_5	0	0	0	$\bar{0}$	$\bar{0}$	0	$\bar{1}$	$\bar{1}$	0	1	0	0	1	1	1	0	0	0	0	0	1	1	1	0	0	1	1	1
e	0	0	1	0	1	1	0	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1

Table 8.4: Spin structure of the worldsheet fermions of the $\mathcal{N} = 0$, $4D$ $SO(6)\otimes S0(4)$ model.

are

$$k_{ij} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{8.4.1}$$

and the full gauge group is given by

$$G = [SO(4)]^3 \otimes [U(1)]^6 \otimes \underbrace{SO(6) \otimes SO(4)}_{\text{contains SM}} \otimes SO(6)\,. \tag{8.4.2}$$

The spectrum for $SO(2N)$ representations can be decomposed under the corresponding $U(N)$ in a complex basis of worldsheet fermions, so that for example the adjoint of $SO(4) \sim SU(2)_L \otimes SU(2)_R$ is a $\mathbf{1} \oplus \mathbf{4} \oplus \overline{\mathbf{1}}$ of $U(2)$ [which is related as $U(2) \supset SU(2) \equiv SU(2)_L$]. As always, the **NS-NS** sector gives rise to the gravity multiplet as well as the adjoint gauge bosons, including the **15** and **6** adjoint gauge bosons of the visible sector. Again the theory is characterised by the absence of a massless gravitino or dilatino and there are no corresponding massless gauginos in the $\overline{V_0 + V_1}$ sector. In addition, the removal of both V_6 and V_7 naturally enables more pairs of light Higgs scalars and singlets to survive the GSO and orbifold projections in the **NS-NS** sector. Therefore, more than one pair of Higgses and singlets remain massless after CDC. Specifically, the complex scalar electroweak doublets $\mathbb{H}$ that survive the GSO and the orbifold projections are

$$\begin{aligned}
\mathbb{H}_1 &\equiv \{H_{U_1}, H_{D_1}\} &=& \ \psi^{56}_{-\frac{1}{2}}\,|0\rangle_R \otimes \overline{\psi}^{4,5}_{-\frac{1}{2}}\overline{\eta}^{1}_{-\frac{1}{2}}\,|0\rangle_L \\
\mathbb{H}_2 &\equiv \{H_{U_2}, H_{D_2}\} &=& \ \psi^{56}_{-\frac{1}{2}}\,|0\rangle_R \otimes \overline{\psi}^{4,5}_{-\frac{1}{2}}\overline{y}^{36,45}_{-\frac{1}{2}}\,|0\rangle_L \\
\mathbb{H}_3 &\equiv \{H_{U_3}, H_{D_3}\} &=& \ \chi^{36}_{-\frac{1}{2}}\,|0\rangle_R \otimes \overline{\psi}^{4,5}_{-\frac{1}{2}}\overline{\eta}^{3}_{-\frac{1}{2}}\,|0\rangle_L \\
\mathbb{H}_4 &\equiv \{H_{U_4}, H_{D_4}\} &=& \ \chi^{36}_{-\frac{1}{2}}\,|0\rangle_R \otimes \overline{\psi}^{4,5}_{-\frac{1}{2}}\overline{\omega}^{45}_{-\frac{1}{2}}\,|0\rangle_L \\
\mathbb{H}_5 &\equiv \{H_{U_5}, H_{D_5}\} &=& \ \chi^{45}_{-\frac{1}{2}}\,|0\rangle_R \otimes \overline{\psi}^{4,5}_{-\frac{1}{2}}\overline{\eta}^{2}_{-\frac{1}{2}}\,|0\rangle_L\,.
\end{aligned} \tag{8.4.3}$$

Note that in these expressions, the labels of the left-moving internal complex fermions are different from those of the SM-like theory defined in Eq. (7.2.25) and (7.2.26). The purpose of this labelling is to ensure that the horizontal symmetries in the PS-model are completely aligned. Similarly, the singlets $\mathbb{X}$ and exotic states $\mathbb{E}$ that survive the projections are

$$\begin{aligned}
\mathbb{X}_1 &\equiv \{X_1, X_1'\} &=& \ \psi^{56}_{-\frac{1}{2}}\,|0\rangle_R \otimes \overline{\eta}^{2}_{-\frac{1}{2}}\overline{\eta}^{3}_{-\frac{1}{2}}\,|0\rangle_L \\
\mathbb{X}_2 &\equiv \{X_2, X_2'\} &=& \ \psi^{56}_{-\frac{1}{2}}\,|0\rangle_R \otimes \overline{\omega}^{45}_{-\frac{1}{2}}\overline{\eta}^{2}_{-\frac{1}{2}}\,|0\rangle_L \\
\mathbb{X}_3 &\equiv \{X_3, X_3'\} &=& \ \chi^{36}_{-\frac{1}{2}}\,|0\rangle_R \otimes \overline{\eta}^{1}_{-\frac{1}{2}}\overline{\eta}^{2}_{-\frac{1}{2}}\,|0\rangle_L \\
\mathbb{X}_4 &\equiv \{X_4, X_4'\} &=& \ \chi^{36}_{-\frac{1}{2}}\,|0\rangle_R \otimes \overline{y}^{36,45}_{-\frac{1}{2}}\overline{\eta}^{2}_{-\frac{1}{2}}\,|0\rangle_L \\
\mathbb{X}_5 &\equiv \{X_5, X_5'\} &=& \ \chi^{45}_{-\frac{1}{2}}\,|0\rangle_R \otimes \overline{\eta}^{1}_{-\frac{1}{2}}\overline{\eta}^{3}_{-\frac{1}{2}}\,|0\rangle_L \\
\mathbb{X}_6 &\equiv \{X_6, X_6'\} &=& \ \chi^{45}_{-\frac{1}{2}}\,|0\rangle_R \otimes \overline{y}^{36,45}_{-\frac{1}{2}}\overline{\omega}^{45}_{-\frac{1}{2}}\,|0\rangle_L \\
\mathbb{X}_7 &\equiv \{X_7, X_7'\} &=& \ \chi^{45}_{-\frac{1}{2}}\,|0\rangle_R \otimes \overline{\omega}^{45}_{-\frac{1}{2}}\overline{\eta}^{1}_{-\frac{1}{2}}\,|0\rangle_L \\
\mathbb{X}_8 &\equiv \{X_8, X_8'\} &=& \ \chi^{45}_{-\frac{1}{2}}\,|0\rangle_R \otimes \overline{y}^{36,45}_{-\frac{1}{2}}\overline{\eta}^{3}_{-\frac{1}{2}}\,|0\rangle_L \\
&\mathbb{E} &=& \ \chi^{45}_{-\frac{1}{2}}\,|0\rangle_R \otimes \overline{\psi}^{1,2,3}_{-\frac{1}{2}}\overline{\omega}^{36}_{-\frac{1}{2}}\,|0\rangle_L\,.
\end{aligned} \tag{8.4.4}$$

Likewise more Higgsino-like states and other Weyl spinors are left massless; these are states which appear in the $\overline{V_0+V_1}$ sector. The resulting mass spectrum from the $\overline{\alpha V} = \mathbf{0}$ and the $\overline{V_0+V_1}$ sectors is summarised in Tables 8.5 and 8.6. Naturally, there are also additional states from the hidden sector that survive the GSO and orbifold projections. However, beyond their contributions to enforcing $N_b^{(0)} = N_f^{(0)}$, these states do not contribute anything phenomenologically and thus will not be considered in this discussion.

Sector	Massless states after CDC	Spin	Representations	Particles
0	$\psi^{34}_{-\frac{1}{2}}\,\lvert 0\rangle_R \otimes X^{34}_{-1}\,\lvert 0\rangle_L$	2 0	(1, 1, 1)	$g_{\mu\nu}$, $B_{[\mu\nu]}$ Dilaton ϕ
	$\psi^{56}_{-\frac{1}{2}}\,\lvert 0\rangle_R \otimes X^{56}_{-1}\,\lvert 0\rangle_L$	0	(1, 1, 1)	Complex radion Φ
	$\psi^{34}_{-\frac{1}{2}}\,\lvert 0\rangle_R \otimes \Psi^i_{-\frac{1}{2}}\Psi^j_{-\frac{1}{2}}\,\lvert 0\rangle_L$	1	(Adj, 1, 1)+ (1, Adj, 1)+ (1, 1, Adj)+ Adj G_{hidden}	Gauge bosons $\mathbb{V}_\mu$
	$\psi^{56}_{-\frac{1}{2}}\,\lvert 0\rangle_R \otimes \Psi^i_{-\frac{1}{2}}\Psi^j_{-\frac{1}{2}}\,\lvert 0\rangle_L$	0	(1, **2**, **2**) (1, 1, 1)	Complex scalar $\mathbb{H}_1$ Complex scalar $\mathbb{X}_1$
	$\chi^{36}_{-\frac{1}{2}}\,\lvert 0\rangle_R \otimes \Psi^i_{-\frac{1}{2}}\Psi^j_{-\frac{1}{2}}\,\lvert 0\rangle_L$	0	(1, **2**, **2**) (1, 1, 1)	Complex scalar $\mathbb{H}_4$ Complex scalar $\mathbb{X}_4$
	$\chi^{45}_{-\frac{1}{2}}\,\lvert 0\rangle_R \otimes \Psi^i_{-\frac{1}{2}}\Psi^j_{-\frac{1}{2}}\,\lvert 0\rangle_L$	0	(1, 1, 1) (1, 1, 1)	Complex scalar $\mathbb{X}_7$ Complex scalar $\mathbb{X}_8$
$\overline{V_0 + V_1}$	$\psi^{56}_{0}\,\lvert \alpha\rangle_R \otimes \Psi^i_{-\frac{1}{2}}\Psi^j_{-\frac{1}{2}}\,\lvert 0\rangle_L$	$\frac{1}{2}$	(1, **2**, **2**) (1, 1, 1)	Weyl spinor $\tilde{\mathbb{H}}_2$ Weyl spinor $\tilde{\mathbb{X}}_2$
	$\chi^{36}_{0}\,\lvert \alpha\rangle_R \otimes \Psi^i_{-\frac{1}{2}}\Psi^j_{-\frac{1}{2}}\,\lvert 0\rangle_L$	$\frac{1}{2}$	(1, **2**, **2**) (1, 1, 1)	Weyl spinor $\tilde{\mathbb{H}}_3$ Weyl spinor $\tilde{\mathbb{X}}_3$
	$\chi^{45}_{0}\,\lvert \alpha\rangle_R \otimes \Psi^i_{-\frac{1}{2}}\Psi^j_{-\frac{1}{2}}\,\lvert 0\rangle_L$	$\frac{1}{2}$	(1, **2**, **2**) (1, 1, 1) (1, 1, 1) (**4**, 1, 1)	Weyl spinor $\tilde{\mathbb{H}}_5$ Weyl spinor $\tilde{\mathbb{X}}_5$ Weyl spinor $\tilde{\mathbb{X}}_6$ Exotic spinor $\tilde{\mathbb{E}}$

Table 8.5: The $\mathbb{Z}_2$-untwisted visible-sector states of the $\mathcal{N} = 1$, $4D$ Pati-Salam theory which remain massless after the CDC. The Ψ_i refer to generic left-moving d.o.f, with indices $i, j = 1, \ldots, 20$. Here $\lvert\alpha\rangle_R$ refers to the remaining unspecified **R** ground states. The massless fields are identified according to the $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ representations.

The fermionic matter arises in the untwisted $\overline{V_0 + V_2}$ sector and is identified by the $(\mathbf{4}, \mathbf{2}, \mathbf{1})$ and $(\mathbf{4}, \mathbf{1}, \mathbf{2})$ representations of $SU(4) \otimes SU(2)_L \otimes SU(2)_R$, where the SM

Sector	States projected after CDC	Spin	Representations	Particles
$\overline{V_0 + V_1}$	$\lvert\alpha\rangle_R \otimes X^{34}_{-1}\lvert 0\rangle_L$	$\frac{3}{2}$	$(1, 1, 1)$	Gravitino ψ_μ
		$\frac{1}{2}$		Dilatino $\widetilde{\phi}$
	$\lvert\alpha\rangle_R \otimes X^{56}_{-1}\lvert 0\rangle_L$	$\frac{1}{2}$	$(1, 1, 1)$	Radino $\widetilde{\Phi}$
	$\lvert\alpha\rangle_R \otimes \Psi^i_{-\frac{1}{2}}\Psi^j_{-\frac{1}{2}}\lvert 0\rangle_L$	$\frac{1}{2}$	$(\text{Adj}, 1, 1)+$ $(1, \text{Adj}, 1)+$ $(1, 1, \text{Adj})+$ Adj G_{hidden}	Gauginos λ_μ
	$\psi^{56}_0\lvert\alpha\rangle_R \otimes \Psi^i_{-\frac{1}{2}}\Psi^j_{-\frac{1}{2}}\lvert 0\rangle_L$	$\frac{1}{2}$	$(1, \mathbf{2}, \mathbf{2})$	Weyl spinor $\tilde{\mathbb{H}}_1$
			$(1, 1, 1)$	Weyl spinor $\tilde{\mathbb{X}}_1$
	$\chi^{36}_0\lvert\alpha\rangle_R \otimes \Psi^i_{-\frac{1}{2}}\Psi^j_{-\frac{1}{2}}\lvert 0\rangle_L$	$\frac{1}{2}$	$(1, \mathbf{2}, \mathbf{2})$	Weyl spinor $\tilde{\mathbb{H}}_4$
			$(1, 1, 1)$	Weyl spinor $\tilde{\mathbb{X}}_4$
	$\chi^{45}_0\lvert\alpha\rangle_R \otimes \Psi^i_{-\frac{1}{2}}\Psi^j_{-\frac{1}{2}}\lvert 0\rangle_L$	$\frac{1}{2}$	$(1, 1, 1)$	Weyl spinor $\tilde{\mathbb{X}}_7$
			$(1, 1, 1)$	Weyl spinor $\tilde{\mathbb{X}}_8$
0	$\psi^{56}_{-\frac{1}{2}}\lvert 0\rangle_R \otimes \Psi^i_{-\frac{1}{2}}\Psi^j_{-\frac{1}{2}}\lvert 0\rangle_L$	$\frac{1}{2}$	$(1, \mathbf{2}, \mathbf{2})$	Complex scalar $\mathbb{H}_2$
			$(1, 1, 1)$	Complex scalar $\mathbb{X}_2$
	$\chi^{36}_{-\frac{1}{2}}\lvert 0\rangle_R \otimes \Psi^i_{-\frac{1}{2}}\Psi^j_{-\frac{1}{2}}\lvert 0\rangle_L$	$\frac{1}{2}$	$(1, \mathbf{2}, \mathbf{2})$	Complex scalar $\mathbb{H}_3$
			$(1, 1, 1)$	Complex scalar $\mathbb{X}_3$
	$\chi^{45}_{-\frac{1}{2}}\lvert 0\rangle_R \otimes \Psi^i_{-\frac{1}{2}}\Psi^j_{-\frac{1}{2}}\lvert 0\rangle_L$	$\frac{1}{2}$	$(1, \mathbf{2}, \mathbf{2})$	Complex scalar $\mathbb{H}_5$
			$(1, 1, 1)$	Complex scalar $\mathbb{X}_5$
			$(1, 1, 1)$	Complex scalar $\mathbb{X}_6$
			$(\mathbf{4}, 1, 1)$	Exotic boson $\mathbb{E}$

Table 8.6: The $\mathbb{Z}_2$-untwisted visible-sector states of the $\mathcal{N} = 1$, $4D$ Pati-Salam theory that will accrue a mass of $\frac{1}{2}\sqrt{R_1^{-2} + R_2^{-2}}$ by the CDC. The Ψ_i refer to generic left-moving d.o.f, with indices $i, j = 1, \ldots, 20$ and the massless fields are identified according to the $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ representations.

matter fields are embedded as

$$\begin{aligned} \mathbb{F}_L &\equiv \{Q_L, L_L\} \\ \mathbb{F}_R &\equiv \{e_R, \nu_R, u_R, d_R\}\,. \end{aligned} \tag{8.4.5}$$

The identification of $SU(2)_L \otimes SU(2)_R \sim SO(4)$ implies then that the $\overline{\mathbf{2}}$ spinor of $SO(4)$ is in the fundamental of $SU(2)_L$ while the $\mathbf{2}$ spinor is in the fundamental of

Sector	States remaining after CDC	Spin	Representations	Particles
$\overline{V_0+V_2}$	$\lvert\alpha\rangle_R \otimes \overline{\psi}^i_0 \overline{\psi}^a_0 \lvert\hat{\alpha}\rangle_L$ $\lvert\alpha\rangle_R \otimes \overline{\psi}^1_0 \overline{\psi}^2_0 \overline{\psi}^3_0 \overline{\psi}^a_0 \lvert\hat{\alpha}\rangle_L$	$\frac{1}{2}$	$(\mathbf{4},\mathbf{2},\mathbf{1})$	$\mathbb{F}_L$
	$\lvert\alpha\rangle_R \otimes \lvert\hat{\alpha}\rangle_L$ $\lvert\alpha\rangle_R \otimes \overline{\psi}^4_0 \overline{\psi}^5_0 \lvert\hat{\alpha}\rangle_L$ $\lvert\alpha\rangle_R \otimes \overline{\psi}^i_0 \overline{\psi}^j_0 \lvert\hat{\alpha}\rangle_L$ $\lvert\alpha\rangle_R \otimes \overline{\psi}^i_0 \overline{\psi}^j_0 \overline{\psi}^4_0 \overline{\psi}^5_0 \lvert\hat{\alpha}\rangle_L$	$\frac{1}{2}$	$(\mathbf{4},\mathbf{1},\mathbf{2})$	$\mathbb{F}_R$
$\overline{V_1+V_2}$	$\lvert\alpha\rangle_R \otimes \lvert\beta\rangle_L$	0	$(\mathbf{4},\mathbf{2},\mathbf{1})$	Exotic spinor $\mathbb{E}$
	$\lvert\alpha\rangle_R \otimes \lvert\beta\rangle_L$	0	$(\mathbf{4},\mathbf{1},\mathbf{2})$	Complex scalar $\mathbb{K}$

Table 8.7: Chiral $\mathbb{Z}_2$-untwisted multiplets of the $\mathcal{N}=1$, $4D$ Pati-Salam theory that remain massless after the CDC. Here $i,j \in SU(4)$ and $a \in SU(2)_L \otimes SU(2)_R$. The $\lvert\alpha\rangle_R$ represent right-moving **R** ground states which are spacetime spinors, while $\lvert\hat{\alpha}\rangle_L$ and $\lvert\beta\rangle_L$ represent different left-moving **R** excitations that do not overlap with the Pati-Salam gauge group. The multiplets are essentially decomposed under the **16** of $SO(10)$, but here the massless fields are identified according to the $SU(4)\otimes SU(2)_L \otimes SU(2)_R$ representations.

$SU(2)_R$. Meanwhile the electroweak Higgses $\mathbb{H}$ are in the fundamental of $SO(4)$ which corresponds to the $(\mathbf{2},\mathbf{2})$. The visible matter in this particular example is still quasi-supersymmetric. The removal of the V_6 and V_7 vectors leaves some of the horizontal symmetries embedded in the gauge group unbroken. As a result, there are more generations of visible matter than in the SM-like theory. In particular, there is an unbroken $SO(4)$ horizontal symmetry, arising from the $\overline{y}^{36}, \overline{y}^{45}$ fermions, which allows four chiral generations from the $\overline{V_0+V_2}$ sector rather than the two of the SM-like theory. Similarly to the SM-like theory, this Pati-Salam-like theory has no massless twisted-sector matter.

Despite the globally supersymmetric matter spectra, the scalar partners would be expected to pick up masses from RG running in the usual way. As a result, the theory is somewhat "no-scale" from the point of view of the visible sector, with gauginos dominating the contributions. It should be noted that the $(\mathbf{4},\mathbf{1},\mathbf{2})$ scalars can play the role of the Higgs field $\mathbb{K}$ in breaking the Pati-Salam gauge symmetry down to the SM gauge symmetry. The mass spectrum for the generations of matter fields in the theory is summarised in Tables 8.7 and 8.8. Of course, in presenting

Sector	States projected after CDC	Spin	Representations	Particles
$\overline{V_1+V_2}$	$\lvert\alpha\rangle'_R \otimes \lvert\beta\rangle_L$	$\frac{1}{2}$	$(\mathbf{4},\mathbf{2},\mathbf{1})$	Spinor $\tilde{\mathbb{E}}$
	$\lvert\alpha\rangle'_R \otimes \lvert\beta\rangle_L$	$\frac{1}{2}$	$(\mathbf{4},\mathbf{1},\mathbf{2})$	Spinor $\tilde{\mathbb{K}}$
$\overline{V_0+V_2}$	$\lvert\alpha\rangle'_R \otimes \overline{\psi}^i_0\overline{\psi}^a_0 \lvert\hat{\alpha}\rangle_L$	0	$(\mathbf{4},\mathbf{2},\mathbf{1})$	$\tilde{\mathbb{F}}_L$
	$\lvert\alpha\rangle'_R \otimes \overline{\psi}^1_0\overline{\psi}^2_0\overline{\psi}^3_0\overline{\psi}^a_0 \lvert\hat{\alpha}\rangle_L$			
	$\lvert\alpha\rangle'_R \otimes \lvert\hat{\alpha}\rangle_L$			
	$\lvert\alpha\rangle'_R \otimes \overline{\psi}^4_0\overline{\psi}^5_0 \lvert\hat{\alpha}\rangle_L$	0	$(\mathbf{4},\mathbf{1},\mathbf{2})$	$\tilde{\mathbb{F}}_R$
	$\lvert\alpha\rangle'_R \otimes \overline{\psi}^i_0\overline{\psi}^j_0 \lvert\hat{\alpha}\rangle_L$			
	$\lvert\alpha\rangle'_R \otimes \overline{\psi}^i_0\overline{\psi}^j_0\overline{\psi}^4_0\overline{\psi}^5_0 \lvert\hat{\alpha}\rangle_L$			

Table 8.8: Chiral $\mathbb{Z}_2$-untwisted multiplets of the $\mathcal{N}=1$, $4D$ Pati-Salam theory which will accrue a mass of $\frac{1}{2}\sqrt{R_1^{-2}+R_2^{-2}}$ by the CDC. As in the previous table $i,j\in SU(4)$ while $a\in SU(2)_L\otimes SU(2)_R$. The $\lvert\alpha\rangle'_R$ represent right-moving **R** ground states that are not spacetime spinors and once again the massless fields are identified according to the $SU(4)\otimes SU(2)_L\otimes SU(2)_R$ representations.

a Pati-Salam-like theory, there is an implicit assumption that the final stage of symmetry breaking can be consigned to the EFT without destabilising the original theory. As is evident above, this Pati-Salam theory has the Higgs fields required for this additional stage of breaking.

To conclude, all four different models presented in this section have $N_b^{(0)}=N_f^{(0)}$ and therefore all have exponentially suppressed one-loop cosmological constants and dilaton tadpoles. Whilst none of these models are completely realistic, it is evident that from a phenomenological point of view each exhibits varying degrees of success. Therefore each model can be viewed as a stepping stone for the potential construction and development of more refined theories with enhanced phenomenological properties.

Chapter 9

Properties of string models with exponentially suppressed cosmological constants

When I am working on a problem I never think about beauty. I only think about how to solve the problem. But when I have finished, if the solution is not beautiful, I know it is wrong.

Buckminster Fuller

9.1 Interpolation Properties

The successful exponential suppression of their cosmological constants sparks a torrent of work on the properties of models with equal numbers of massless bosons and fermions. Given that these non-supersymmetric, tachyon-free models belong to the class of interpolating models, it is a priority to investigate their interpolation associated behaviour. There are several subjects in the agenda including: the mass spectrum behaviour at large radii, and if one expects the small radius limit to exhibit the same form; the interpolation of the cosmological constant Λ from large to small radii of compactification; an examination of whether there is a possibility of tachyons appearing at some critical radius $R \sim 1$, signalling a Hagedorn-like instability and last but perhaps not least, whether there is restoration of gauge symmetry and/or SUSY.

9.1.1 Behaviour of mass spectrum

In the limit of large interpolating radius, *i.e.* $a \equiv R^{-1} \to 0$, the low-energy spectrum of models with exponentially suppressed cosmological constants is divided into a visible and a hidden sector. In general, "visible" refers to the states associated with the SM (or one of its unified extensions) and "hidden" refers to states which do not carry SM gauge quantum numbers. Along with these states, both sectors also contain their would-be superpartners whose masses are lifted by the CDC, so that they acquire a mass $\sim (2R)^{-1}$. As a result, neither sector is supersymmetric (except in the infinite-radii limit where the theory becomes higher dimensional and SUSY is restored). At low energies, the particle landscape is poor of variety, in the sense that the lightest states in each sector consist of the massless ($n = 0$) string states, along with their KK excitations. This picture is altered dramatically at higher mass levels $M > M_{string}$ because these sectors also contain string oscillator states, winding states and of course the twisted sectors contribute additional string states. This is a generic behaviour of the mass spectrum which applies for *all* semi-realistic interpolating models. However, interpolating models with suppressed cosmological constants have a distinct feature: the visible sector contributes exactly equal and opposite net (bosonic minus fermionic) numbers of string states with masses $M < M_{string}$ to those from the hidden sector. This property holds for all sufficiently small radii, even if the strict $R \to \infty$ limit is not satisfied. This general structure of the mass spectrum for the semi-realistic interpolating models is depicted schematically in Fig. 9.1.

A remark worth emphasising is that for the *lightest physical states in the spectrum*, *i.e.* $M < M_{string}$, the cancellation of net physical-state degeneracies between the visible and hidden sectors does not imply that this is the result of any spacetime SUSY, either exact or approximate, in the string spectrum. Nonetheless, this

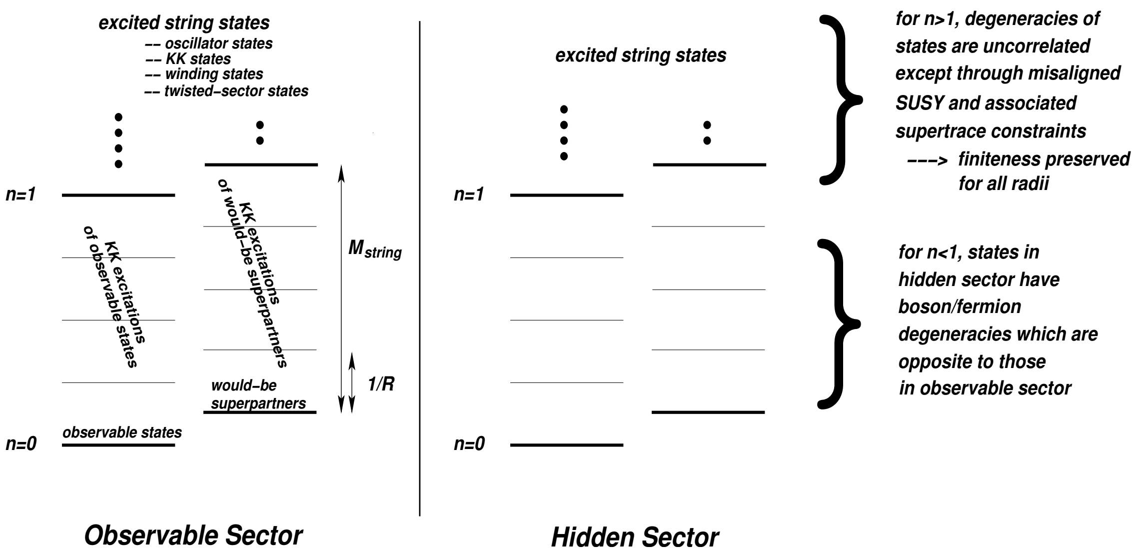


Figure 9.1: A schematic illustration of the mass spectrum structure for a generic interpolating model with suppressed cosmological constant in the limit of large interpolating radius. At mass scales below M_{string} (or below $n = 1$) the spectrum consists of massless states coming from the visible and hidden sectors, their would-be superpartners and their lightest KK excitations. For these lightest states, the net physical-state degeneracies between the visible and hidden sectors cancel for all large radii. This property leads to the exponential suppression of the cosmological constant and hence the dilaton tadpole, so that the stability properties of the theory are enhanced. At mass scales above M_{string} the mass spectrum includes also string oscillator, winding and twisted sector states. Therefore, there are no longer cancellations between the net numbers of d.o.f from the visible and hidden sectors. Nevertheless, the behaviour of these states is governed by misaligned supersymmetry which guarantees the finiteness of the theory. This figure is adapted from Refs. [15, 16].

cancellation has a vital role because it sits at the root of the one-loop cosmological constant's exponential suppression. Consequently, this guarantees the suppression of the dilaton tadpole, thereby enhancing the stability properties of these strings. By contrast, for the heavier states, *i.e.* $M > M_{string}$, the visible and hidden sectors can no longer contribute equal and opposite numbers of d.o.f due to the variety of states in the spectrum. Nevertheless, the properties of these sectors are governed by misaligned SUSY constraints, and the entire string spectrum continues to satisfy the supertrace relations in Eq. (4.3.6). These relations ensure that the overall string theory maintains its UV finiteness even without spacetime SUSY.

These properties of the mass spectrum could be better perceived if one rewinds back to Chapter 5, where the general framework of the $\mathbb{Z}_2$ interpolating models is presented. Recall that the interpolating models constructed via a compactification on a $\mathbb{Z}_2$ orbifold have four sectors whose states contribute to the partition function traces $Z^{(a)}$, with $a = 1, 2, 3, 4$. The general form of the partition function for such interpolating models is then found to be the one given in Eq. (5.2.4). Under the assumption that SUSY is restored in the $R \to \infty$ limit it is deduced that at the level of q-expansions $Z^{(2)} = -Z^{(1)}$. Thus the partition function takes the form given in Eq. (5.4.1). The $Z^{(1)}$ contains the contributions from the massless states coming from the visible sectors, while the $Z^{(2)}$ contains the contributions from their would-be superpartners. For low-lying mass levels, the functions $\mathcal{E}_0$ and $\mathcal{E}_{1/2}$ serve to tally the KK excitations of the would-be superpartner states, which are shifted relative to each other by masses $\sim (2R)^{-1}$. Depending on the low-energy EFT, it is expected that the massless states of the visible sector do not contain an equal number of bosonic and fermionic d.o.f. It is then concluded that the only way to achieve $N_b^{(0)} = N_f^{(0)}$ is for the $Z^{(1)}$ itself to contain also contributions from a separate hidden sector, which is not necessarily related to the visible sector. The only requirement is for the hidden sector to provide the additional bosonic and fermionic d.o.f needed so as to fulfil the condition $N_b^{(0)} = N_f^{(0)}$ in $Z^{(1)}$. Since $Z^{(2)} = -Z^{(1)}$, the same must also be true of the would-be superpartners, whereupon their respective multiplications by the $\mathcal{E}_0$ and $\mathcal{E}_{1/2}$ functions implies that the same will also be true for their associated low-lying KK states.

At energy scales above M_{string}, $Z^{(1)}$ may generally have a non-zero net number of physical states. Thus, there are no exactly equal and opposite numbers of bosonic ($N_b^{(n)}$) and fermionic ($N_f^{(n)}$) d.o.f with masses m_n, where n is the energy level, and hence a condition of the form $N_b^{(n)} = N_f^{(n)}$ cannot be fulfilled. However, these heavier states are subjected to the misaligned SUSY constraint, and along with states across the entire string spectrum they contribute to the cancellation of UV divergences.

Based on these outcomes, it is easy to examine the behaviour of the actual physical-state degeneracies a_{nn} as a function of n for such interpolating models with suppressed one-loop cosmological constants. The entire spectrum of the Pati-

Salam-like theory defined by the spin structure in Table 8.4 is displayed in a plot of $\pm\log(|a_{nn}|)$ versus n. Such plots demonstrate the Bose-Fermi non-degeneracies of this theory and are shown in Fig. 9.2. The results are presented for several different values of the dimensionless inverse radii $a_{1,2} \equiv \sqrt{\alpha'} r_{1,2}^{-1}$ [15], and can be compared with the results for general interpolating models which are illustrated in Fig. 5.1. As required by misaligned SUSY, there are distinct bosonic/fermionic oscillations and for $a \sim \mathcal{O}(1)$, the physical-state degeneracies oscillate within smoothly growing exponential functions which are determined by the oscillator states. For $a_{1,2} \sim \mathcal{O}(1)$ the KK and winding states have masses similar to those of the oscillator states and thus their contributions are not readily distinguishable from those of the oscillator states. However, as $a_{1,2} \to 0$, it is observed that the KK states begin to separate out from the oscillator states, which leads to the step-wise growth in the envelope function shown in the last three panels of Fig. 9.2. From those three panels it is also deduced that for sufficiently large radii, the physical-state degeneracies a_{nn} for all relevant mass levels $n < 1$ develop exact Bose-Fermi degeneracies. Despite the absence of spacetime SUSY, the lightest physical-state degeneracies vanish exactly; this confirms the cancellation of bosonic against fermionic d.o.f between the observable and hidden sectors respectively, as illustrated in Fig. 9.1. The vanishing or "evacuation" of net physical-state degeneracies below $n = 1$ as $a_{1,2} \to 0$ is thus the hallmark of interpolating string models with exponentially suppressed cosmological constants and therefore enhanced stability properties.

There is another way to understand the behaviour of the mass spectrum of such non-supersymmetric theories. This is through the expressions in Eq. (7.2.11), where at large r_1, r_2 the non-zero j_1, j_2 terms can be neglected in the evaluation of the cosmological constant because they are heavily suppressed and can be considered negligible. So, at low energies the mass spectrum is almost identical to the massless spectrum. The difference is that the KK numbers of the lightest states are shifted by $\mathbf{e} \cdot \mathbf{Q}$ due to the CDC effects. However, $\mathbf{e} \cdot \mathbf{Q}$ shifts an equal number of bosons and fermions in the massless sector, as well as all of their KK modes. As a result, despite the fact that the spectrum is *completely* non-supersymmetric, it

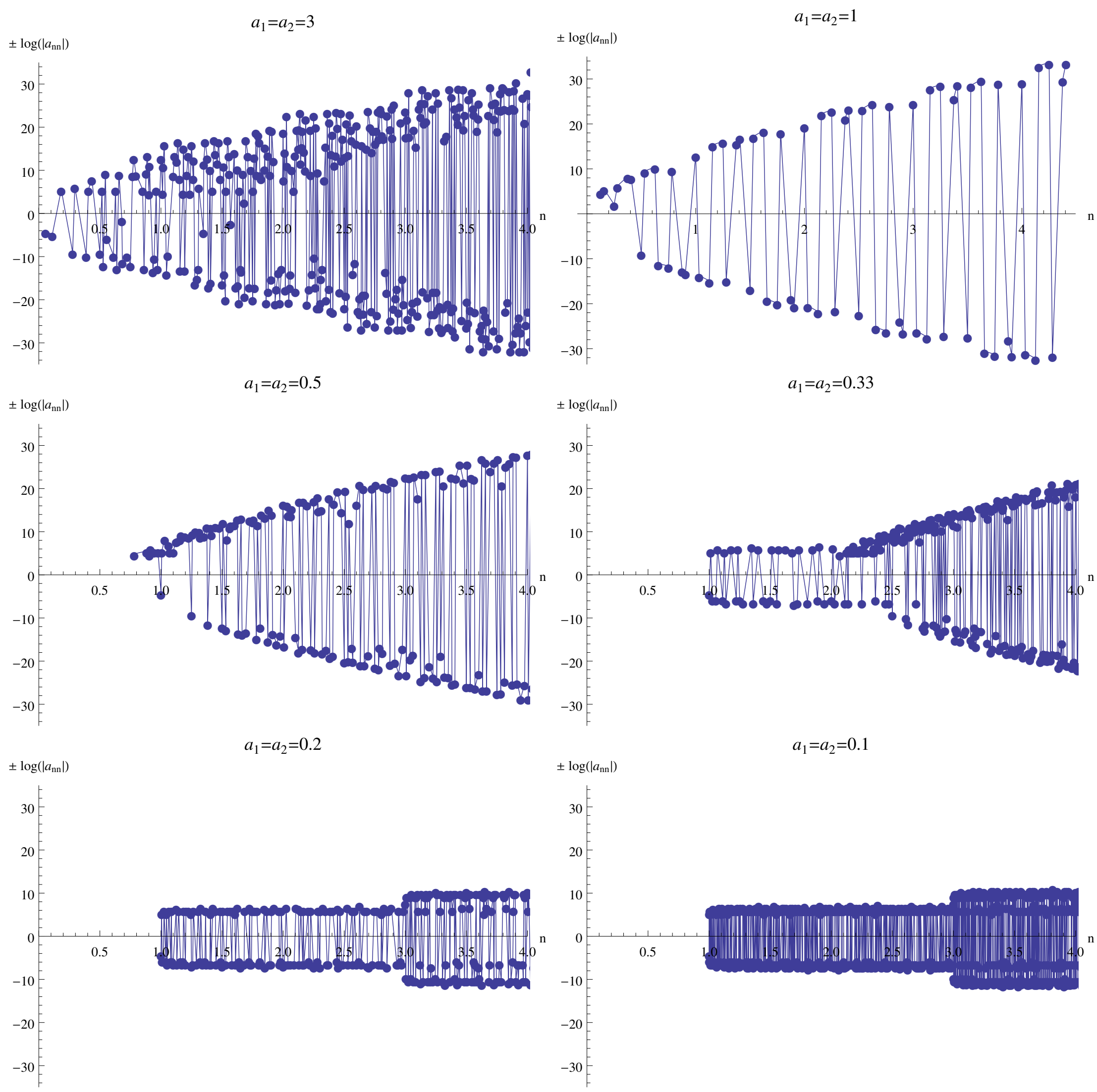


Figure 9.2: Degeneracies of physical states for the Pati-Salam-like theory defined in Table 8.4. The results are shown for inverse radii values that vary from $a_1 = a_2 = 3$ (upper left) to $a_1 = a_2 = 0.1$ (lower right). Comparing these plots with those in Fig. 5.1, it is evident that all the features associated with general interpolating models survive the CDC: First, there is a smoothly growing exponential envelope function for $a \sim \mathcal{O}(1)$ which slowly deforms into a discretely step-wise growing exponential function as $a \to 0$, reflecting the emerging hierarchy between KK and oscillator states. Second, it is observed that there are no net state degeneracies a_{nn} for $n \leq 1$ for sufficiently large radius. This critical feature implies that at this limit, the spectrum of such models develops exact Bose-Fermi degeneracies for all $n < 1$, even though spacetime SUSY is absent, whereupon it confirms the cosmological constant's exponential suppression. As illustrated in Fig. 9.1, this degeneracy is the result of cancelling non-zero net degeneracies associated with a non-supersymmetric *visible* sector against the degeneracies associated with a non-supersymmetric *hidden* sector. The plots are adapted from Refs. [15, 16].

actually exhibits $N_b = N_f$ *for all states up to the first-excited string oscillator mass level*.

Up until now, the analysis of the mass spectrum is performed in the large radii limit of the theories where the exponential suppression of the cosmological constant is achieved. At the beginning of this chapter, it was questioned whether the spectrum exhibits the same behaviour at the small radii limit. Considering instead this opposite limit, it is found that the $\jmath_1$, $\jmath_2$ become closely packed and the states with non-zero "net KK number" $\ell_{1,2} + \mathbf{e}\cdot\mathbf{Q} - \frac{1}{2}(\jmath_1 + \jmath_2)$ become very heavy. In order to understand this better, it is useful to consider first the winding modes of the massless sector that remains unshifted by CDC, *i.e.* $\mathbf{e}\cdot\mathbf{Q} = 0$, and that satisfy $N_b^{(0)} = N_f^{(0)}$. Denoting by $\mathbf{Q}^0$ the charges in this sector, it is observed that at small r_1, r_2, different winding modes can be light if they have a shift $\mathbf{Q} = \mathbf{Q}^0 + \mathbf{e}(\jmath_1 + \jmath_2)$, provided also that the corresponding KK numbers have a shift $\ell_i = -\frac{1}{2}(\jmath_1 + \jmath_2)$ so as to cancel the net contribution of the KK modes. Hence at generic but small r_1, r_2 only the even $\jmath_1 + \jmath_2$ combinations of the $\mathbf{e}\cdot\mathbf{Q} = 0$ states are light. Note that this statement does not mean that one simply takes the physical states with $\mathbf{e}\cdot\mathbf{Q}^0 = 0$ and maps them to a set of even winding modes with charge and net KK number given respectively by $\mathbf{Q} = \mathbf{Q}^0 + \mathbf{e}(\jmath_1 + \jmath_2)$ and $\ell_i = -\frac{1}{2}(\jmath_1 + \jmath_2)$. This would imitate what happens for the KK modes at large radii but it is incorrect to assume that it also happens in the small radii limit. The explanation for this lies in the fact that in the small radii limit, any shift in $\mathbf{Q}$ affects the factor g (and subsequently affects the GSO projection) in Eq. (7.2.9) which includes a phase $2\pi i\beta V\cdot\mathbf{Q}$. This phase is shifted by a factor $2\pi i(\jmath_1 + \jmath_2)\beta V\cdot\mathbf{e}$ w.r.t the non-winding sector, and some of the overlaps $V_i\cdot\mathbf{e}$ generate $\frac{1}{4}$-integer values. Thus, while the subset of winding modes that satisfies $\jmath_1 + \jmath_2 = 0 \bmod (4)$ still exhibits the $N_b^{(0)} = N_f^{(0)}$ cancellation of the massless sector, the subset of winding modes that satisfies $\jmath_1 + \jmath_2 = 4k + 2 \bmod (4)$, $k \in \mathbb{Z}$ has different projections and generally does not exhibit this cancellation. This could be seen within the large-($a_1 = a_2$) plot in the first two panels of Fig. 9.2, where it is evident that the light states do not exhibit Bose-Fermi degeneracies. It is therefore concluded that maintaining the exponential suppression of the cosmological constant when the model interpolates to $r_{1,2} \to 0$ (or $a_{1,2} \to \infty$) is no longer feasible.

Meanwhile, the winding modes of the states that satisfy $\mathbf{e} \cdot \mathbf{Q} \neq 0$ and thus become massive at the large radii limit, behave differently at the opposite limit. At the small radii limit these states can have low-lying *odd* winding modes while the KK numbers are shifted, with the $\ell_{1,2}$ again compensating to make the net KK contribution vanish. Denoting by $\mathbf{Q}^1$ the charges of the original non-winding states, it is observed that the charges of the low-energy winding states are shifted as: $\mathbf{Q} = \mathbf{Q}^1 + \mathbf{e}(\jmath_1 + \jmath_2)$, with

$$\ell_{1,2} + \mathbf{e} \cdot \mathbf{Q} - \frac{1}{2}(\jmath_1 + \jmath_2)\mathbf{e}^2 = 0\,. \tag{9.1.1}$$

This confirms that there are indeed odd-winding/KK states with no net KK number.

A notable observation is that due to the $2\pi i(\jmath_1 + \jmath_2)\beta V \cdot \mathbf{e}$ shift in the GSO phase the generalised GSO projections in the V_0 sector remain the same as before the shift, whereas the generalised GSO projections in the V_1 sector are reversed for the low-energy odd-winding modes. Thus the projection in Eq. (E.0.2) at small radii removes the odd winding modes of the gravitinos but *in principle* allows the odd winding modes of the **NS**-**NS** tachyons. The squared masses of these tachyons which could be found in the low-energy states of the theory are given by

$$\frac{\alpha' M^2}{4} = -\frac{1}{2} + \frac{\jmath_1^2 r_1^2 + \jmath_2^2 r_2^2}{4}\,, \tag{9.1.2}$$

with $r_1, r_2 < \sqrt{2}$. If such states existed, they would have $\mathbf{e} \cdot \mathbf{Q} = \frac{1}{2}$ mod (1). However, as discussed above, in these models the phase in the generalised GSO projections is altered by shifts in the odd-winding sectors. Because of this modification, the **NS**-**NS** tachyons are projected out by the generalised GSO projections in the V_4 sector. Thus, one does not expect to find a tachyon-induced (Hagedorn-like) instability at $r_1 = r_2 \sim 1$, and this can indeed be verified through an inspection of the partition function. The absence of such tachyon-induced Hagedorn-like instabilities is in fact a special property of all generic non-supersymmetric string models that interpolate between a supersymmetric $r \to \infty$ endpoint (a.k.a. M_1) and a non-supersymmetric $r \to 0$ endpoint (a.k.a. M_2) which is also tachyon-free. However, it is necessary to investigate this property further as there is strongly suggestive evidence that it might actually be a special feature of all interpolating models with $N_b^{(0)} = N_f^{(0)}$.

Continuing on the analysis of the generic spectrum as a function of radius, the point $T = U = i$ (or $r_1 = r_2 = 1$) is normally a point of enhanced gauge symmetry for T_2 compactifications. At this point the entire theory can be fermionized so as to take the form of a fermionic string, as explained in Ref. [152]. Indeed, the $4D$ theory obtained without CDC carries additional massless states which appear with $\ell_{1,2} = \jmath_{1,2} = \pm 1$ when either $r_1 = 1$ or $r_2 = 1$; something that can also be deduced from Eq. (7.2.11). However, the $4D$ theory obtained via the CDC, does not allow symmetry enhancements of this form. This enhancement is impossible for states with $\mathbf{e} \cdot \mathbf{Q} = 0 \bmod (1)$ because it is required to have $\jmath_1 + \jmath_2 = odd$, which counterintuitively results in a non-zero net KK number. Alternatively, this argument could be realised through the lightest squared masses of the would-be additional massless states:

$$\begin{aligned}
\frac{\alpha' M_L^2}{4} &= \frac{1}{4}\left(-\frac{\jmath_1}{2r_1}\right)^2 + \frac{1}{4}\left(-\frac{\jmath_1}{2r_1} - \jmath_1 r_1\right)^2 \\
\frac{\alpha' M_R^2}{4} &= \frac{1}{4}\left(-\frac{\jmath_1}{2r_1}\right)^2 + \frac{1}{4}\left(-\frac{\jmath_1}{2r_1} + \jmath_1 r_1\right)^2 .
\end{aligned} \tag{9.1.3}$$

For the level-matching constraints to be satisfied, there must be equal numbers of left- and right-moving excitations. This condition is clearly not fulfilled for Eq. (9.1.3), so these states are eliminated from the spectrum through the generalised GSO projections in the V_0 sector of the theory. By similar arguments, winding modes of states with $\mathbf{e} \cdot \mathbf{Q}^1 = \frac{1}{2} \bmod (1)$ cannot yield any additional massless states either. Therefore, it is concluded that at the traditional enhanced symmetry point of the CDC'd theory, there does not seem to be a direct link to the $4D$ fermionic string. It could be conjectured though that the model at that point corresponds to a conventional $4D$ model broken to a tachyon-free non-supersymmetric model by discrete torsion.

9.1.2 Behaviour of one-loop cosmological constant

Given these observations about the physical mass spectrum of the Pati-Salam-like theory obtained via a CDC, it is easy to evaluate the one-loop cosmological constant Λ as a function of the *generic* inverse radius $a = r^{-1}$. A plot of the results is illustrated in Fig. 9.3, from which one can actually deduce *all* the gross features described in the above discussion. For large a, the rescaled cosmological constant tends to a large but constant value, a behaviour which is similar to that in Fig. 5.2. This outcome forms compelling evidence for the existence of a zero-radius end-point model which is entirely non-supersymmetric and tachyon-free (a.k.a. M_2); such a model is a $6D$ fermionic string which could be constructed with discrete torsion. Since the entire curve is finite, it is deduced that no tachyons emerge at any intermediate radii and therefore there are no Hagedorn-like instabilities. However, at the limit of small a, the curve is radically different from that in Fig. 5.2. The first immediate observation is that the Λ is not power-law suppressed as in Fig. 5.2, but rather it undergoes an exponential suppression. Second, it is observed that Λ changes sign as $a \to 0$ increases to $a \to \infty$. A finding that comes as a surprise is the stable anti-de Sitter minimum which appears just above the self-dual radius. Despite not being clear what the physical interpretation of this minimum could be, a possible scenario is to consider this minimum as an indication for gauge symmetry and/or SUSY restoration. This situation is similar to the one encountered in the Type II models considered in Ref. [56]. Moreover, the cosmological constant crosses zero at yet another, slightly higher radius than the self-dual, at which point it is also unclear whether there might exist a hidden unbroken SUSY.

Although all these features are significant and suggest the possibility of stabilising the *radius* modulus near $a \sim \mathcal{O}(1)$, the main interest is in the large-radii limit where the exponential suppression of the one-loop cosmological constant, and hence of the one-loop dilaton tadpole, is successfully achieved. It is only under such conditions that the purpose of this thesis manifests itself by neatly addressing the momentous *dilaton*-related stability issues associated with non-supersymmetric strings. Moreover, in this limit, $\partial\Lambda(r)/dr$ is also suppressed yielding an effectively flat potential for the radius modulus in this region.

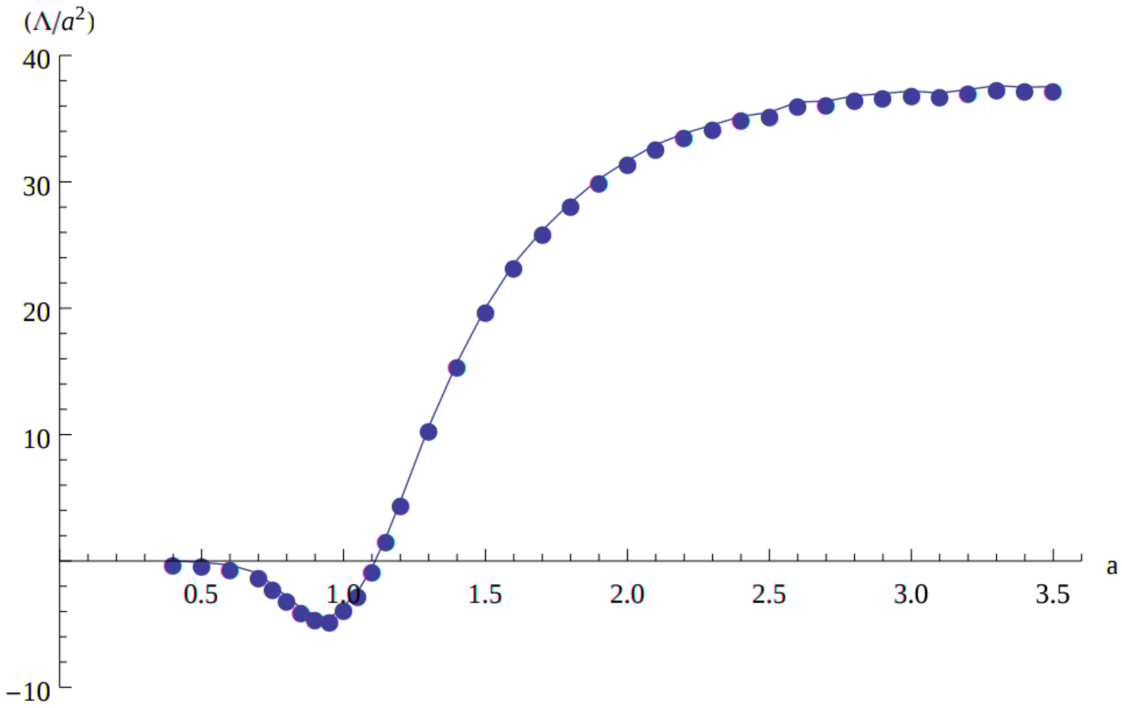


Figure 9.3: The rescaled cosmological constant Λa^{-2} for the Pati-Salam-like theory defined in Table 8.4. The results are plotted versus $a \equiv \sqrt{\alpha'} r^{-1}$. It is evident that the entire curve remains finite which means that there are no tachyons emerging in the theory, at any radii values. As expected, at the small but finite a limit the cosmological constant Λ is exponentially suppressed. For finite a values Λ remains finite but forms a stable minimum just above the self-dual radius which might be a signal for a gauge symmetry or SUSY restoration. At a slightly higher radius Λ crosses zero and changes sign so that at the large a limit tends to a large non-zero but constant value, indicating that the $a \to \infty$ limit of this model is a $6D$ non-supersymmetric and tachyon-free theory. This plot is adapted from Refs. [15, 16].

9.2 Phenomenological Properties

Another priority of equal importance is the phenomenological study of the models with exponentially suppressed cosmological constants, especially the aspects which relate directly to hierarchy and stability issues. Some phenomenological properties are quite general and would apply to any non-supersymmetric string model of this type. However there are others, like the natural particle assignments and Yukawa couplings which depend on the structure of a specific model and are best analysed within the context of the associated theory. A remarkable feature that is closely related to the behaviour of the cosmological constant is the behaviour of the scalar masses. In particular, as it is discussed below, there is a possibility to also suppress the radiative contributions to scalar masses, thereby paving a new path for tackling those issues related to hierarchy.

9.2.1 Natural particle assignments

Among the more model-independent phenomenological aspects is the identification of the SM particles in terms of specific string states. Recall that for the SM-like theory specified by the spin structure in Table 7.1, the SSSB is manifest only for the untwisted generations at leading order. This SSSB will inevitably appear in the twisted generations as well due to RG running but the breaking in these generations will almost certainly be smaller than that in the untwisted ones, assuming that untwisted matter exists at all.

As it is argued in Refs. [153,154], Scherk-Schwarz configurations may enhance "naturalness" in the sense advanced in Ref. [155]. From this standpoint, a natural assignment is to take the untwisted generations to be the first and second generations of the SM and one of the twisted generations to be the third. The large SUSY-splitting within the first generation does indeed then indicate a certain degree of naturalness. This observation seems like a good starting point for having reduced third generation masses while having relatively partial cancellations of radiative contributions to the third generation. Note that since the second generation is also relatively light, there are many flavour-related issues that would need to be carefully examined. Within QFT, these would be difficult questions to address as there are threshold contributions from the entire tower of states in the spectrum. Probably one would have to resort to setting soft terms as boundary conditions at the compactification scale. By contrast, one appealing aspect of the present constructions is that one does not have to rely on field theory because all the leading effects, including thresholds and RG running can in principle be simply computed from scratch within string perturbation theory.

9.2.2 Yukawa couplings

Even though the SM-like theory specified by the spin structure in Table 7.1 does not successfully meet the $N_b^{(0)} = N_f^{(0)}$ condition, it is useful to study the relevant Yukawa couplings. This study will serve as a benchmark for the Pati-Salam-like theory with realistic Higgs sectors.

• **The SM-like theory with $N_b^{(0)} \neq N_f^{(0)}$:** The possible Yukawa couplings are determined by the non-SM charges under horizontal $U(1)$ symmetries. The number of horizontal symmetries depends on the assignment of the boundary conditions in the defining spin structure. Every model has at least three horizontal $U(1)_{L_k}$ ($k = 1, 2, 3$) symmetries for the left-moving worldsheet currents and three matching global $U(1)_{R_k}$ ($k = 1, 2, 3$) symmetries from the right-moving worldsheet currents. With $J_\psi \equiv \frac{1}{2\pi} \int dz\, \psi^\dagger_{-\frac{1}{2}}(z)\psi_{-\frac{1}{2}}(z)$, these worldsheet currents may be denoted

$$J_{\overline{\eta}_{i=1,2,3}} \to U(1)_{L_{i=1,2,3}}\, ; \qquad J_{\psi^{56},\chi^{34},\chi^{56}} \to U(1)_{R_{1,2,3}}\, . \tag{9.2.1}$$

A cursory examination of the spectrum reveals that the chiral matter states arising from the V_2, b_3, and b_4 sectors as well as the Higgses $H_{U_{i=1,2,3}}$ and $H_{D_{i=1,2,3}}$, carry charges under $U(1)_{L_{i=1,2,3}}$ and $U(1)_{R_{i=1,2,3}}$ respectively. Given that SSSB affects only the untwisted sectors, and in this case is concentrated in the V_2 sector, then it is safe to assume that the states coming from the V_2 sector are the first and second SM generations.

In addition to these symmetries there are other horizontal $U(1)_{L_{k=4,5,...}}$ symmetries which arise when pairs of real worldsheet fermions are complexified. The real left-moving worldsheet fermions which can be complexified belong in the subsets $\{\overline{y}^{3,4,5,6}\}$, $\{\overline{\omega}^{3,4}\}$ and $\{\overline{\omega}^{5,6}\}$. Correspondingly, the complexified right-moving fermions from the subsets $\{y^{3,4,5,6}\}$, $\{\omega^{3,4}\}$ and $\{\omega^{5,6}\}$ give rise to four $U(1)_{R_{k=4',4,5,6}}$ symmetries assigned as

$$J_{\overline{y}^{34},\overline{y}^{56},\overline{\omega}^{56},\overline{\omega}^{34}} \to U(1)_{L_{4',4,5,6}}\, ; \qquad J_{y^{34},y^{56},\omega^{56},\omega^{34}} \to U(1)_{R_{4',4,5,6}}\, . \tag{9.2.2}$$

The non-vanishing Yukawa couplings for the states from the sectors V_2, b_3, and b_4

then depend on the boundary conditions assigned to the real fermions of the V_7 sector. The ultimate requirement that must be imposed for the correct assignment of the Yukawa couplings, is that the charge vectors for the states in a given coupling sum up to zero, $\mathbf{Q}_1 + \mathbf{Q}_2 + \mathbf{Q}_3 = \mathbf{0}$. The states with non-vanishing Yukawa couplings are then determined according to the value of [147]

$$|V_{7,R_{k+3}} - V_{7,L_{k+3}}|\,, \tag{9.2.3}$$

where the subscript refers to the element of V_7 corresponding to that particular worldsheet fermion. The value of this parameter can be either 0 or $\frac{1}{2}$ and determines which type of coupling is generated, *i.e.* involving d_R, e_R, or u_R, ν_R respectively. Both couplings cannot be present for the same generation. As mentioned in the previous section, the V_2 sector gives rise to two generations which are correlated with the $U(1)_4$ and $U(1)_{4'}$ horizontal symmetries. The b_3 and b_4 sectors give rise to more generations but the generations from each twisted sector are correlated with the existence of $U(1)_5$ and $U(1)_6$, respectively. In total, from all the sectors of the unbroken $\mathcal{N} = 1$ theory the only non-vanishing couplings are

$$\begin{aligned} W \;\supset\;& u_{R_1}H_{U_1}Q_{L_1} + \nu_{R_1}H_{U_1}L_{L_1} + d_{R_{1'}}H_{D_1}Q_{L_{1'}} + e_{R_{1'}}H_{D_1}L_{L_{1'}} + d_{R_2}H_{D_2}Q_{L_2} + e_{R_2}H_{D_2}L_{L_2} \\ +\;& u_{R_3}H_{U_3}Q_{L_3} + \nu_{R_3}H_{U_3}L_{L_3} + H_{U_1}H_{D_2}\Xi_3 + H_{U_2}H_{D_3}\Xi_1 + H_{U_1}H_{D_3}\Xi_2 + H_{D_1}H_{U_2}\Xi'_3 \\ +\;& H_{D_2}H_{U_3}\Xi'_1 + H_{D_1}H_{U_3}\Xi'_2 + \Xi_1\Xi'_2\Xi_3 + \Xi'_1\Xi_2\Xi'_3\,, \end{aligned} \tag{9.2.4}$$

where indices '1' and '1′' on the matter fields label the two generations from the V_2 sector while indices '2' and '3' correspond to the twisted-sector generations. All these Yukawa couplings arise with the same degenerate magnitude and despite being written in the form of superpotential terms, all the relevant would-be superpartners are massive.

For the original $\mathcal{N} = 0$ theory defined in Table 7.1, the non-vanishing Yukawa couplings are given by

$$\begin{aligned} W \;\supset\;& u_{R_1}H_{U_1}Q_{L_1} + \nu_{R_1}H_{U_1}L_{L_1} + d_{R_{1'}}H_{D_1}Q_{L_{1'}} + e_{R_{1'}}H_{D_1}L_{L_{1'}} + H_{U_2}H_{D_3}\Xi_1 + H_{D_2}H_{U_3}\Xi'_1 \\ +\;& \Xi_1\Xi'_2\Xi_3 + \Xi'_1\Xi_2\Xi'_3\,, \end{aligned} \tag{9.2.5}$$

where they are written again as superpotential terms. From this result, it is clear that out of the three Higgses in this model, the best suited to be the *actual* Higgs is the H_{U_1}. This is the only Higgs that remains massless after CDC, while H_{D_2} and H_{U_3} become massive. Interestingly, this generic situation is quite similar to the phenomenological 'one-Higgs-doublet' model [156], which was adopted in the context of Scherk-Schwarz breaking in Ref. [154]. The masses for the bottom and tau can then be generated via the 'wrong-Higgs' couplings coming from the Kähler potential. As usual the charm mass can be calculated by higher-order corrections. Note that any superpotential-like terms would violate the usual non-renormalization theorems, therefore even though they would be present they are subjected to a suppression given by the scale of SSSB.

• **The Pati-Salam Theory with $N_b^{(0)} = N_f^{(0)}$**: This model admits a greater number of horizontal $U(1)$ symmetries than those of the SM-like theory discussed above, but it contains the same $U(1)_{L_{1,2,3,5,6}}$ and $U(1)_{R_{1,2,3,5,6}}$ symmetries as expected. In this case, there is no V_7, so there is no extra condition imposed for determining the Yukawa couplings, except from the condition that the overall charge ($\mathbf{Q}_{\rm total}$) of the coupled states should vanish. It turns out that there exist many possible trilinear terms. Just as for the SM-like theory, the actual Higgs which remains massless after CDC is $\mathbb{H}_1$, and the non-vanishing Yukawas associated with it are given in the form of superpotential terms as

$$\mathbb{W} \supset \mathbb{F}_{R_1}\mathbb{H}_1\mathbb{F}_{L_1} + \mathbb{H}_3\mathbb{X}_5\mathbb{H}_1 + \mathbb{H}_5\mathbb{X}_3\mathbb{H}_1 + \mathbb{H}_4\mathbb{X}_7\mathbb{H}_1 \,. \tag{9.2.6}$$

Here, each coupled term involves the relevant component fields, *i.e.* the fields of the SM-like theory, as defined in Eqs. (8.4.3)-(8.4.5). For convenience, the label '1' here represents *all* the generations of matter fields arising from the V_2 sector. For future purposes, it is also useful to consider the larger set of all the Yukawa couplings involving the $\mathbb{H}_1$ Higgs and other Higgs states:

$$\begin{aligned} \mathbb{W} \supset\ & \mathbb{F}_{R_1}\mathbb{H}_1\mathbb{F}_{L_1} + \mathbb{F}_{R_2}\mathbb{H}_4\mathbb{F}_{L_2} + \mathbb{H}_3\mathbb{X}_5\mathbb{H}_1 + \mathbb{H}_5\mathbb{X}_3\mathbb{H}_1 + \mathbb{H}_2\mathbb{X}_6\mathbb{H}_4 + \mathbb{H}_5\mathbb{X}_2\mathbb{H}_4 \\ +\ & \mathbb{H}_3\mathbb{H}_5\mathbb{X}_1 + \mathbb{H}_2\mathbb{H}_5\mathbb{X}_4 + \mathbb{H}_2\mathbb{H}_3\mathbb{X}_8 + \mathbb{X}_5\mathbb{X}_3\mathbb{X}_1 + \mathbb{X}_2\mathbb{X}_6\mathbb{X}_4 + \mathbb{X}_2\mathbb{X}_3\mathbb{X}_7 \,. \end{aligned} \tag{9.2.7}$$

Note that the Higgsinos ($\tilde{\mathbb{H}}$) as well as the spinorial singlet fields ($\tilde{\mathbb{X}}$) that remain massless after the CDC can be made massive by their Yukawa couplings to the scalar fields that also remain massless after CDC, provided that these fields acquire a VEV.

9.2.3 Scalar masses

The Scherk-Schwarz mechanism applied to string compactification enables the calculation of all threshold corrections, including those that generate the scalar masses. For convenience, this discussion is focused on the mass-squared of the Higgs-like states that always remain massless in the CDC'd theory. The mass-squared operator for the states is given by

$$H_{U_1,D_1} \equiv \psi^{56}_{-\frac{1}{2}}\,|0\rangle_R \otimes \tilde{\psi}^j_{-\frac{1}{2}}\tilde{\psi}^k_{-\frac{1}{2}}\,|0\rangle_L\,. \tag{9.2.8}$$

In principle, the typical contributions to the scalar two-point functions can be calculated either in field theory or in string theory. At one-loop order, the leading contribution to scalar mass-squared at large radius can be computed through a field-theoretic calculation because it is dominated by the physical modes propagating in the loops. The same calculation can be performed string-theoretically by directly determining the two-point function for the scalar, but with the appropriate Scherk-Schwarz modified partition function. The result no longer vanishes, and the amplitude can be written as

$$A(k,-k) = -\,(2\pi)^4\,\frac{g_{YM}^2}{16\pi^2}\int_{\mathcal{F}}\frac{d^2\tau}{4\tau_2}\sum_{\alpha,\beta,\ell}\left(\frac{Y^2}{g_{YM}^2}-\frac{1}{4\pi\tau_2}\right)\frac{|\vec{\ell}|^2}{\tau_2^2}Z_{\ell,0}\,Z\begin{bmatrix}\alpha\\ \beta\end{bmatrix}, \tag{9.2.9}$$

where the term $Y^2 g_{YM}^{-2}$ includes those states coupling to the Higgs. The computation includes the gauge fields, for which $Y \equiv g_{YM}$, and closely follows that of the cosmological constant. The term $(4\pi\tau_2)^{-1}$ will be proportional to the overall cosmological constant and is therefore exponentially suppressed. The contribution from the massless physical states to the canonically normalised $4D$ Higgs mass-squared is given by

$$
\begin{aligned}
M_{H_1}^2 &= \frac{1}{16\pi^2}\int_{\frac{1}{\mu^2}\approx 1}^{\infty}\frac{d\tau_2}{4\tau_2^5}\sum_{\ell=odd,\,i} Y^2\,(N_{fH}^{(i)}-N_{bH}^{(i)})\,|\vec{\ell}|^2\,e^{-\frac{\pi}{\tau_2}|\vec{\ell}|^2}e^{-\pi\tau_2\alpha' m_i^2} \\
&\approx \frac{2}{\alpha'}\frac{Y^2}{16\pi^2}\,(N_{fH}^{(0)}-N_{bH}^{(0)})\,\frac{\pi^2 a^6}{320}\,,
\end{aligned}
\tag{9.2.10}
$$

where the sum is divided into mass-levels m_i. By contrast, the contribution from the massive states is given by

$$
M_{H_1}^2 = \frac{2}{\alpha'}\frac{Y^2}{16\pi^2}(N_{fH}^{(i)}-N_{bH}^{(i)})\sum_{\ell=odd}|\vec{\ell}|^{-5/2}(\sqrt{\alpha'}m_i)^{7/2}e^{-2\pi\sqrt{\alpha'}\,m_i\,|\vec{\ell}|}\,. \tag{9.2.11}
$$

Note that the result in Eq. (9.2.10) does not necessarily vanish even if its analogue does for the cosmological constant. This is because the Higgs couples differently to the states that become massive by the CDC. Thus, even though $N_b^{(0)} = N_f^{(0)}$, the condition $(N_{bH}^{(0)} - N_{fH}^{(0)}) = 0$ may in general not be fulfilled. Note, however, that *if* this particular condition is successfully met, then the Higgs mass will also be exponentially suppressed. This has some very significant implications regarding the hierarchy issues as it provides a 'natural' way to suppress the large radiative corrections to the Higgs mass.

The above results can be shown explicitly for the Pati-Salam-like theory. Inspecting the Yukawa and gauge couplings, it is found that the matter fields ($\mathbb{F}$) and their scalar superpartners ($\tilde{\mathbb{F}}$) both remain in the massless spectrum at leading order and therefore do not contribute to the mass-squared of $\mathbb{H}_1$. On the other hand, both the gauge fields and singlets are removed by the spectrum in a non-supersymmetric fashion, with Y^2 involving contraction over the massless pairs $\tilde{\mathbb{H}}_3\tilde{\mathbb{X}}_5$, $\tilde{\mathbb{H}}_5\tilde{\mathbb{X}}_3$, $\mathbb{H}_4\mathbb{X}_7$ and $A^\mu\mathbb{H}_1$. This yields a net result which is basically the coefficient of the one-loop quadratic divergence of the Higgs mass in the EFT of the massless d.o.f:

$$
Y^2(N_{fH}^{(i)}-N_{bH}^{(i)}) \equiv C_2(\Box)\,g_{SU(2)_L}^2 + C_2(\Box)\,g_{SU(2)_R}^2 - Y^2 = \frac{g_{\rm YM}^2}{2}\,. \tag{9.2.12}
$$

On a final note, the other scalars, and in particular the superpartners of the matter multiplets, naturally receive similar contributions. A detailed analysis of the threshold contributions to scalar masses is presented in Section IX.C of Ref. [15].

Chapter 10

Consistency of stable non-supersymmetric models with naturalness

The opposite of a correct statement is a false statement. But the opposite of a profound truth may well be another profound truth.

Neils H. D. Bohr

10.1 The "decompactification" problem

The main purpose thus far has been to establish a framework for constructing *stable* non-supersymmetric models and studying their associated phenomenological features. A natural development of this work is the surface of many questions regarding the energy and mass scales involved in the phenomenological structure of such models. An issue of special concern is the all-important question of whether some scalar fields, and most specifically the Higgs discussed in Section 9.2.2 of Chapter 9, might naturally remain light. It is indeed possible to suppress the mass-squared corrections to the Higgs field that survives the CDC procedure and remains in the massless spectrum, provided that $(N_{bH}^{(0)} - N_{fH}^{(0)}) = 0$. However, the question that remains here is whether it is feasible to construct such a model that satisfies both this condition as well as the condition $N_b^{(0)} = N_f^{(0)}$, required for the suppression of the cosmological constant. There are also other pressing scale-related phenomenological issues, such as the the large-volume "decompactification" problem which is discussed in Refs. [157–161].

The decompactification problem encompasses the contributions to the gauge-coupling beta functions due to the predominance of KK modes that necessarily appear in such models. In the weakly coupled heterotic string these contributions yield large gauge couplings at large volumes (or large compactification radii). In order to understand the impact of a parametrically large volume on the gauge couplings recall that the coupling expansion for n–point diagrams in heterotic string theory behaves as

$$V_{D-4}\, g_c^{n-\chi}\,, \tag{10.1.1}$$

where V_{D-4} is the compactification volume. This then gives the tree-level result

$$\begin{aligned} g_{tree}^{-2} &= g_c^{-2}\, v_{D-4} \\ M_P^2 &= g_c^{-2}\, v_{D-4}\alpha'^{-1}, \end{aligned} \tag{10.1.2}$$

where $v_{D-4} = V_{D-4}\alpha'^{-3}$ is the volume normalised w.r.t the fundamental string scale and g_{tree} is the tree-level Yang-Mills coupling. Note that the expansion parameter is g_c while the volume contributes with a single factor. One assumption, that is *not* applicable in a large volume scenario, is that $g_{tree}^2 \approx g_{GUT}^2 \approx 0.5$. From this result it is deduced that for the weakly coupled heterotic string theory is required to have $v_{D-4} \sim 1$, while

$$M_P^2 = g_{tree}^{-2}\, \alpha'^{-1}\,, \tag{10.1.3}$$

indicates a string scale that is within an order of magnitude of the Planck scale. If one wishes to allow a parametrically large volume but maintain a weak coupling $g_c \lesssim 1$, then it is clear that is required to have $g_{tree} \ll 1$.

Assuming that the $D = (4 + d)$-dimensional theory prior the CDC is supersymmetric implies that, from a $4D$ point of view, the KK modes of the CDC'd theory fall into incomplete multiplets of a 'parent' $\mathcal{N} = 2$ SUSY, that is ultimately broken by the action of the $\mathbb{Z}_2$ orbifold prior the CDC. The $\mathcal{N} = 2$ multiplets are split by the Scherk-Schwarz mechanism and the $\mathcal{N} = 2$ sectors feel the entirety of the large extra dimensions. A rough approximation of the number of KK modes in the theory below the string scale is given by the volume (v_{D-4}), while the expansion parameter $g_c^2 = N g_{tree}^2$ evidently plays the role of the $4D$ 't Hooft coupling. The one-loop gauge coupling can then be estimated within field theory as presented in Refs. [162, 163]. For this estimate the logarithmic contribution is ignored, and *only* the $\mathcal{N} = 2$ sector contribution between the string scale $M_s = \frac{1}{\sqrt{\alpha'}}$ and the KK scale $M_R = R^{-1}$ is considered. Then for d large dimensions it is obtained that

$$\alpha^{-1}(\mu) = \alpha^{-1}(M_R) + \frac{\tilde{b}\,\mathrm{Vol}(S_d)}{2\pi d}\left[\left(\frac{\mu}{M_R}\right)^d - 1\right], \tag{10.1.4}$$

where $\tilde{b} \equiv 2N_c - N_f$ is the $\mathcal{N} = 2$ beta function coefficient from a single set of KK modes. This expression can be obtained in string perturbation theory by performing the one-loop integral on a curved background related to μ as in Refs. [164, 165]. From this standpoint, the six-dimensional EFT flows from a UV fixed point [162, 163]. Setting $\mu = M_s$ the generic result is then obtained:

$$\alpha^{-1}(M_R) \approx \frac{4\pi v_d}{g_c^2} - \frac{\tilde{b} v_d}{2\pi d}\,. \tag{10.1.5}$$

Despite the fact that this result might be altered, depending on the model under study, it is still evident that phenomenology requires $\alpha^{-1}(M_R) \sim 1$. This means that in order to achieve order-one couplings in a generic theory, a fine-tuning of one-loop corrections against tree-level ones is required. In the context of Scherk-Schwarz breaking of SUSY, this constitutes to a certain degree a dimensionally transmuted naturalness problem. However, a possible solution to overcome this fine-tuning problem is based on a dynamical mechanism that requires a g_{YM} to become strong in the IR. This idea will be presented in forthcoming work [166], which to a great extent utilises ideas from Ref. [167] and particularly from Ref. [162].

10.2 "GUT precursors"

As it is established, for the construction of 'stable' non-supersymmetric theories, it is mandatory to have a perturbative UV complete framework and at the same time have parametrically large volumes in order to have exponentially suppressed cosmological constants and hence dilaton tadpoles. Following the discussion above, in order to make such models consistent with the naturalness problem it is imperative that the couplings flow as a power-law in a unified way from order-one values at the weak scale to extremely small values at the string scale. This in turn implies that there is a restoration of the GUT symmetry at the compactification scale, with KK and winding modes falling into complete GUT multiplets. The attainment of this scenario is allowed through the existence of configurations known as GUT precursor models [162, 163].

The need for a model which accommodates GUT precursors becomes apparent if one revisits the assumption that $g_c \sim 1$ at the string scale where the Yang-Mills couplings are tiny. Thus, the only couplings which can be significant at low scales are those which are asymptotically free. In addition, any IR free couplings would also become very small at low scales. Recall that the gauge couplings are universal at the string scale, therefore the one-loop corrections governed by $\tilde{b}$ must be asymptotically free and also universal. This requirement is trivially satisfied if the KK modes fall into complete GUT multiplets at the compactification scale. Consequently, this requirement can be achieved if the theory has a GUT precursor structure. The net effect from the EFT perspective is one in which the SM gauge couplings flow from a Gaussian fixed point in the UV to order-one values at the compactification scale. What is remarkable is that below this scale, the gauge couplings obey the normal logarithmic power-law running.

An interesting remark is that one might naively and erroneously suppose that an outcome in which a one-loop contribution balances a tree-level one must necessarily be non-perturbative and receive even more pronounced corrections at higher loops. However, as explained in Refs. [162, 163, 168], this is clearly not the case because the subsequent two-loop and higher-loop contributions do not require the same degree of tuning as the one-loop ones. The reason lies in the fact that from a field theory perspective the L-loop diagrams that involve only KK modes are $\mathcal{N} = 2$ and hence are cancelled for $L \geq 2$. Thus a non-vanishing contribution is one that has at least one $\mathcal{N} = 1$ propagator, implying from Eq. (10.1.1) that higher order diagrams are only proportional to g_c^{2L-2}. An additional remark is that the one-loop approximation works more effectively for larger compactification volumes and for $g_c \sim 1$ a single tuning at one-loop is the only thing required. Indeed, the higher order contributions do not necessitate further tuning.

On a final note, the GUT precursor structure is achievable only if the KK modes feel the entire volume of the compactification manifold. Taking also into account the fact that the $\mathcal{N} = 2$ sectors typically feel the volume of complex sub-planes on the compactification manifold, it is inferred that there is only one way to form a consistent theory: The first step is to consider compactification to an $\mathcal{N} = 1$ six-dimensional theory on a string-sized orbifold or manifold; this will yield a model with a similar spin structure as the one defined in Table 6.1, but the overall gauge group of this theory will include the $SU(5)$ GUT group. This step is then followed by compactification on a large two-dimensional orbifold down to four dimensions; this will yield an $\mathcal{N} = 1$ $SU(5)$ GUT. The final stage of compactification is then given a coordinate dependence which breaks spontaneously the $\mathcal{N} = 1$ SUSY yielding an entirely non-supersymmetric theory.

10.3 A SM-like theory from a GUT precursor model

Having established the general phenomenological framework, it is now presented how an explicit SM-like theory that displays Bose-Fermi degeneracy of the massless modes, and hence enhanced stability, can be obtained from a GUT precursor model. As alluded to above, the starting point is an $\mathcal{N} = 1$ six-dimensional theory which is first compactified on a freely acting $\mathbb{Z}_2$ orbifold with a b_3 action, and without the deformations of CDC. This yields an $\mathcal{N} = 1$ four-dimensional theory which includes the $SU(5)$ GUT group; an example that serves as a benchmark for the purpose of this section is specified by the spin structure displayed in Table 10.1.

Sector	ψ^{34}	ψ^{56}	χ^{34}	y^{34}	ω^{34}	χ^{56}	y^{56}	ω^{56}	$\bar{y}^{34}$	$\bar{\omega}^{34}$	$\bar{y}^{56}$	$\bar{\omega}^{56}$	$\bar{\psi}^1$	$\bar{\psi}^2$	$\bar{\psi}^3$	$\bar{\psi}^4$	$\bar{\psi}^5$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^1$	$\bar{\phi}^2$	$\bar{\phi}^3$	$\bar{\phi}^4$	$\bar{\phi}^5$	$\bar{\phi}^6$	$\bar{\phi}^7$	$\bar{\phi}^8$
V_0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_1	0	0	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_2	0	0	1	0	1	1	0	1	0	1	0	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
b_3	1	0	1	0	0	0	0	1	0	0	0	1	1	1	1	1	1	0	1	0	1	0	1	0	0	0	1	1
V_5	0	0	0	0	0	0	1	1	1	0	0	1	0	0	0	0	0	0	1	1	0	1	1	1	1	0	1	1
V_7	0	0	0	1	1	0	0	0	0	1	0	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1	1	1	1	0	1	0

Table 10.1: Spin structure of the worldsheet fermions of the $\mathcal{N} = 1$, $4D$ $SU(5)$ GUT model before applying the CDC. This model serves as a benchmark so as to demonstrate the GUT precursor structure of the theory once the effects of CDC are turned on. This spin structure is accompanied by two bosonic d.o.f compactified on a $\mathbb{Z}_2$ orbifold with twist action corresponding to the vector b_3.

As always, along with the spin structure is the matrix k_{ij} which specifies the phases involved in the corresponding GSO projections:

$$k_{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{3}{4} \\ 0 & 0 & 0 & 0 & 0 & \frac{3}{4} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} . \tag{10.3.1}$$

The full gauge group is found to be

$$SU(5) \otimes SO(6)^2 \otimes [U(1)]^9 \, . \tag{10.3.2}$$

The physical spectrum from both the twisted and untwisted sectors of the GUT theory is presented in Table 10.2. As always, the **NS-NS** sector gives rise to the gravity multiplet as well as the massless scalar states required to build $\mathcal{N} = 2$ gauge multiplets and hypermultiplets. There are five untwisted **10** representations of the $SU(5)$. These **10**s along with the $(\overline{\mathbf{5}} \oplus \mathbf{1})$s form complete chiral and vector-like **16** spinorial representations of the $SO(10)$. As a result, this model has *four net generations of chiral matter fields*: two of them coming directly from the $\overline{V_0 + V_2}$ while the remaining two come from the $\overline{V_0 + V_1 + V_2 + \alpha V_7}$ untwisted sectors (where $\alpha = 1, 3$), and with the scalar superpartners in the $\overline{V_1 + V_2}$ and $\overline{V_2 + \alpha V_7}$

Fields	State	$U(1)$	$U(1)$	$U(1)$	$SO(6)$	$U(5)$	$U(1)$	$U(1)$	$U(1)$	$U(1)$	$SO(6)$	$U(1)$	$U(1)$
	$A^{(1)}$	−1/2	.	1/2	.	$\mathbf{10}$	−1/2	.	.	.	.	.	.
	$A^{(2)}$	1/2	.	1/2	.	$\mathbf{10}$	−1/2	.	.	.	.	.	.
	$A^{(3)}$	.	.	.	.	$\mathbf{10}$	−1/2	1/2	1/2	.	.	.	.
	$A^{(4)}$	.	.	.	.	$\mathbf{10}$	−1/4	1/4	1/4	.	.	−1/2	−1/2
	$A^{(5)}$	.	.	.	.	$\mathbf{10}$	−1/4	1/4	1/4	.	.	−1/2	1/2
L / R - handed matter fields + Higgs fields	$Q^{c(1)}$	−1/2	.	−1/2	.	$\overline{\mathbf{5}}$	−1/2	.	.	.	.	.	.
	$Q^{c(2)}$	1/2	.	−1/2	.	$\overline{\mathbf{5}}$	−1/2	.	.	.	.	.	.
	$Q^{c(3)}$	1/2	.	−1/2	.	$\overline{\mathbf{5}}$	.	−1/2	1/2	.	.	.	.
	$Q^{c(4)}$	−1/2	.	−1/2	.	$\overline{\mathbf{5}}$	.	−1/2	1/2	.	.	.	.
	$Q^{c(5)}$	.	1/2	1/2	.	$\overline{\mathbf{5}}$	1/4	1/4	1/4	−1/2	.	.	−1/2
	$Q^{c(6)}$	.	−1/2	1/2	.	$\overline{\mathbf{5}}$	1/4	1/4	1/4	−1/2	.	.	−1/2
	$Q^{c(7)}$	.	1/2	1/2	.	$\overline{\mathbf{5}}$	1/4	1/4	1/4	−1/2	.	.	1/2
	$Q^{c(8)}$	.	−1/2	1/2	.	$\overline{\mathbf{5}}$	1/4	1/4	1/4	−1/2	.	.	1/2
	$Q^{c(9)}$	.	.	.	.	$\overline{\mathbf{5}}$	1	.	.	.	.	.	.
	$Q^{c(10)}$	.	.	.	.	$\overline{\mathbf{5}}$	−1/2	−1/2	−1/2	.	.	.	.
	$Q^{c(11)}$	.	.	.	.	$\overline{\mathbf{5}}$	−3/4	−1/4	−1/4	.	.	1/2	−1/2
	$Q^{c(12)}$	.	.	.	.	$\overline{\mathbf{5}}$	−3/4	−1/4	−1/4	.	.	1/2	1/2
	$Q^{c(13)}$	.	.	.	.	$\overline{\mathbf{5}}$	1/4	3/4	−1/4	.	.	1/2	1/2
	$Q^{c(14)}$	.	.	.	.	$\overline{\mathbf{5}}$	1/4	3/4	−1/4	.	.	1/2	−1/2
	$Q^{c(15)}$	−1/2	−1/2	.	.	$\overline{\mathbf{5}}$	.	.	.	−1/2	.	−1/2	.
	$Q^{c(16)}$	−1/2	1/2	.	.	$\overline{\mathbf{5}}$	.	.	.	−1/2	.	−1/2	.
	$Q^{c(17)}$	1/2	−1/2	.	.	$\overline{\mathbf{5}}$	.	.	.	−1/2	.	−1/2	.
	$Q^{c(18)}$	1/2	1/2	.	.	$\overline{\mathbf{5}}$	.	.	.	−1/2	.	−1/2	.
	$A^{c(1)}$	.	.	.	.	$\overline{\mathbf{10}}$	1/2	−1/2	−1/2	.	.	.	.
R / L - handed matter fields + Higgs fields	$Q^{(1)}$	1/2	.	1/2	.	$\mathbf{5}$	.	−1/2	1/2	.	.	.	.
	$Q^{(2)}$	−1/2	.	1/2	.	$\mathbf{5}$	.	−1/2	1/2	.	.	.	.
	$Q^{(3)}$	.	−1/2	−1/2	.	$\mathbf{5}$	−1/4	−1/4	−1/4	−1/2	.	.	−1/2
	$Q^{(4)}$	.	1/2	−1/2	.	$\mathbf{5}$	−1/4	−1/4	−1/4	−1/2	.	.	−1/2
	$Q^{(5)}$	.	−1/2	−1/2	.	$\mathbf{5}$	−1/4	−1/4	−1/4	−1/2	.	.	1/2
	$Q^{(6)}$	.	1/2	−1/2	.	$\mathbf{5}$	−1/4	−1/4	−1/4	−1/2	.	.	1/2
	$Q^{(7)}$	.	.	.	.	$\mathbf{5}$	−1	.	.	.	.	.	.
	$Q^{(8)}$	.	.	.	.	$\mathbf{5}$	1/2	1/2	1/2	.	.	.	.
	$Q^{(9)}$	.	.	.	.	$\mathbf{5}$	−1/4	1/4	−3/4	.	.	−1/2	1/2
	$Q^{(10)}$	.	.	.	.	$\mathbf{5}$	−1/4	1/4	−3/4	.	.	−1/2	−1/2
	$Q^{(11)}$	−1/2	1/2	.	.	$\mathbf{5}$	.	.	.	−1/2	.	1/2	.
	$Q^{(12)}$	−1/2	−1/2	.	.	$\mathbf{5}$	.	.	.	−1/2	.	1/2	.
	$Q^{(13)}$	1/2	1/2	.	.	$\mathbf{5}$	.	.	.	−1/2	.	1/2	.
	$Q^{(14)}$	1/2	−1/2	.	.	$\mathbf{5}$	.	.	.	−1/2	.	1/2	.

Table 10.2: The $\mathbb{Z}_2$ massless GUT states of the $\mathcal{N} = 1$, $4D$ model derived from the twisted and untwisted sectors. Some of the states that give rise to the $\mathbf{5}$ and the $\overline{\mathbf{5}}$ of the $SU(5)$ GUT group produce the Higgs doublets. The remaining states that give rise to the $\mathbf{16}$ representation produce the left- and right-handed chiral matter fields as well as a vector-like set of fermionic matter.

sectors respectively. There is also a vector-like generation of matter fields coming directly from the $\overline{V_0 + V_1 + 2V_7}$ untwisted sector, with the scalar superpartners in the $\overline{2V_7}$ sector. The remaining $\mathbf{5}$ and $\overline{\mathbf{5}}$ pairs can be identified as Higgs fields, with A and Q labelling antisymmetrics and $\mathbf{5}$'s respectively.

The final stage of compactification is then, as described above, given a coordinate dependence so as to obtain a chiral $\mathcal{N}=1\to\mathcal{N}=0$, $4D$ SM-like theory. The resulting CDC preserves all the vital features of the original theory, in particular modular invariance and hence finiteness. As discussed in Section 8.1 of Chapter 8, there are multiple conditions involved in the construction of stable non-supersymmetric theories, which add up to an extremely constraining set of requirements for any consistent model. Therefore, finding working examples is non-trivial as all of these constraints must be appreciated. On top of all the above, there are also imposed two additional constraints for the present context:

- In order to have GUT precursor structure, only the orbifold action b_3 and/or the CDC itself may break a GUT symmetry;
- The $\mathcal{N}=0$ model interpolates between the $\mathcal{N}=1$ $6D$ model presented above in the $R\to\infty$ limit and a $6D$ model in the $R\to 0$ limit which in this case may not be necessarily non-supersymmetric. In the $R\to 0$, the CDC vector $\mathbf{e}$ reverts to the role of a normal vector and is added in the spin structure of that model (a more detailed explanation of this argument will be presented in Ref. [169]). Therefore, the $\mathbf{e}$ vector must obey the *same* modular invariance constraints as the other V_i vectors that define the model in four dimensions. These constraints are given in Eq. (4.1.7).

Despite all these constraints it is still possible to construct a model that in the low energy limit bears a resemblance to the SM. Before presenting its details, it is useful to make some very general statements about where the matter generations and the Higgses may appear. In order to do this it is a requisite to extend the discussion of the previous Chapters so as to include cases where both the CDC vector ($\mathbf{e}$) and the orbifold vector (b_3) can overlap with more general complex GSO phases in the spin structure. Under such an extension the light states that survive the orbifold projections are formed by a linear combination of excitations in different winding sectors, namely

$$|\psi_{phys}\rangle=\frac{1}{\sqrt{2}}\left(|\psi^{(\mathcal{N}=2)}_{q_e,J,\ell}\rangle+|\psi^{(\mathcal{N}=2)}_{-q_e,-J,-\ell}\rangle\right),\tag{10.3.3}$$

where $q_e = \mathbf{e} \cdot \mathbf{Q}$ is the CDC charge of the associated state. As stated above, in order to have a consistent projection this must be conjugated under the orbifolding. There are various consistent bases that one could work with but they are not quite obvious. Despite being relevant, this issue is beyond the scope of this discussion, but is extensively treated in the Appendix A of Ref. [166]. For a consistent model, it is then required that there is at least one pair of complete worldsheet fermions whose phases must overlap as follows:

$$
\begin{aligned}
V_n &= -\tfrac{1}{2}\Big[\ \ldots\ldots\ldots\ldots \mid \ldots\ \tfrac{1}{4}\ \tfrac{1}{4}\ \ldots\ \Big]; \\
b_3 &= -\tfrac{1}{2}\Big[\ 0\,0\,1\,0\,0\,0\,0\,1 \mid \ldots\ 0\ \tfrac{1}{2}\ \ldots\ \Big]; \\
\mathbf{e} &= +\tfrac{1}{2}\Big[\ 0\,0\,1\,0\,1\ 1\,0\,1 \mid \ldots\ \tfrac{1}{2}\ \tfrac{1}{2}\ \ldots\ \Big].
\end{aligned}
\tag{10.3.4}
$$

A general form about how the phases of the vectors should overlap so as to obtain a SM-like theory from a GUT precursor model, is then given in Table 10.3. The phases of the left-moving fermions in the basis vectors $V_{i=2,\ldots,n-1}$ are assigned so as to act degenerately on the components of the GUT group that must be preserved and be asymptotically free. This breaks the gauge symmetry at the string scale, to $SU(5) \otimes U(1)_1 \otimes U(1)_2 \otimes U(1)_3 \ldots$. The $*$ in Table 10.3 is used as wildcard for $-\frac{1}{2}$ or 0 phases, allowing these vectors to break the $SO(6)$ factor at the string scale. Finally, the vector V_n is required to break the theory to unitary groups, with the use of $-\frac{1}{4}$ complex phases on the worldsheet fermions. This is precisely the form of the supersymmetric model specified in Table 10.1, but the overlap of $\mathbf{e}$ is prescribed by the form of Eq. (10.3.4).

The final breaking of the $SU(5)$ gauge symmetry occurs through the $\mathbf{e}$, which at the same time breaks SUSY spontaneously. By choosing the $\mathbf{e}$ for the breaking of the gauge symmetry, it allows the retainment of an untwisted Higgs state which appears in the **NS-NS** sector as a bi-fundamental of the general form

$$
\begin{aligned}
H_U &\sim \psi^{56}_{-\frac{1}{2}}\,|0\rangle \otimes \tilde{\psi}^{U(2)}_{-\frac{1}{2}}\tilde{\psi}^{U(1)^{\dagger}_{1,3}}_{-\frac{1}{2}}\,|0\rangle \\
H_D &\sim \psi^{56}_{-\frac{1}{2}}\,|0\rangle \otimes \tilde{\psi}^{U(2)^{\dagger}}\tilde{\psi}^{U(1)_{1,3}}_{-\frac{1}{2}}\,|0\rangle\,.
\end{aligned}
\tag{10.3.5}
$$

Sector	ψ_μ	ψ_{56}	... ...	$U(3)\otimes U(2)$	$U(1)^3$	... ...
V_0	1	1	... ...	1 1 1 1 1	1 1 1	... ...
V_1	0	0	... ...	1 1 1 1 1	1 1 1	... ...
$V_{i=2,\ldots,n-1}$	0	0	... ...	0 0 0 0 0	$*$ $*$ $*$	... ...
V_n	0	0	... ...	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	... ...
b_3	0	0	... ...	1 1 1 1 1	0 1 0	... ...
e	0	0	... ...	0 0 0 1 1	1 0 1	... ...

Table 10.3: A general spin structure for a chiral SM-like theory obtained from a GUT precursor model in $4D$. The $*$ is used as a wildcard '0' or '1'. As with the previous spin structure specifications, all entries are understood to be multiplied by a factor of $-\frac{1}{2}$.

The generalised GSO projections in this sector are very general but the important one is that from the orbifold which takes the form $b_3 \cdot N_{\mathbf{0}} = \frac{1}{2}$. Because of this projection, no Higgs states involving $\tilde{\psi}_{U(1)_2}$ are allowed to remain in the spectrum; light vector-like triplets of the form

$$T \quad \sim \quad \psi^{56}_{-\frac{1}{2}}\,|0\rangle \otimes \tilde{\psi}^{U(3)}_{-\frac{1}{2}}\tilde{\psi}^{U(1)_2^\dagger}_{-\frac{1}{2}}\,|0\rangle\,, \tag{10.3.6}$$

plus its conjugate, are however allowed. The benefit of choosing **e** to break the GUT symmetry is now apparent: States which satisfy $\mathbf{e}\cdot\mathbf{Q} = \frac{1}{2} \bmod (1)$ become massive by the CDC, so if the **e** is chosen so that it has degenerate entries across $SU(3)\otimes SU(2)$ it will inevitably either make all Higgs states massive or leave a larger $SU(4)$ symmetry unbroken. The resulting CDC'd model is then defined by the spin structure in Table 10.4, with structure constants as in Eq. (10.3.1).

Evidently, the vector **e** breaks the gauge group down to $G_{visible}\otimes G_{semi-hidden}\otimes G_{hidden}$, where $G_{semi-hidden} = [U(1)]^{11}$ and the visible sector contains the SM gauge group. The convention for the hypercharge of the SM particles is now chosen as

$$\frac{1}{2}U(1)_Y \equiv -\frac{1}{3}\left[U(1)_{\overline{\psi}^1} + U(1)_{\overline{\psi}^2} + U(1)_{\overline{\psi}^3}\right] + \frac{1}{2}\left[U(1)_{\overline{\psi}^4} + U(1)_{\overline{\psi}^5}\right]. \tag{10.3.7}$$

As always, the twisted sectors remain globally supersymmetric. In the untwisted sectors all states satisfying $Q_e = \mathbf{e}\cdot\mathbf{Q} \neq 0 \bmod (1)$ become massive, while those states remaining massless satisfy $Q_e = 0$. The massless particle content from both the twisted and untwisted sectors is summarised in Tables 10.5-10.8. Even though

Sector	ψ^{34}	ψ^{56}	χ^{34}	y^{34}	ω^{34}	χ^{56}	y^{56}	ω^{56}	$\overline{y}^{34}$	$\overline{\omega}^{34}$	$\overline{y}^{56}$	$\overline{\omega}^{56}$	$\overline{\psi}^1$	$\overline{\psi}^2$	$\overline{\psi}^3$	$\overline{\psi}^4$	$\overline{\psi}^5$	$\overline{\eta}^1$	$\overline{\eta}^2$	$\overline{\eta}^3$	$\overline{\phi}^1$	$\overline{\phi}^2$	$\overline{\phi}^3$	$\overline{\phi}^4$	$\overline{\phi}^5$	$\overline{\phi}^6$	$\overline{\phi}^7$	$\overline{\phi}^8$
V_0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_1	0	0	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_2	0	0	1	0	1	1	0	1	0	1	0	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
b_3	1	0	1	0	0	0	0	1	0	0	0	1	1	1	1	1	1	0	1	0	1	0	1	0	0	0	1	1
V_5	0	0	0	0	0	0	1	1	1	0	0	1	0	0	0	0	0	0	1	1	0	1	1	1	1	0	1	1
V_7	0	0	0	1	1	0	0	0	0	1	0	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1	1	1	1	0	1	0
e	0	0	1	0	1	1	0	1	1	0	0	0	0	0	0	1	1	1	0	1	0	1	1	0	0	0	1	0

Table 10.4: Spin structure of the $\mathcal{N} = 0$, $4D$ chiral SM-like theory obtained from a GUT precursor model after applying the CDC. This theory has Bose-Fermi degeneracy at the massless level which guarantees the exponential suppression of the one-loop cosmological constant and dilaton tadpole, thereby having enhanced stability and finiteness properties. Note also that the entries of the CDC vector **e** are assigned so that it spontaneously breaks SUSY and the $SU(5)$ gauge symmetry to $SU(3) \otimes SU(2)$, while at the same time it obeys the modular invariance constraints applied to the other vectors.

the graviton and gauge bosons of the theory are not listed, they can be generally identified in the **NS**-**NS** sector in the usual way, along with the complex Radion. Conversely, the gravitino as well as the gauginos become massive after the CDC. It should be noted that linear combinations of the basis vectors $\{V_0, \ldots, V_7\}$ can potentially produce sectors that yield gauge bosons in the spinorial representations of the observable $SU(3) \otimes SU(2)$ and/or the hidden gauge group, indicating unwanted gauge enhancement. These states can be projected from the massless physical spectrum of the theory through a consistent set of generalised GSO projections, which greatly depend on the value of the structure constants. Such states are non existent for the GUT precursor model presented here.

This model should be thought of more as an existence proof rather than the finished product. Nevertheless, to a certain extent it resembles the actual SM. From Table 10.5, it is observed that there are 18 sets of Higgs pairs. Explicitly in the notation of Ref. [135], the Higgs states remaining in the **NS**-**NS** sector are

$$\begin{aligned} H_U^{(1),(2)} &= \{b, d\}^{56}_{-\frac{1}{2}} \, |0\rangle_R \otimes \tilde{b}^{4,5}_{-\frac{1}{2}} \, \tilde{d}^{1}_{-\frac{1}{2}} \, |0\rangle_L \\ H_D^{(1),(2)} &= \{b, d\}^{56}_{-\frac{1}{2}} \, |0\rangle_R \otimes \tilde{d}^{4,5}_{-\frac{1}{2}} \, \tilde{b}^{1}_{-\frac{1}{2}} \, |0\rangle_L \, . \end{aligned} \tag{10.3.8}$$

The other Higgses as well as the singlets are produced from various untwisted or twisted sectors and they all carry charges under the semi-hidden sector's gauge

Higgs fields																
State	$U(1)$	$U(1)$	$U(1)$	$U(1)$	$U(3)$	$U(2)$	$U(1)$	$U(1)$	$U(1)$	$U(1)$	$U(1)$	$SO(4)$	$SO(4)$	$U(1)$	$U(1)$	Y
$H_U^{(1)}$	.	.	.	.	.	**2**	−1	.	.	.	.	.	.	.	.	1/2
$H_U^{(2)}$	.	.	.	.	.	**2**	−1	.	.	.	.	.	.	.	.	1/2
$H_U^{(3)}$	.	.	.	.	.	**2**	1/2	1/2	1/2	.	.	.	.	.	.	1/2
$H_U^{(4)}$	.	.	.	.	.	**2**	1/2	1/2	1/2	.	.	.	.	.	.	1/2
$H_U^{(5)}$	.	.	.	.	.	**2**	3/4	1/4	1/4	.	.	.	.	−1/2	1/2	1/2
$H_U^{(6)}$	.	.	.	.	.	**2**	−1/4	−3/4	1/4	.	.	.	.	−1/2	1/2	1/2
$H_U^{(7)}$	.	.	.	.	.	**2**	−1/4	1/4	−3/4	.	.	.	.	−1/2	−1/2	1/2
$H_U^{(8)}$	−1/2	−1/2	.	.	.	**2**	.	.	.	1/2	.	.	.	1/2	.	1/2
$H_U^{(9)}$	−1/2	1/2	.	.	.	**2**	.	.	.	−1/2	.	.	.	1/2	.	1/2
$H_U^{(10)}$	1/2	1/2	.	.	.	**2**	.	.	.	1/2	.	.	.	1/2	.	1/2
$H_U^{(11)}$	1/2	−1/2	.	.	.	**2**	.	.	.	−1/2	.	.	.	1/2	.	1/2
$H_U^{(12)}$	1/2	.	1/2	.	.	**2**	1/2	.	.	.	.	.	.	.	.	1/2
$H_U^{(13)}$	1/2	.	1/2	.	.	**2**	.	1/2	−1/2	.	.	.	.	.	.	1/2
$H_U^{(14)}$	−1/2	.	1/2	.	.	**2**	.	−1/2	1/2	.	.	.	.	.	.	1/2
$H_U^{(15)}$	.	1/2	−1/2	.	.	**2**	−1/4	−1/4	−1/4	1/2	.	.	.	.	−1/2	1/2
$H_U^{(16)}$	.	−1/2	−1/2	.	.	**2**	−1/4	−1/4	−1/4	−1/2	.	.	.	.	−1/2	1/2
$H_U^{(17)}$	.	−1/2	−1/2	.	.	**2**	−1/4	−1/4	−1/4	1/2	.	.	.	.	1/2	1/2
$H_U^{(18)}$	.	1/2	−1/2	.	.	**2**	−1/4	−1/4	−1/4	−1/2	.	.	.	.	1/2	1/2
$H_D^{(1)}$	.	.	.	.	.	**2**	1	.	.	.	.	.	.	.	.	−1/2
$H_D^{(2)}$	.	.	.	.	.	**2**	1	.	.	.	.	.	.	.	.	−1/2
$H_D^{(3)}$	.	.	.	.	.	**2**	−1/2	−1/2	−1/2	.	.	.	.	.	.	−1/2
$H_D^{(4)}$	.	.	.	.	.	**2**	−1/2	−1/2	−1/2	.	.	.	.	.	.	−1/2
$H_D^{(5)}$	.	.	.	.	.	**2**	−3/4	−1/4	−1/4	.	.	.	.	1/2	−1/2	−1/2
$H_D^{(6)}$	.	.	.	.	.	**2**	1/4	3/4	−1/4	.	.	.	.	1/2	−1/2	−1/2
$H_D^{(7)}$	.	.	.	.	.	**2**	1/4	−1/4	3/4	.	.	.	.	1/2	1/2	−1/2
$H_D^{(8)}$	−1/2	1/2	.	.	.	**2**	.	.	.	1/2	.	.	.	−1/2	.	−1/2
$H_D^{(9)}$	−1/2	−1/2	.	.	.	**2**	.	.	.	−1/2	.	.	.	−1/2	.	−1/2
$H_D^{(10)}$	1/2	−1/2	.	.	.	**2**	.	.	.	1/2	.	.	.	−1/2	.	−1/2
$H_D^{(11)}$	1/2	1/2	.	.	.	**2**	.	.	.	−1/2	.	.	.	−1/2	.	−1/2
$H_D^{(12)}$	−1/2	.	−1/2	.	.	**2**	−1/2	.	.	.	.	.	.	.	.	−1/2
$H_D^{(13)}$	1/2	.	−1/2	.	.	**2**	.	1/2	−1/2	.	.	.	.	.	.	−1/2
$H_D^{(14)}$	−1/2	.	−1/2	.	.	**2**	.	−1/2	1/2	.	.	.	.	.	.	−1/2
$H_D^{(15)}$	.	−1/2	1/2	.	.	**2**	1/4	1/4	1/4	1/2	.	.	.	.	−1/2	−1/2
$H_D^{(16)}$	.	1/2	1/2	.	.	**2**	1/4	1/4	1/4	−1/2	.	.	.	.	−1/2	−1/2
$H_D^{(17)}$	.	1/2	1/2	.	.	**2**	1/4	1/4	1/4	1/2	.	.	.	.	1/2	−1/2
$H_D^{(18)}$	.	−1/2	1/2	.	.	**2**	1/4	1/4	1/4	−1/2	.	.	.	.	1/2	−1/2

Table 10.5: The scalar Higgs states of the SM-like theory obtained from a GUT precursor model. These states come from the twisted and untwisted sectors, and survive in the physical massless spectrum after the CDC. The $H_U^{(1),(2)}$ and the $H_D^{(1),(2)}$ come from the **NS-NS** sector of the theory and are explicitly identified in the text.

group. Note that although the supersymmetric counterparts of the Higgses in Table 10.5 become massive, there are other Higgsino states that remain massless even after the CDC, but can be lifted by their Yukawa couplings. Once the $SU(5)$ GUT symmetry is broken by the CDC there are only two net chiral fermion generations from the original four supermultiplets. One generation is in the $\overline{V_0 + V_2}$ and the other is in the $\overline{V_0 + V_1 + V_2 + \alpha V_7}$ untwisted sectors. These states are listed in

L / R - handed matter fields																
State	$U(1)$	$U(1)$	$U(1)$	$U(1)$	$U(3)$	$U(2)$	$U(1)$	$U(1)$	$U(1)$	$U(1)$	$U(1)$	$SO(4)$	$SO(4)$	$U(1)$	$U(1)$	Y
$q^{(1)}$	1/2	.	1/2	.	$\mathbf{3}$	$\mathbf{2}$	−1/2	.	.	.	.	.	.	.	.	1/6
$q^{(2)}$	.	.	.	.	$\mathbf{3}$	$\mathbf{2}$	−1/2	1/2	1/2	.	.	.	.	.	.	1/6
$q^{(3)}$	.	.	.	.	$\mathbf{3}$	$\mathbf{2}$	−1/4	1/4	1/4	.	.	.	.	−1/2	1/2	1/6
$\ell^{(1)}$	1/2	.	−1/2	.	.	$\mathbf{2}$	−1/2	.	.	.	.	.	.	.	.	−1/2
$\ell^{(2)}$	1/2	.	−1/2	.	.	$\mathbf{2}$	.	−1/2	1/2	.	.	.	.	.	.	−1/2
$\ell^{(3)}$	.	−1/2	1/2	.	.	$\mathbf{2}$	1/4	1/4	1/4	−1/2	.	.	.	.	−1/2	−1/2
$\ell^{(4)}$	.	1/2	1/2	.	.	$\mathbf{2}$	1/4	1/4	1/4	−1/2	.	.	.	.	1/2	−1/2
$\ell^{(5)}$	.	.	.	.	.	$\mathbf{2}$	−3/4	−1/4	−1/4	.	.	.	.	1/2	1/2	−1/2
$\ell^{(6)}$	.	.	.	.	.	$\mathbf{2}$	1/4	3/4	−1/4	.	.	.	.	1/2	1/2	−1/2
$\ell^{(7)}$	−1/2	1/2	.	.	.	$\mathbf{2}$	.	.	.	−1/2	.	.	.	−1/2	.	−1/2
$\ell^{(8)}$	1/2	−1/2	.	.	.	$\mathbf{2}$	.	.	.	−1/2	.	.	.	−1/2	.	−1/2
$u^{c(1)}$	−1/2	.	1/2	.	$\overline{\mathbf{3}}$	.	−1/2	.	.	.	.	.	.	.	.	−2/3
$u^{c(2)}$	.	.	.	.	$\overline{\mathbf{3}}$	.	−1/4	1/4	1/4	.	.	.	.	−1/2	−1/2	−2/3
$d^{c(1)}$	−1/2	.	−1/2	.	$\overline{\mathbf{3}}$	.	−1/2	.	.	.	.	.	.	.	.	1/3
$d^{c(2)}$	−1/2	.	−1/2	.	$\overline{\mathbf{3}}$	.	.	−1/2	1/2	.	.	.	.	.	.	1/3
$d^{c(3)}$	.	1/2	1/2	.	$\overline{\mathbf{3}}$	.	1/4	1/4	1/4	−1/2	.	.	.	.	−1/2	1/3
$d^{c(4)}$	.	−1/2	1/2	.	$\overline{\mathbf{3}}$	.	1/4	1/4	1/4	−1/2	.	.	.	.	1/2	1/3
$d^{c(5)}$	.	.	.	.	$\overline{\mathbf{3}}$	.	1	.	.	.	.	.	.	.	.	1/3
$d^{c(6)}$	.	.	.	.	$\overline{\mathbf{3}}$	.	−1/2	−1/2	−1/2	.	.	.	.	.	.	1/3
$d^{c(7)}$	.	.	.	.	$\overline{\mathbf{3}}$	.	−3/4	−1/4	−1/4	.	.	.	.	1/2	−1/2	1/3
$d^{c(8)}$	.	.	.	.	$\overline{\mathbf{3}}$	.	1/4	3/4	−1/4	.	.	.	.	1/2	−1/2	1/3
$d^{c(9)}$	−1/2	−1/2	.	.	$\overline{\mathbf{3}}$	.	.	.	.	−1/2	.	.	.	−1/2	.	1/3
$d^{c(10)}$	1/2	1/2	.	.	$\overline{\mathbf{3}}$	.	.	.	.	−1/2	.	.	.	−1/2	.	1/3
$e^{c(1)}$	−1/2	.	1/2	.	.	.	−1/2	.	.	.	.	.	.	.	.	1
$e^{c(2)}$	.	.	.	.	.	.	−1/4	1/4	1/4	.	.	.	.	−1/2	−1/2	1
$\nu^{c(1)}$	−1/2	.	1/2	.	.	.	1	1/2	−1/2	.	.	.	.	.	.	.
$\nu^{c(2)}$	.	1/2	−1/2	.	.	.	3/4	−1/4	−1/4	1/2	.	.	.	.	1/2	.
$\nu^{c(3)}$	.	−1/2	−1/2	.	.	.	3/4	−1/4	−1/4	1/2	.	.	.	.	−1/2	.
$\nu^{c(4)}$	.	.	.	.	.	.	3/4	−3/4	1/4	.	.	.	.	−1/2	−1/2	.
$\nu^{c(5)}$	1/2	−1/2	.	.	.	.	1	.	.	1/2	.	.	.	1/2	.	.
$\nu^{c(6)}$	−1/2	1/2	.	.	.	.	1	.	.	1/2	.	.	.	1/2	.	.
$\nu^{c(7)}$	.	.	.	.	.	.	.	−1	1	.	.	.	.	.	.	.
$\nu^{c(8)}$	.	.	.	.	.	.	.	1	−1	.	.	.	.	.	.	.

Table 10.6: The chiral matter fields of the SM-like theory obtained from a GUT precursor model. These fields come from the untwisted sectors of the theory and remain massless after the CDC. There are only two net generations of chiral matter fields; each one remains from an initial two in the $\overline{V_0 + V_2}$ and $\overline{V_0 + V_1 + V_2 + \alpha V_7}$ ($\alpha = 1, 3$) sectors once the CDC is performed. The third generation is vector-like and does not fall into complete multiplets.

Tables 10.6, 10.7 along with their SM representations and hypercharges.

Moreover, there are Smatter chiral fields surviving in the massless spectrum, as shown in Table 10.8. It should be emphasised that the charges of those fields under the horizontal gauge groups are different from those of the matter fields in Table 10.6. This outcome forms another remarkable piece of evidence that spacetime SUSY is indeed absent from the mass spectrum of this theory.

L / R - handed *partner* matter fields																
State	$U(1)$	$U(1)$	$U(1)$	$U(1)$	$U(3)$	$U(2)$	$U(1)$	$U(1)$	$U(1)$	$U(1)$	$U(1)$	$SO(4)$	$SO(4)$	$U(1)$	$U(1)$	Y
$q^{c(1)}$	.	.	.	.	$\overline{\mathbf{3}}$	$\mathbf{2}$	1/2	−1/2	−1/2	.	.	.	.	.	.	−1/6
$\ell^{c(1)}$	1/2	.	1/2	.	.	$\mathbf{2}$	.	−1/2	1/2	.	.	.	.	.	.	1/2
$\ell^{c(2)}$	.	1/2	−1/2	.	.	$\mathbf{2}$	−1/4	−1/4	−1/4	−1/2	.	.	.	.	−1/2	1/2
$\ell^{c(3)}$	.	−1/2	−1/2	.	.	$\mathbf{2}$	−1/4	−1/4	−1/4	−1/2	.	.	.	.	1/2	1/2
$\ell^{c(4)}$	.	.	.	.	.	$\mathbf{2}$	−1/4	1/4	−3/4	.	.	.	.	−1/2	1/2	1/2
$\ell^{c(5)}$	−1/2	−1/2	.	.	.	$\mathbf{2}$	.	.	.	−1/2	.	.	.	1/2	.	1/2
$\ell^{c(6)}$	1/2	1/2	.	.	.	$\mathbf{2}$	.	.	.	−1/2	.	.	.	1/2	.	1/2
$d^{(1)}$	−1/2	.	1/2	.	$\mathbf{3}$	.	.	−1/2	1/2	.	.	.	.	.	.	−1/3
$d^{(2)}$	.	−1/2	−1/2	.	$\mathbf{3}$	.	−1/4	−1/4	−1/4	−1/2	.	.	.	.	−1/2	−1/3
$d^{(3)}$	.	1/2	−1/2	.	$\mathbf{3}$	.	−1/4	−1/4	−1/4	−1/2	.	.	.	.	1/2	−1/3
$d^{(4)}$	.	.	.	.	$\mathbf{3}$	.	−1	.	.	.	.	.	.	.	.	−1/3
$d^{(5)}$	.	.	.	.	$\mathbf{3}$	.	1/2	1/2	1/2	.	.	.	.	.	.	−1/3
$d^{(6)}$	.	.	.	.	$\mathbf{3}$	.	−1/4	1/4	−3/4	.	.	.	.	−1/2	−1/2	−1/3
$d^{(7)}$	−1/2	1/2	.	.	$\mathbf{3}$	.	.	.	.	−1/2	.	.	.	1/2	.	−1/3
$d^{(8)}$	1/2	−1/2	.	.	$\mathbf{3}$	.	.	.	.	−1/2	.	.	.	1/2	.	−1/3
$\nu^{(1)}$	−1/2	.	−1/2	.	.	.	−1	1/2	−1/2	.	.	.	.	.	.	.
$\nu^{(2)}$	.	−1/2	1/2	.	.	.	−3/4	1/4	1/4	1/2	.	.	.	.	1/2	.
$\nu^{(3)}$	.	1/2	1/2	.	.	.	−3/4	1/4	1/4	1/2	.	.	.	.	−1/2	.
$\nu^{(4)}$	.	.	.	.	.	.	−3/4	−1/4	3/4	.	.	.	.	1/2	−1/2	.
$\nu^{(5)}$	1/2	1/2	.	.	.	.	−1	.	.	1/2	.	.	.	−1/2	.	.
$\nu^{(6)}$	−1/2	−1/2	.	.	.	.	−1	.	.	1/2	.	.	.	−1/2	.	.

Table 10.7: The chiral *partner* matter fields in the untwisted sectors of the SM-like theory obtained from a GUT precursor model. These fields remain massless after the CDC.

L / R - handed Smatter fields																
State	$U(1)$	$U(1)$	$U(1)$	$U(1)$	$U(3)$	$U(2)$	$U(1)$	$U(1)$	$U(1)$	$U(1)$	$U(1)$	$SO(4)$	$SO(4)$	$U(1)$	$U(1)$	Y
$\tilde{q}^{(1)}$	.	.	.	.	$\mathbf{3}$	$\mathbf{2}$	−1/4	1/4	1/4	.	.	.	.	−1/2	−1/2	1/6
$\tilde{q}^{(2)}$	−1/2	.	1/2	.	$\mathbf{3}$	$\mathbf{2}$	−1/2	.	.	.	.	.	.	.	.	1/6
$\tilde{u}^{c(1)}$	.	.	.	.	$\overline{\mathbf{3}}$	.	−1/2	1/2	1/2	.	.	.	.	.	.	−2/3
$\tilde{u}^{c(2)}$	.	.	.	.	$\overline{\mathbf{3}}$	.	−1/2	1/2	1/2	.	.	.	.	.	.	−2/3
$\tilde{u}^{c(3)}$	.	.	.	.	$\overline{\mathbf{3}}$	.	−1/4	1/4	1/4	.	.	.	.	−1/2	1/2	−2/3
$\tilde{u}^{c(4)}$	1/2	.	1/2	.	$\overline{\mathbf{3}}$	.	−1/2	.	.	.	.	.	.	.	.	−2/3
$\tilde{d}^{c(1)}$	.	.	.	.	$\overline{\mathbf{3}}$	.	−3/4	−1/4	−1/4	.	.	.	.	1/2	1/2	1/3
$\tilde{d}^{c(2)}$	.	.	.	.	$\overline{\mathbf{3}}$	.	1/4	3/4	−1/4	.	.	.	.	1/2	1/2	1/3
$\tilde{d}^{c(3)}$	.	.	.	.	$\overline{\mathbf{3}}$	.	1/4	−1/4	3/4	.	.	.	.	1/2	−1/2	1/3
$\tilde{d}^{c(4)}$	−1/2	−1/2	.	.	$\overline{\mathbf{3}}$	.	.	.	.	1/2	.	.	.	−1/2	.	1/3
$\tilde{d}^{c(5)}$	−1/2	1/2	.	.	$\overline{\mathbf{3}}$	.	.	.	.	−1/2	.	.	.	−1/2	.	1/3
$\tilde{d}^{c(6)}$	1/2	1/2	.	.	$\overline{\mathbf{3}}$	.	.	.	.	1/2	.	.	.	−1/2	.	1/3
$\tilde{d}^{c(7)}$	1/2	−1/2	.	.	$\overline{\mathbf{3}}$	.	.	.	.	−1/2	.	.	.	−1/2	.	1/3
$\tilde{d}^{c(8)}$	1/2	.	−1/2	.	$\overline{\mathbf{3}}$	.	−1/2	.	.	.	.	.	.	.	.	1/3
$\tilde{d}^{c(9)}$	1/2	.	−1/2	.	$\overline{\mathbf{3}}$	.	.	−1/2	1/2	.	.	.	.	.	.	1/3
$\tilde{d}^{c(10)}$	−1/2	.	−1/2	.	$\overline{\mathbf{3}}$	.	.	1/2	−1/2	.	.	.	.	.	.	1/3
$\tilde{d}^{c(11)}$	.	1/2	1/2	.	$\overline{\mathbf{3}}$	.	1/4	1/4	1/4	1/2	.	.	.	.	−1/2	1/3
$\tilde{d}^{c(12)}$	.	−1/2	1/2	.	$\overline{\mathbf{3}}$	.	1/4	1/4	1/4	−1/2	.	.	.	.	−1/2	1/3
$\tilde{d}^{c(13)}$	.	−1/2	1/2	.	$\overline{\mathbf{3}}$	.	1/4	1/4	1/4	1/2	.	.	.	.	1/2	1/3
$\tilde{d}^{c(14)}$	.	1/2	1/2	.	$\overline{\mathbf{3}}$	.	1/4	1/4	1/4	−1/2	.	.	.	.	1/2	1/3
$\tilde{e}^{c(1)}$	.	.	.	.	.	.	−1/2	1/2	1/2	.	.	.	.	.	.	1
$\tilde{e}^{c(2)}$	.	.	.	.	.	.	−1/2	1/2	1/2	.	.	.	.	.	.	1
$\tilde{e}^{c(3)}$	.	.	.	.	.	.	−1/4	1/4	1/4	.	.	.	.	−1/2	1/2	1
$\tilde{e}^{c(4)}$	1/2	.	1/2	.	.	.	−1/2	.	.	.	.	.	.	.	.	1

Table 10.8: The chiral Smatter fields produced in the untwisted sectors of the theory that remain massless after the CDC.

In summary, this particular model has a total of $N_b^{(0)} = N_f^{(0)} = 576$ complex d.o.f, thus the cosmological constant is exponentially suppressed making this the first construction of a *stable, non-supersymmetric* SM-like theory which addresses the decompactification problem, albeit having only two generations of chiral matter fields.

10.4 Effective Field Theory

Before bringing the narrative of this chapter to an end, it is worth examining the EFT description of the SM-like theory presented above. As mentioned, a key aspect of this framework is that the non-supersymmetric four-dimensional theory interpolates between two six-dimensional theories, one of which is supersymmetric. This sets the compactification scale as a large free parameter by which the one-loop cosmological constant can be exponentially suppressed in theories that have equal numbers of massless bosons and fermions, $N_b^{(0)} = N_f^{(0)}$. A scale of R^{-1} provides a dimensionful order parameter for SUSY breaking, which in conjuction with the UV finiteness (due to misaligned supersymmetry) guarantees that all stringy threshold corrections appear as soft terms in the EFT. Despite the absence of spacetime SUSY, this is beneficial for writing down a spontaneously broken SUGRA theory for the low-lying spectrum, with parametrically large SUSY breaking terms.

A SSSB does not yield any contributions at tree-level to the kinetic terms and as a result the symmetries of the Kähler manifold are the same in the broken theory as they are in the theory without the CDC. Therefore the EFT can be built around the structure of the unbroken $\mathcal{N} = 1$ SUGRA; a review of which can be found in Ref. [170]. The metric of a general $\mathbb{T}_2$ torus in the absence of Wilson lines is given by Eq. (D.0.9), Appendix D. The supermultiplets containing the Higgs scalars are associated with continuous Wilson line moduli along the two cycles of the $\mathbb{T}_2$. The U modulus remains unaffected by the Wilson lines, while the T_2 combines with them in the Kähler metric according to

$$T_2 \to T_2 - \frac{\Phi\bar{\Phi}}{U_2}\,, \tag{10.4.1}$$

where Φ is a particular linear combination of the untwisted Higgs fields that can be identified as $2\Phi = \left(h_u + \bar{h}_d\right)$ [171–174]. In order to be in agreement with most of the SUGRA literature, the convention used here is

$$\begin{aligned} iU &= U_1 + iU_2 \\ iT &= T_1 + iT_2 , \end{aligned} \tag{10.4.2}$$

and thus $2T_2 = T + \bar{T}$, and $\det G = T_2^2$. The Kähler potential is then found to be

$$K = \log Y - \log 4(T_2 U_2 - \Phi\bar{\Phi})\,. \tag{10.4.3}$$

The dilaton combination (Y) generally includes a term from the (Heterotic) Green-Schwarz mechanism given by

$$Y = S + \bar{S} + \frac{1}{8\pi^2}\delta_{GS}\log 4(T_2 U_2 - \Phi\bar{\Phi})\,, \tag{10.4.4}$$

where S is the holomorphic tree coupling. Note that the low-energy effective action obeys the shift-symmetry

$$h_u \to h_u + C\,; \qquad h_d \to h_d - C^*\,, \tag{10.4.5}$$

for constant (superfield) C. This symmetry is allowed by the partition function of the original theory which is modular invariant. The latter has been introduced so that the overall effect will result in a massless Higgs state, given by the linear combination $h = \frac{1}{\sqrt{2}}\left(h_u - \bar{h}_d\right)$, while the orthogonal combination $H = \frac{1}{\sqrt{2}}\left(h_u + \bar{h}_d\right)$ can be massive [175–179]. As per Ref. [180], a shift symmetry can be protected to some extent beyond tree-loop if one takes highly asymmetric configurations, but in general these configurations do not hold at one-loop order. Nevertheless, there is a larger symmetry in the kind of theories discussed in this work that protects the masses of both h and H at tree-level. This larger symmetry is in fact an $\mathcal{N} = 2$ sub-sector of the theory living on the $\mathbb{T}_2$ torus. Even though in this set-up the gauge kinetic term of the original $\mathcal{N} = 1$ SUGRA theory is not discussed, the avid readers can find it in the review of Ref. [170].

Based on the previous work of Ref. [52], the shift in the masses of the light d.o.f induced by the CDC results in a superpotential that is dependent on the U modulus. To demonstrate this, one considers the gravitino mass which in the EFT is determined by

$$m^2_{\frac{3}{2}} = e^K |W|^2 \,. \tag{10.4.6}$$

It is known that the Scherk-Schwarz compactification causes a half-integer shift in both of the KK numbers ℓ_1, ℓ_2, so that adding left- and right-moving contributions, the total mass-squared of the gravitino becomes

$$\begin{aligned} \left(m^2_{\frac{3}{2}}\right)^{(string)} &= \frac{1}{\alpha'} \hat{m}_i G^{ij} \hat{m}_j \\ &= \frac{1}{4\alpha'} \frac{1}{T_2 U_2 - \Phi\bar{\Phi}} |1 - iU|^2 \,. \end{aligned} \tag{10.4.7}$$

As deduced in the Appendix D, the square $\mathbb{T}_2$ with radii (normalised to the string length) $r_{1,2}$ has $T_2 = r_1 r_2$, $U_2 = r_1 r_2^{-1}$. Therefore, the gravitino mass is actually determined by

$$\left(m^2_{\frac{3}{2}}\right)^{(string)} = \frac{1}{4\alpha'} \frac{1}{r_1^2} \left(1 + \frac{r_1^2}{r_2^2}\right), \tag{10.4.8}$$

The physical mass of the gravitino can then be identified as

$$m^2_{\frac{3}{2}} = \frac{1}{2Y \left(T_2 U_2 - \Phi\bar{\Phi}\right)} |iU - 1|^2 \,, \tag{10.4.9}$$

which in turn yields

$$W = \sqrt{2}(iU - 1) \,. \tag{10.4.10}$$

As expected, there are no small parameters in the superpotential because the large radii of compactification in the Scherk-Schwarz mechanism allow the suppression of soft terms, while SUSY can be restored in the infinite radii limit. Note that the expression in Eq. (10.4.10) is similar to the expression in Refs. [63,86]; the difference being that in this case the Scherk-Schwarz twists act along both large extra dimensions. In the string spectrum the tree-level gaugino masses are degenerate with the gravitino, and it is indeed checked in Ref. [166] that this effective SUGRA theory agrees with this statement.

There is at least one Higgs state coming from the untwisted sectors that remains massless, while the corresponding Higgsino (as well as others) pick up a mass which is equal to that of the gravitino. Explicitly, for the Higgsinos, the Kähler potential is expanded as

$$\begin{aligned} K &\supset -\log\left(4T_2U_2 - |h_u + \bar{h}_d|^2\right) \\ &= -\log 4T_2U_2 + \frac{1}{4T_2U_2}(|h_u|^2 + |h_d|^2 + h_u h_d + \bar{h}_u\bar{h}_d)\,. \end{aligned} \tag{10.4.11}$$

Using the result in Eq. (10.4.10), the effective tree-level fermion mass terms (i.e. μ-terms) are calculated as

$$\mu = m_{\frac{3}{2}}\left(4T_2U_2\frac{W_{ij}}{W} - \frac{\bar{W}}{W}\right). \tag{10.4.12}$$

In the case that there are no explicit W_{ij} μ-terms in the superpotential, this automatically has the same magnitude as the gaugino and gravitino masses in accord with the string theory spectrum. Finally, it is worth checking that all the scalars remain massless. The only possible non-zero contributions after SUSY breaking could come from U derivatives of W. However, using $K^{\bar{j}i}K_i = -2(S_2, T_2, U_2, \bar{\Phi}, \Phi)$ it is found that these contributions vanish as well resulting in

$$\begin{aligned} V &\supset e^K\left(K_{\bar{j}}\bar{W}K^{\bar{j}U}W_U + \bar{W}_{\bar{U}}K^{\bar{U}i}K_iW + \bar{W}_{\bar{U}}K^{\bar{U}U}W_U\right) \\ &= e^K\left[-(U+\bar{U})(\bar{U}-i) - (U+\bar{U})(U+i) + (U+\bar{U})^2\right] = 0\,. \end{aligned} \tag{10.4.13}$$

It is therefore inferred that the SUGRA effective theory discussed provides an accurate description of the SUSY breaking up to the compactification scale $\sim R_c^{-1}$ at which the lightest stringy modes appear.

Chapter 11

Conclusions

> *Nature uses only the longest threads to weave her patterns, so that each small piece of her fabric reveals the organisation of the entire tapestry.*
>
> **Richard P. Feynman**

In our current era, theoretical particle physicists are in a relentless pursuit of elegant and intelligible theories that would elucidate the mechanisms governing the inner workings of our universe. It is an irrefutable fact though that up until now they have managed to successfully build only a partially comprehensive picture of the observable world; having seen only a part of the bigger picture, there are still many unavoidable questions that demand lucid answers. Fortunately, theorists are not alone in this pursuit; the rapid technological advancements of our time have led to the development of powerful accelerators which have experimentally validated theories, discarded others and altered the direction of other theoretical pursuits. A prominent accelerator is the LHC, which sits at the root of diverse experimental collaborations with ambitions to tackle some of the most urgent issues head on. One of the challenges that the LHC faces is the discovery of spacetime SUSY, which is the strongest contender for addressing the gauge hierarchy issue. Indeed, the victorious discovery of the Higgs boson makes the need for stabilising the gauge hierarchy mandatory, and has thus sparked a huge surge in experimental activity. At the same time, the stark absence of spacetime SUSY from the currently accessible energy scales has forced theorists to resort to finding alternative methods which could serve as its coequal substitutes.

One of the most disconcerting aspects of strings is their lack of stability once they are formulated in the absence of spacetime SUSY. While most perturbative strings have unfixed moduli, non-supersymmetric strings carry an extra degree of difficulty: they admit non-zero dilaton tadpoles, which shift the vacuum and destabilise the theory. This is a general feature of all non-supersymmetric strings, and afflicts even those which are free of physical tachyons at tree level. This represents an inescapable fundamental obstacle for the use of such strings as the basis for constructing consistent non-supersymmetric string theories with viable phenomenologies.

In this thesis, great care has been taken to demonstrate that this obstacle can be overcome within a class of perturbative four-dimensional heterotic strings constructed through CDCs. Specifically, these strings belong in a class of models that interpolate between two different six-dimensional theories at $R \to \infty$ and $R \to 0$, where R is the generic radius of compactification from six to four dimensions.

The discussion in this work begins with a concise description of the prerequisite theoretical background. This entails details that appear throughout the narrative of the entire thesis, and set the stage for presenting new ideas. A more extensive description of the basic mathematical framework used for the construction of all models presented in this work then follows. The end of this discussion signals the beginning of the original work in this thesis. The main part starts with a presentation of several crucial aspects associated with the spectra of non-supersymmetric string models. Included among these aspects is the study of the one-loop partition functions, the existence of off-shell states such as the proto-gravitons and proto-gravitinos, as well as the study of the leading and subleading contributions from these and other states to the corresponding one-loop cosmological constants. A particular emphasis is placed on how such strings preserve their finite properties when spacetime SUSY is no longer present in the spectrum. This led to the introduction of "misaligned SUSY", the mechanism which in conjuction with modular invariance maintains the finiteness of the strings. The generic class of models that interpolate between supersymmetric and non-supersymmetric models at their two endpoints is then explored. In particular, it is demonstrated how the one-loop cosmological constant, and hence the dilaton tadpole of such models, can be exponentially suppressed, thus enhancing the stability properties of the associated theory.

It is then outlined how one can actually construct phenomenologically appealing models that have equal numbers of massless bosonic and fermionic d.o.f. Indeed, this is the condition that must be satisfied for the exponential suppression of the cosmological constant. As demonstrated, such models are most easily built by starting with existing self-consistent supersymmetric four-dimensional string theories, and lifting them to $\mathcal{N} = 1$ in six dimensions. The higher-dimensional supersymmetric theory then undergoes a CDC on a $\mathbb{Z}_2$ orbifold, thus yielding a four-dimensional theory in which SUSY is spontaneously broken. The CDC is considered to be a generalisation of a Scherk-Schwarz compactification. It should be stressed that even though the breaking of SUSY is spontaneous, from a four-dimensional perspective this SUSY breaking is not "soft" in the usual sense of the term. Actually, there is no longer an exact cancellation between the numbers of bosonic and fermionic d.o.f at every energy level, but instead there is a net degeneracy of bosonic minus fermionic d.o.f at adjacent energy levels. These degeneracies grow exponentially with energy therefore such models are genuinely non-supersymmetric by construction and at all energy scales.

All models are constructed in the so-called free-fermionic formalism, but there is no reason why this procedure cannot be duplicated within other formalisms. In addition, all the interpolating models in four dimensions are derived from the same $\mathcal{N} = 1$ six-dimensional theory. Each model can be constructed so as to have a different gauge group and particle content. Most importantly, the models can be constructed so as to have chiral generations coming from either both the twisted and untwisted sectors or only from one of them. This is achieved by altering the boundary conditions assigned to the vectors of the theory as well as adjusting the choice of CDC vector $\mathbf{e}$. An $SO(10)$ grand-unified model, a flipped $SU(5)$, a Pati-Salam and finally a SM-like theory are then presented.

Despite the fact that all these models exhibit equal numbers of massless bosonic and fermionic d.o.f and thus suppressed instabilities, each one of these demonstrates a different degree of phenomenological success. However, none has all of the desirable phenomenological features one would want in order to serve as the starting point for a detailed phenomenological study. All their properties were examined in three independent ways: through their partition functions, through their Poisson-resummed large-radius expansions, and finally through explicit construction and study of their low-energy spectra. These models could be considered as a stepping stone in the fabrication of non-supersymmetric string models which are not plagued by instabilities, and in the evolution of string phenomenology. They are distinctly characterised by a Bose-Fermi degeneracy exhibited by the KK excitations of the massless modes. This is essentially a cancellation of an aggregate of bosonic states in an observable sector against an aggregate of fermionic states in a presumably hidden sector, and vice versa. Since this is a feature applicable only to the massless modes, it implies that there is no remnant spacetime SUSY in the spectrum. Indeed, the massive string-oscillator excitations do not exhibit any such bosonic/fermionic degeneracies, and instead it is only through misaligned SUSY at all mass levels that the finiteness of such strings is ensured.

A general phenomenological feature of these models is that SUSY can be spontaneously broken only in the untwisted sectors of the theory, while the twisted sectors remain globally supersymmetric. Depending on the choice of CDC vector $\mathbf{e}$, the states in the gauge sector and the untwisted matter sectors that do not fulfil the condition $\mathbf{e} \cdot \mathbf{Q} = 0$ gain masses of order R^{-1}. By contrast, the twisted sectors are initially unaffected, with states gaining masses only radiatively. One appealing aspect of this setup is that all radiative terms, including their RG running, are completely calculable within the string theory, and are finite.

Aside from general phenomenological features which are in fact derived from the interpolating structure of the model, there is also a discussion on some model-dependent phenomenological properties. These include the natural particle assignment, the Yukawa couplings, and the masses of the scalar particles. Specifically for the latter, it is shown that it might be possible to suppress the radiative corrections to the scalar particles that survive in the massless spectrum of the theory after CDC, thereby achieving naturally light scalars and addressing the issue of naturalness. There can be a range of mass scales for which non-supersymmetric string theories can be consistent and produce interesting, sufficiently small, cosmological-constant values and at the same time reasonable radiative physical masses for scalars. However, for such models there appear to be low string scales, implying ultimately either some form of strong coupling or large gauge threshold corrections from KK modes. This occurrence is more commonly referred to as the "decompactification problem". There are configurations which prevent these unwanted scenarios, and may be compatible with perturbative unification near the canonical heterotic string scale. The configurations are known as GUT precursors and their existence provide the freedom to construct a non-supersymmetric string theory with small couplings at large volumes.

The issues addressed in this thesis regarding the cosmological constant, vacuum stability, and the mass hierarchy for light scalars such as the Higgs are some of the most challenging and unique problems of non-supersymmetric string models. It must not come as a surprise that they are all related within this framework. This is precisely because there is only one source of SSSB which is applicable for all of them and hence they are all intrinsically non-supersymmetric, even at the string scale. Moreover, the tight self-consistency constraints of the string constructions unavoidably tie the resolutions of these different problems to each other.

In particular, the exponential suppression of the cosmological constants suggests that there is hope of stabilising such models within field theory. This is due to the fact that stability issues in general are considered to have the same status and degree of severity as in a SUGRA theory in which SUSY is softly broken by non-perturbative field-theory effects. A possible scenario is that field theory effects can dominate over the exponentially suppressed dilaton tadpole, or that there is some interesting interplay between the two effects.

Significantly, it was suggested a long time ago that non-supersymmetric string compactifications would benefit from exponentially suppressed cosmological constants if there were an *exact cancellation* between the massless bosonic and fermionic d.o.f. This thesis has explored the ideas relevant to this suggestion, has presented their development and eventually has demonstrated the successful construction of actual models which exhibit enhanced stability. Therefore, this work paves a new pathway for studying non-supersymmetric models built entirely within a heterotic string framework. In regards to the phenomenological viability of the resulting models, it should be emphasised that they stand on equal grounds with non-stabilised supersymmetric strings; when the latter undergoes a SUSY breaking typically, a runaway dilaton potential appears which is comparable to the potential of these models. In order to verify the full stability of theories with exponentially suppressed cosmological constants, it will be necessary to study their complete moduli spaces, also taking extreme care to prevent the F-flat directions from becoming tachyonic.

Clearly, all the ideas encompassed in this thesis are still in their infancy, which implies that they require a perpetual and meticulous approach in order to develop into tools for powerful theories. Consequently, at this stage there are many possibilities for future work, beyond those related to the purely technical side (construction) of the models. First, it is important to understand the precise nature of the model under interpolation, which requires the identification of the complete structure of the tachyon-free non-supersymmetric six-dimensional theory at the $R \to 0$ endpoint. This would help to clarify how the six-dimensional theory at $R \to 0$ relates to the six-dimensional theory at $R \to \infty$, it would provide more details on the phenomenological structure of the interpolating theory and it would also shed light on the full effects of CDC at that endpoint. Second, one issue of particular interest is to identify the precise form of the potential and how it is associated to an EFT in which SUSY is spontaneously broken. It is equally important to understand and interpret the various features related to potentials, such as those in Fig. 9.3, as they would undoubtedly provide useful details about the dynamics of the theory.

From a phenomenological perspective, it would be ideal if a three-generation SM-like theory with an exponentially small one-loop cosmological constant and a working Higgs sector could be formulated. The benefit of constructing such a model is that all scalar masses would be calculable and finite. In addition, being non-supersymmetric by construction, the Yukawa couplings would also receive radiative corrections suppressed by powers of $R\mathcal{M}^{-1}$. This outcome brings forth the question of whether these radiative corrections would have any impact on the SM hierarchies. Furthermore, it would be interesting to examine how one can relate the suppressed Yukawa terms with those terms obtained in field theories with softly-broken SUSY.

One other issue that certainly deserves further investigation is whether the exponential suppression of the cosmological constant continues beyond one-loop order. In general, higher-order contributions depend not only on the spectra of the theories but also on the particle content interactions. One would speculate that as long as there are no contributions from the massless d.o.f to the cosmological constant, then the suppressions exhibited at one-loop would continue to persist to higher loops. In the EFT, all loops would then be expected to experience the same cancellations, while the couplings exhibit a high degree of degeneracy. However, this is just speculation and needs to be handled with extra care, requiring careful analysis before being turned into a definite fact. Another issue, on a par with the former, that requires clarification, is whether there can be scalar particles in the theory which remain naturally light. As demonstrated, the contributions to the mass-squared of the Higgs field that survives in the massless spectrum could be suppressed, provided that the numbers of massless bosonic and fermionic states that couple to the Higgs cancel each other. This finding is truly significant as it would undoubtedly help to elucidate the naturalness problem in string theory. Hence, finding a model that satisfies this condition and that also has an exponentially suppressed cosmological constant is of utmost priority. It should be noted that in the construction of such theories there are various different scales involved, such as the Planck scale, the string scale, the unification scale and of course the compactification scale $\mathcal{O}(R^{-1})$. Generally, these scales are related to each other through the string coupling and the volume of compactification. In light of the above discussion about ensuring a naturally light Higgs, great attention must be paid to what constitutes reasonable energy and mass scales for the phenomenologies of such models.

Finally, this setup was focused on model building and phenomenology in weakly coupled heterotic strings. However, there is no restriction on applying these ideas to other types of string theories so as to yield more diverse results. These ideas could also be extended into QFT by attempting to reproduce at least to some extent some of the most significant aspects of the theories presented. There is also potential for such theories to be applied in string cosmology, opening new doors in the field of inflation and other relevant studies.

In conclusion, then, there are numerous compelling pieces of evidence to suggest that the models and methods presented here could be considered as a benchmark for the emergence and evolution of a genuine non-supersymmetric string phenomenology. This work has inevitably triggered an avalanche of issues, both on a theoretical and a phenomenological level; some of them are presently under investigation while others clearly remain an attractive challenge for future works. Although this could be considered as downside, it still cannot outshine the evident existence of numerous models which are fundamentally non-supersymmetric and at the same time admit suppressed dilaton tadpoles. Inevitably, the existence of such models correlates with the existence of a vast landscape of finite non-supersymmetric theories which bear enhanced stability properties. All such models carry a promising theoretical background as well as fascinating phenomenological prospects. Because of this, they can be considered as suitable candidates for providing a good description of nature, especially at the low energies where our observable non-supersymmetric world resides. Therefore, it is imperative that the landscape must be explored with every tool that string phenomenologists have at their disposal in order to determine the extent to which a remarkably accurate non-supersymmetric string phenomenology is achievable. This thesis is coming to an end, however as is always the case, this end marks a new beginning! This is a turning point for bringing together auspicious, ambitious and perhaps even revolutionary ideas in order to target some of the most crucial and perpetuating issues in theoretical particle physics. It goes without saying that nature will never cease to amaze us and that there are certainly exciting times to look forward to.

September 20, 2016

Appendix A

Basic theoretical tools for the Standard Model

A.1 The full form of the Standard Model Lagrangian

In this section the explicit form of each component that constitutes the SM Lagrangian, as per Eq. (2.1.8) of Chapter 2 is presented. The first component is the $\mathcal{L}$ for the SM gauge bosons:

$$\begin{aligned}\mathcal{L}^{SM}_{gauge\,bosons} &= -\tfrac{1}{2}Tr\,[F_{\mu\nu}F^{\mu\nu}] \\ &= -\tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} - \tfrac{1}{4}F^{a}_{\mu\nu}F^{a\,\mu\nu} - \tfrac{1}{4}F^{A}_{\mu\nu}F^{A\,\mu\nu} + \mathcal{L}_{\substack{gauge\\ fixing}} + \mathcal{L}_{\substack{FP\\ ghosts}}\,,\end{aligned} \tag{A.1.1}$$

where $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ is the hypercharge field strength. The second term corresponds to the $SU(2)$ field strength with $a = 1, 2, 3$ corresponding to each $SU(2)$ vector boson. The third term is the gluon kinetic term with $A = 1, \ldots, 8$ for each gluon. For explicit perturbative calculations, it is necessary to do some gauge fixing. This requires the consideration of additional terms which include the gauge fixing terms and the Fadeev-Popov (FP) ghosts. Since the form of these terms depends on the choice of gauge, the final form of the fourth and fifth terms in Eq. (A.1.1) is not predefined.

The second component of the $\mathcal{L}^{SM}$ contains the kinetic terms of the SM chiral matter fields:

$$\begin{aligned}
\mathcal{L}^{SM}_{fermion\,KT} &= \overline{\Psi}_{L\hat{c}}\gamma^\mu D_\mu \Psi_{L\hat{c}} \\
&= i\,\bar{l}^T_{L\hat{c}}\gamma^\mu \mathbf{D}_\mu l_{L\hat{c}} + i\,\bar{e}_{R\hat{c}}\gamma^\mu D_\mu e_{R\hat{c}} + i\,\bar{\nu}_{R\hat{c}}\gamma^\mu \partial_\mu \nu_{L\hat{c}} \\
&+ i\,\overline{Q}^T_{L\hat{c}}\gamma^\mu \mathbf{D}_\mu Q_{L\hat{c}} + i\,\overline{d}_{R\hat{c}}\gamma^\mu \mathbf{D}_\mu d_{R\hat{c}} + i\,\overline{u}_{R\hat{c}}\gamma^\mu \partial_\mu u_{R\hat{c}}\,. \qquad \text{(A.1.2)}
\end{aligned}$$

The covariant derivatives are defined as

$$\mathbf{D}_\mu = \partial_\mu + ig\mathbf{T}^a W^a_\mu + ig' Y_{l_L} B_\mu \qquad \text{for the } l_L \text{ and } Q_L \text{ fields} \qquad \text{(A.1.3)}$$

$$\mathbf{D}_\mu = \partial_\mu + ig_s \mathbf{T_s}^A G^A_\mu + ig' Y_{d_R} B_\mu \qquad \text{for the } d_R \text{ fields} \qquad \text{(A.1.4)}$$

$$D_\mu = \partial_\mu + ig' Y_{e_R} B_\mu \qquad \text{for the } e_R \text{ fields} \qquad \text{(A.1.5)}$$

where g, g' and g_s are the weak and strong coupling respectively, $\mathbf{T_s}^A$ are the $SU(3)$ generators, G^A_μ are the gluon fields, W^a_μ are the $SU(2)_L$ gauge fields, and Y_f is the hypercharge of fermion f. Similarly, the third component is the Lagrangian that determines the masses of the SM chiral matter fields, through their Yukawa couplings:

$$\begin{aligned}
\mathcal{L}^{SM}_{fermion\,masses} &= -\tfrac{1}{2}\Psi^T_{L\hat{c}}\,\mathbf{Ch}\,\Phi\Psi_{L\hat{c}} + h.c \\
&= -\hat{h}_{e\hat{c}\hat{g}}\bar{l}_{L\hat{c}}\Phi e_{R\hat{g}} - \hat{h}_{d\hat{c}\hat{g}}\overline{q}_{L\hat{c}}\Phi d_{R\hat{g}} - \hat{h}_{u\hat{c}\hat{g}}\overline{q}_{L\hat{c}}\tilde{\Phi} u_{R\hat{g}} + h.c\,, \qquad \text{(A.1.6)}
\end{aligned}$$

where $\hat{c}$ (and $\hat{g}$) $= 1, 2, 3$ are the fermion families, $\hat{h}$ are the Yukawa couplings and $\tilde{\Phi} = i\sigma_2\Phi^*$. Note that in this case a neutrino Yukawa term is omitted because neutrinos do not get their masses from Yukawa interactions. Finally, the last piece of the SM Lagrangian is the Higgs part:

$$\mathcal{L}^{SM}_{Higgs} = |D_\mu\Phi|^2 - \mu^2\Phi^\dagger\Phi - \lambda_H(\Phi^\dagger\Phi)^2\,. \qquad \text{(A.1.7)}$$

The Lagrangian associated with the the spontaneous breaking of the electroweak symmetry is found by substituting Eq. (2.1.9) to the equation above. The result is

$$\begin{aligned} \mathcal{L}^{SSB}_{Higgs} &= \tfrac{1}{2}(\partial_\mu H)^2 + \lambda_H v^2 H^2 + \frac{g^2 v^2}{4} W^{+\mu} W^-_\mu + \frac{g^2 v^2}{8\cos\theta_W} Z^\mu Z_\mu \\ &+ \text{interactions}\,. \end{aligned} \tag{A.1.8}$$

The expression $\lambda_H v^2 H^2 = \frac{1}{2} m_H^2 H^2$ defines the mass of the Higgs boson (m_H), and λ_H is the Higgs coupling. The weak mixing angle, θ_W, is defined by the expression

$$\tan\theta_W = \frac{g_1}{g_2}\,, \tag{A.1.9}$$

where g_1, g_2 are the coupling strengths of the $U(1)$ and $SU(2)$ gauge bosons. The g_2 is directly the coupling strength of the $W^\pm$ bosons and is analogous to the value of the electromagnetic coupling for photons, $e = g_2 \sin\theta_W = \frac{1}{\sqrt{g_1^2+g_2^2}} g_1 g_2$.

A.2 Masses of particles in the Standard Model

The mass of the physical gauge bosons $W^\pm$ and Z^0, is determined by the kinetic term of the $\mathcal{L}^{SM}_{Higgs}$,

$$m^2_{W^\pm} = \tfrac{1}{4} g^2 v^2 \qquad\qquad m^2_{Z^0} = \tfrac{1}{4}(g'^2 + g^2) v^2\,. \tag{A.2.1}$$

where $v = 246.22$ GeV is the VEV.

The masses of the chiral matter fields, except neutrino masses, are determined by $\mathcal{L}^{SM}_{fermion\ masses}$,

$$m_f = \frac{1}{\sqrt{2}} \hat{h}_f v\,. \tag{A.2.2}$$

where f labels the fermions.

A.3 One-loop corrections to the Higgs mass

As it is seen in Eq. (A.1.7), the $\mathcal{L}$ of the scalar Higgs boson includes a term $m^2 \Phi^\dagger \Phi$. The m^2 parameter is the bare mass. At tree level it obeys $m^2 = m^2_{th}$ and the value is equivalent to the experimental result m_H. At one-loop the theoretical mass is

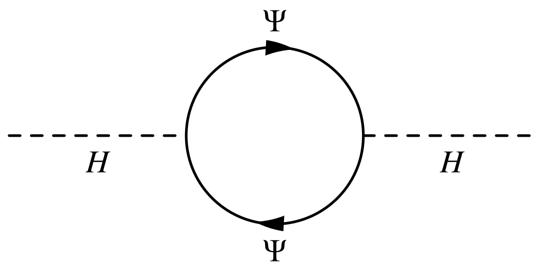


Figure A.1: Higgs mass renormalisation from a fermion loop.

subjected to quantum corrections so the total Higgs mass is given by

$$m^2_{bare} = m^2_{renormalised} + \delta m^2 \,. \tag{A.3.1}$$

The quantum corrections are due to scalar and fermion loops generated by Higgs self-interactions as it is demonstrated in the Feynman diagrams depicted in Figs. A.1-A.3. Considering the renormalisation of the scalar mass from a fermion loop shown in Fig. A.1 using the $\mathcal{L}^{SM}$ there is a resulting contribution [106]:

$$\begin{aligned}
-i\Sigma(Q^2) \equiv (\delta m_H^2)_F &= i\left(\frac{-i\,g_F}{\sqrt{2}}\right)^2 \int^{\lambda} \frac{d^4p}{(2\pi)^4} \frac{\mathrm{Tr}\,[(\not{p}+m_F)((\not{p}-\not{Q})+m_F)]}{(p^2-m_f^2)[(p-Q)^2-m_F^2]} \\
&= -\frac{g_F^2}{8\pi^2}\Bigg\{\lambda^2 + (Q^2-6m_F^2)\,ln\left(\frac{\lambda}{m_F}\right) \\
&+ \left(2m_F^2 - \tfrac{1}{2}Q^2\right)\left[1 + I_1\left(\frac{Q^2}{m_F^2}\right)\right]\Bigg\} + O\left(\frac{1}{\lambda^2}\right),
\end{aligned} \tag{A.3.2}$$

where $I_1(x) = \int_0^1 dy\, log\,[1 - xy(1-y)]$ and m_F is the physical mass of the fermion Ψ.

From the diagram of Fig. A.2, the contribution to the Higgs mass renormalisation due to scalar fields ϕ interacting with the Higgs boson is calculated as in [106]:

$$\begin{aligned}
(\delta m_H^2)_{S1} &= -g_S \int \frac{d^4p}{(2\pi)^4}\left[\frac{i}{p^2-m_1^2} + \frac{i}{p^2-m_2^2}\right] \\
&= \frac{g_S}{16\pi^2}\left\{2\lambda^2 - 2m_1^2\,ln\left(\frac{\lambda}{m_1}\right) - 2m_2^2\,ln\left(\frac{\lambda}{m_2}\right)\right\} + O\left(\frac{1}{\lambda^2}\right),
\end{aligned} \tag{A.3.3}$$

where g_S is the scalar coupling and $m_{1,2}$ are the physical masses of the scalar par-

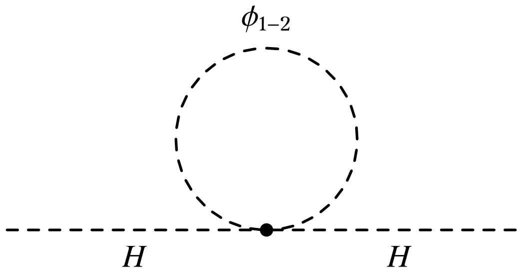


Figure A.2: Higgs mass renormalisation from a scalar loop.

ticles $\phi_{1,2}$.

After SSB, the part of the $\mathcal{L}^{SM}$ that determines the renormalisation of the Higgs mass parameter from scalar loops leads to a cubic interaction shown in the diagram of Fig. A.3 which yields a non quadratically divergent contribution, as calculated in [106]:

$$\begin{aligned}(\delta m_H^2)_{S2} &= -g_S \int \frac{d^4p}{(2\pi)^4}\Big[\frac{1}{(p^2-m_1^2)^2}+\frac{1}{(p^2-m_2^2)^2}\Big] \\ &= \frac{g_S^2 v^2}{16\pi^2}\Big\{-1+2\,ln\Big(\frac{\lambda}{m_1}\Big)-I_1\Big(\frac{m_H^2}{m_1}\Big)\Big\}+(m_1\to m_2)+O\Big(\frac{1}{\lambda^2}\Big). \quad \text{(A.3.4)}\end{aligned}$$

Even though the calculations presented above are applied to the case of the Higgs boson mass, it is worth emphasising that they are valid for any *light, scalar* particle.

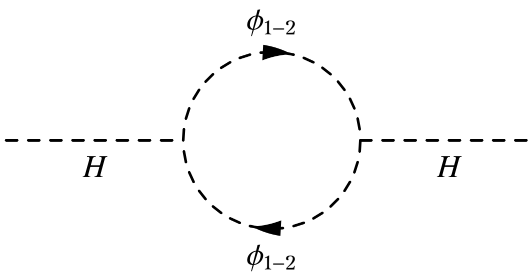


Figure A.3: Higgs mass renormalisation from a scalar loop of cubic interaction.

Appendix B

The MSSM Higgs sector

As described in Section 2.2.1 of Chapter 2, in the MSSM it is necessary to add a second complex Higgs doublet field with opposite hypercharge $\left(Y = -\frac{1}{2}\right)$. Therefore, the two complex Higgs doublet fields are postulated as

$$H_d = \begin{pmatrix} h_d^0 \\ h_d^- \end{pmatrix}_{Y=-\frac{1}{2}} \quad \text{and} \quad H_u = \begin{pmatrix} h_u^+ \\ h_u^0 \end{pmatrix}_{Y=+\frac{1}{2}} . \tag{B.0.1}$$

The Higgs scalar potential (at tree level) in the MSSM is determined by

$$\begin{aligned} V &= (\mu^2 + m_d^2)\,|H_d|^2 + (\mu^2 + m_u^2)\,|H_u|^2 - \sum_{ij} \mu B\,(\epsilon_{ij} H_d^i H_u^j + h.c) \\ &+ \frac{g^2 + g'^2}{8}\,(|H_d|^2 - |H_u|^2)^2 + \frac{g^2}{2}\,|H_d^\dagger H_u|^2 . \end{aligned} \tag{B.0.2}$$

The terms proportional to μ^2 come from F-terms. Similarly, the terms proportional to $(g^2 + g'^2)$ come from D-terms, and the remaining ones are contributions from the soft breaking terms [181]. The scalar potential also includes many other terms that are contributions from squark and slepton fields. However, these terms are omitted in Eq. (B.0.2) since they are not relevant for the upcoming discussion.

The potential has a trivial minimum $V = 0$ which is satisfied for $H_d = H_u = 0$, so no spontaneous breaking takes place. To achieve a spontaneous breaking of the

$SU(2) \otimes U(1)$ symmetry to the $U(1)_{em}$, it is essential to break first SUSY[1]. Without loss of generality, one can set $h_d^- = h_u^+ = 0$ as justified in Ref. [114]. The scalar potential is now modified to

$$\begin{aligned} V &= (\mu^2 + m_d^2)\,|h_d^0|^2 + (\mu^2 + m_u^2)\,|h_u^0|^2 - \sum_{ij} \mu B\,(\epsilon_{ij} h_d^0 h_u^0 + h.c) \\ &+ \frac{g^2 + g'^2}{8}\,(|h_d^0|^2 - |h_u^0|^2)^2 + \frac{g^2}{2}\,|h_d^{0\dagger} h_u^0|^2 \,. \end{aligned} \tag{B.0.3}$$

Requiring gauge breaking and having a potential that is necessarily bounded from below yields the following constraints:

$$(\mu B)^2 > (|\mu|^2 + m_d^2)(|\mu|^2 + m_u^2) \tag{B.0.4a}$$

$$2\,|\mu B| < (|\mu|^2 + m_d^2) + (|\mu|^2 + m_u^2)\,, \tag{B.0.4b}$$

which surprisingly are fulfilled provided that the condition $m_d^2 \neq m_u^2$ is satisfied. In GMSB and mSUGRA models, $m_d^2 \neq m_u^2$ at tree level so that the electroweak breaking is driven by the different evolution of the m_d^2 and m_u^2. This is the mechanism of radiative electroweak symmetry breaking. Once this breaking occurs the two Higgs fields obtain a VEV:

$$\langle H_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_d \\ 0 \end{pmatrix} \qquad \text{and} \qquad \langle H_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \tag{B.0.5}$$

where both v_d and v_u are taken to be real and positive. Note that the two VEVs add up quadratically to a net VEV value, as in the SM, and their ratio is written as

$$\tan\beta \equiv \frac{v_u}{v_d}\,, \qquad v^2 \equiv \sqrt{v_d^2 + v_u^2}\,. \tag{B.0.6}$$

The value of $\tan\beta$ is largely dependent on the parameters of the MSSM Lagrangian and as a result can be determined quantitatively. The condition

[1]This is defined to be a radiative breaking of the gauge group.

$$\frac{\partial V}{\partial h_d^0} = \frac{\partial V}{\partial h_d^0} = 0\,, \tag{B.0.7}$$

minimises the potential of Eq. (B.0.3) yielding:

$$\mu^2 + m_d^2 = (-B\mu)^2 \tan\beta + \frac{g^2 + g'^2}{4}\,(v_d^2 + v_u^2) \tag{B.0.8a}$$

$$\mu^2 + m_u^2 = (-B\mu)^2 \cot\beta + \frac{g^2 + g'^2}{4}\,(v_d^2 + v_u^2)\,. \tag{B.0.8b}$$

These expressions relate the parameters on which $\tan\beta$ depends to the gauge couplings, the soft breaking terms and the mass parameter μ. They also eliminate the parameters B and μ from the Lagrangian but do not produce any value for the latter. From this result it is concluded that the ratio of v_d and v_u, along with the mass parameter μ are significant parameters in describing the phenomenology of the Higgs sector.

The complex scalar Higgs field has originally eight *real* scalar d.o.f. After electroweak SSB, three d.o.f correspond to the Goldstone bosons G^0 and $G^\pm$ which are absorbed into the longitudinal modes of the Z^0 and $W^\pm$ massive vector bosons under a unitary gauge transformation. This leaves five d.o.f which are actually five Higgs physical states. Two of them are identified as charged: the H^+ with charge $+1$ along with its conjugate state H^- with charge -1. The remaining three are identified as neutral states: the h^0 and H^0 with $CP = +1$ and the A^0 with $CP = -1$. The eigenstates of the Goldstone and Higgs fields is a mixture of the mass eigenstates which have the same charge and CP quantum numbers:

$$\begin{pmatrix} G^- \\ H^- \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} h_d^- \\ h_u^+ \end{pmatrix}, \tag{B.0.9a}$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} Im\,(h_d^0) \\ Im\,(h_u^0) \end{pmatrix}, \tag{B.0.9b}$$

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} Re\,(h_d^0) - v_d \\ Re\,(h_u^0) - v_u \end{pmatrix}, \tag{B.0.9c}$$

$$G^+ = (G^-)^\dagger\,, \qquad H^+ = (H^-)^\dagger\,, \tag{B.0.9d}$$

where α is the mixing angle between the two Higgs physical states with $CP = +1$ quantum number. Provided that the scalar potential in Eq. (B.0.3) is minimised and with the Goldstone masses being $m^2_{G^0} = m^2_{G^\pm} = 0$, the mass eigenvalues of the eigenstates in Eq. (B.0.9) are found to be:

$$m^2_{H^\pm} = \tfrac{1}{4}\left[g^2 + 2\frac{(\mu B)^2}{v_d v_u}\right](v_d^2 + v_u^2)\,, \tag{B.0.10a}$$

$$m^2_{A^0} = \frac{(\mu B)^2}{v_d v_u}(v_d^2 + v_u^2) = (\mu B)^2(\tan\beta + \cot\beta) = m^2_{H^\pm} - m^2_{W^\pm}\,, \tag{B.0.10b}$$

$$m^2_{H^0,\,h^0} = \tfrac{1}{2}\left(m^2_{A^0} + m^2_{Z^0} \pm \sqrt{m^2_{A^0} + m^2_{Z^0} - 4m^2_{Z^0}m^2_{A^0}\cos^2 2\beta}\right). \tag{B.0.10c}$$

The mixing angle α is then determined at tree level in terms of the eigenvalues in Eq. (B.0.10) as shown:

$$\sin 2\alpha = -\sin 2\beta\left(\frac{m^2_{h^0} + m^2_{H^0}}{m^2_{h^0} - m^2_{H^0}}\right), \qquad \cos 2\alpha = -\cos 2\beta\left(\frac{m^2_{A^0} + m^2_{Z^0}}{m^2_{h^0} - m^2_{H^0}}\right). \tag{B.0.11}$$

These ingredients can now be used to determine the masses of the SM gauge bosons and fermions. The procedure is the same as described in Appendix A.2 and yields:

$$m^2_{W^\pm} = \sqrt{\frac{g^2}{4}(v_d^2 + v_u^2)}\,, \qquad m_{Z^0} = \sqrt{\frac{g^2 + G'^2}{4}(v_d^2 + v_u^2)}\,, \tag{B.0.12}$$

$$m_l = \frac{\hat{h}_l}{\sqrt{2}}v_1\,, \qquad m_d = \frac{\hat{h}_d}{\sqrt{2}}v_1\,, \qquad m_u = \frac{\hat{h}_u}{\sqrt{2}}v_2\,. \tag{B.0.13}$$

As deduced from Eq. (B.0.9), all three neutral Higgs physical states are a mixture of the two Higgs fields. This property allows the couplings of A^0, h^0 and H^0 with the up- and down-type quarks. The coupling of the pseudoscalar A^0 with the gauge bosons violates CP invariance at tree level and hence is not allowed. A comparison between the masses of all the Higgs and gauge bosons which have the same charge in the MSSM theory, as presented in Eq. (B.0.10) and (B.0.12) respectively, yields:

$$m_{H^0} \leq (m_{A^0}, m_{Z^0}) \leq m_{h^0}\,, \qquad m_{W^\pm} \leq m_{H^\pm}\,. \tag{B.0.14}$$

An interesting outcome from this relation is that the lightest neutral Higgs state is predicted to be lighter than the Z^0 boson. This is a perplexing outcome since the successful experimental discovery of Higgs boson, and subsequently the accurate measurements of its mass, rule out any possibility of a Higgs boson being lighter than Z^0. The outcome ceases to be perplexing when one realises that this is a result derived at tree level where radiative corrections due to higher order loop diagrams are neglected. If one takes the radiative corrections into account, the upper bound for the mass of the lightest Higgs boson is significantly lifted. Nevertheless, and despite the contribution from the radiative corrections, the mass of the Higgs is found to have the definite value of $m_H = 125.09 \pm 0.24$ GeV.

Appendix C

Quantising the bosonic string

The classical bosonic string theory and all of its properties and structure are studied in Section 3.2.1 of Chapter 3. Even though this type of string theory is not quite realistic, it provides a good understanding on the type of analysis that takes place in string theory. Therefore, the next step of this analysis is the study of the quantised theory. The best course of action is to consider first what options are available for quantising this classical theory. It is found that there are three options:

1. Covariant quantisation
2. Lightcone quantisation
3. BRST quantisation

Each one of them bears different features and each points out some hurdles that must be overcome if the quantisation is to be considered successful. The following discussion is focused only on the main characteristics of each quantisation procedure and to the most important problems that arise in each.

C.1 Covariant Quantisation

The first step is to promote the fields X^μ and their canonical momenta $P^\mu = \frac{1}{2\pi\alpha'}\dot{X}^\mu$, to operator valued fields which obey the canonical equal-time commutation relations,

$$
\begin{aligned}
&[\hat{X}^{\mu}(\tau,\sigma),\hat{P}^{\nu}(\tau,\sigma')] = i\,\eta^{\mu\nu}\delta(\sigma-\sigma') \\
&[\hat{X}^{\mu}(\tau,\sigma),\hat{X}^{\nu}(\tau,\sigma')] = [\hat{P}^{\mu}(\tau,\sigma),\hat{P}^{\nu}(\tau,\sigma')] = 0\,. \qquad \text{(C.1.1)}
\end{aligned}
$$

Plugging into Eq. (C.1.1) the mode expansions for the X^{μ} and P^{μ} one obtains the canonical equal-time commutation relations in terms of the Fourier modes $x^{\mu}, p^{\mu}, \alpha^{\mu}_{n}$, which are now also promoted to operators:

$$
\begin{aligned}
&[\hat{x}^{\mu},\hat{p}^{\nu}] = i\,\eta^{\mu\nu} \\
&[\hat{\tilde{\alpha}}^{\mu}_{m},\hat{\tilde{\alpha}}^{\nu}_{n}] = [\hat{\alpha}^{\mu}_{m},\hat{\alpha}^{\nu}_{m}] = m\,\eta^{\mu\nu}\delta_{m+n,0} \\
&[\hat{\tilde{\alpha}}^{\mu}_{m},\hat{\alpha}^{\nu}_{n}] = 0\,. \qquad \text{(C.1.2)}
\end{aligned}
$$

By defining new operators: $\hat{a}^{\mu}_{n} \equiv \frac{1}{\sqrt{n}}\hat{\alpha}^{\mu}_{n}$ and $\hat{a}^{\mu\dagger}_{n} \equiv \frac{1}{\sqrt{n}}\hat{\alpha}^{\mu\dagger}_{-n}$ $\forall\, n > 0$; the expressions in Eq. (C.1.2) clearly satisfy

$$
[\hat{\tilde{a}}^{\mu}_{m},\hat{\tilde{a}}^{\nu\dagger}_{n}] = [\hat{a}^{\mu}_{m},\hat{a}^{\nu\dagger}_{n}] = \eta^{\mu\nu}\delta_{m,n} \qquad \forall\, m,n > 0\,. \qquad \text{(C.1.3)}
$$

From the results presented so far, it is inferred that from each scalar field's left- and right-moving modes emerge two infinite towers of creation and annihilation operators. The vacuum state of the string ($|0\rangle$) is then defined as the state which is annihilated by all of the annihilation operators $\hat{\tilde{a}}^{\mu}_{n}$ or $\hat{a}^{\mu}_{n}$,

$$
\hat{\tilde{a}}^{\mu}_{n}\,|0\rangle = \hat{a}^{\mu}_{n}\,|0\rangle = 0 \qquad \forall\, n > 0\,. \qquad \text{(C.1.4)}
$$

The physical states of the string are states that are constructed by acting on the ground state with the creation operators $\hat{\tilde{a}}^{\mu\dagger}_{n}$ or $\hat{a}^{\mu\dagger}_{n}$,

$$
|\phi\rangle = \hat{a}^{\mu_1\dagger}_{n_1}\,\hat{a}^{\mu_2\dagger}_{n_2}\ldots\hat{\tilde{a}}^{\nu_1\dagger}_{m_1}\,\hat{\tilde{a}}^{\nu_2\dagger}_{m_2}\ldots|0;k\rangle\,. \qquad \text{(C.1.5)}
$$

The physical states are momentum eigenstates, *i.e.* $p^{\mu}\,|0;k\rangle = k^{\mu}\,|0;k\rangle$, where k^{μ} is the momentum eigenvalue. Since there are infinite towers of operators, there is an infinite number of string excitations. All the excitations are different states of the string and their physical interpretation is that they are in fact different particles in the spectrum of the string theory. The set of all physical states in the theory is known as the *Fock space*.

The problem with this quantisation procedure arises in the Fock space of the theory. Due to the signature of the Minkowski metric, the commutation relations in Eq. (C.1.3) yield a negative sign for the $\mu = \nu = 0$ modes,

$$[\hat{\tilde{a}}^0_m, \hat{\tilde{a}}^{0\dagger}_n] = [\hat{a}^0_m, \hat{a}^{0\dagger}_n] = \eta^{00}\delta_{m,n} = -\delta_{m,n} \,. \tag{C.1.6}$$

This outcome has dire implications because it leads to the prediction of negative norm physical states,

$$\langle k'; 0|\, \hat{a}^0_n\, \hat{a}^{0\dagger}_n \,|0; k\rangle \propto -\delta^D(k - k')\,, \tag{C.1.7}$$

known also as ghost states. These states are unacceptable in the theory as they are unphysical. However, it is possible to remove these unphysical states at the expense of imposing a constraint on the number of background spacetime dimensions.

A direct implication of the Fourier modes promotion to operators is that the generators $\tilde{L}_n$, L_n are also upgraded to the status of operators. The expressions for the classical theory, given in Eq. (3.2.17) are now modified to

$$\hat{\tilde{L}}_n = \frac{1}{2}\sum_n : \hat{\tilde{\alpha}}_{m-n} \cdot \hat{\tilde{\alpha}}_n : \qquad \text{and} \qquad \hat{L}_n = \frac{1}{2}\sum_n : \hat{\alpha}_{m-n} \cdot \hat{\alpha}_n : \,, \tag{C.1.8}$$

where the : : indicates normal ordering. Similarly to the classical strings, the $\hat{\tilde{L}}_0$ and $\hat{L}_0$ operators play a pivotal role in determining the physical spectrum of the quantised string theory. The operators carry an ambiguity as a result of the normal ordering and hence are determined by

$$\hat{\tilde{L}}_0 = \frac{1}{2}\hat{\tilde{\alpha}}^2_0 + \sum_{n=1}^{\infty} \hat{\tilde{\alpha}}_{-n} \cdot \hat{\tilde{\alpha}}_n \,; \qquad \hat{L}_0 = \frac{1}{2}\hat{\alpha}^2_0 + \sum_{n=1}^{\infty} \hat{\alpha}_{-n} \cdot \hat{\alpha}_n \,. \tag{C.1.9}$$

The mass-shell conditions for the quantised closed strings become

$$(\hat{\bar{L}}_0 - z)\,|\phi\rangle = 0 \tag{C.1.10a}$$

$$(\hat{L}_0 - z)\,|\phi\rangle = 0\,, \tag{C.1.10b}$$

where z is a constant which arises due to the normal ordering ambiguity of $\hat{L}_0, \hat{\bar{L}}_0$. Subsequently, the mass of the physical states of a quantised closed string is given by

$$\begin{aligned} m_S^2 &= \frac{4}{\alpha'}\left(\sum_{n=1}^{\infty} \hat{\bar{\alpha}}_{-n}\cdot\hat{\bar{\alpha}}_n - z\right) = \frac{4}{\alpha'}(\hat{\bar{N}} - z) \\ &= \frac{4}{\alpha'}\left(\sum_{n=1}^{\infty} \hat{\alpha}_{-n}\cdot\hat{\alpha}_n - z\right) = \frac{4}{\alpha'}(\hat{N} - z)\,, \end{aligned} \tag{C.1.11}$$

with the $\hat{\bar{N}}$ and $\hat{N}$ defined as the number operators for the left- and right-moving modes respectively. Back to Eq. (C.1.10), subtracting the condition for the left-moving physical state from the right-moving one implies that the numbers of left- and right-moving oscillator modes are equal, *i.e.*

$$(\hat{L}_0 - \hat{\bar{L}}_0)\,|\phi\rangle = 0 \Rightarrow \hat{\bar{N}} = \hat{N}\,; \tag{C.1.12}$$

thus *setting the level-matching condition for a quantised bosonic string theory with closed strings*. In turn, the generators $\hat{L}_0$, $\hat{\bar{L}}_0$, form an algebra which is a central extension of the Witt algebra. It is known as the *Virasoro* algebra and the generators are called *Virasoro operators*, satisfying the following general commutation relations

$$\begin{aligned} [\hat{L}_m, \hat{L}_n] &= (m-n)\,\hat{L}_{m+n} + \frac{c}{12}\,m\,(m^2-1)\,\delta_{m,-n} \\ [\hat{\bar{L}}_m, \hat{\bar{L}}_n] &= (m-n)\,\hat{\bar{L}}_{m+n} + \frac{c}{12}\,m\,(m^2-1)\,\delta_{m,-n}\,, \end{aligned} \tag{C.1.13}$$

where c is the central charge and is equivalent to the dimension of the background spacetime for the bosonic string theory. Even though in the covariant quantisation string theory is plagued by unphysical states, these states are eliminated if $z = 1$

and $c = 26$. These constraints imply that the background spacetime dimension of a covariantly quantised bosonic string theory free from unphysical states and manifestly Lorentz invariant is in fact *twenty-six*.

C.2 Lightcone quantisation

An advantage of this choice over the previous one is that there are no negative norm states predicted. This advantage is eclipsed by a serious disadvantage; the theory is no longer manifestly Lorentz invariance as shall be shortly demonstrated.

As was presented in Section 3.2.1, even though the spacetime dynamical metric $h^{\alpha\beta}$ becomes flat, the bosonic string theory still carries residual gauge symmetries which are the conformal transformations. Under conformal transformations, the lightcone coordinates on the worldsheet are reparametrized as

$$\sigma^{\pm} \mapsto \sigma'^{\pm} = \xi^{\pm}(\sigma^{\pm}) , \tag{C.2.1}$$

leaving the bosonic string action invariant. The conformal symmetry allows the freedom of choosing a particular noncovariant gauge, known as the *lightcone gauge* which removes the negative norm states from the theory. Following the definition of the lightcone coordinates on the worldsheet, the corresponding coordinates on the background spacetime are defined to be the set $\{X^-, X^+, X^i\}$ for $i = 1, \ldots, D-2$ and with

$$X^{\pm} \equiv \frac{1}{\sqrt{2}}(X^0 \pm X^{D-1}) . \tag{C.2.2}$$

Here the X^0 is the choice for the time direction and the X^{D-1} is the choice for the transverse spatial direction. Due to the different treatment of the lightcone spacetime coordinates, the manifest Lorentz invariance of the symmetry does not hold any longer. Instead, the new Lorentz symmetry becomes $SO(D-2)$ and the spacetime Minkowski metric now is defined as

$$ds^2 = -2\, dX^+ dX^- + \sum_i^{D-2} dX^i dX^i . \tag{C.2.3}$$

The solution to the equation of motion of the X^+ now becomes

$$X^+ = X_L^+(\sigma^+) + X_R^+(\sigma^-)\,; \tag{C.2.4}$$

with

$$X_L^+(\sigma^+) = \tfrac{1}{2}x^+ + \tfrac{1}{2}\alpha' p^+ \sigma^+ \qquad \text{and} \qquad X_R^+(\sigma^-) = \tfrac{1}{2}x^+ + \tfrac{1}{2}\alpha' p^+ \sigma^-\,, \tag{C.2.5}$$

which results in

$$X^+ = x^+ + \alpha' p^+ \tau\,. \tag{C.2.6}$$

This is the *lightcone gauge* in which all the oscillator modes for the X^+ of the closed string are set to zero. Subsequently, the form of the X^- coordinates is expected to be similar to that of Eq. (C.2.5) and is derived using the lightcone gauge in conjuction with the constraints in Eq. (3.2.15). These coordinates differ from the X^+ as they carry mode expansions:

$$\begin{aligned} X^-(\sigma^+)_L &= \tfrac{1}{2}x^- + \tfrac{1}{2}\alpha' p^- \sigma^+ + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{n}\, \tilde{\alpha}_n^-\, e^{-in\sigma^+} \\ X^-(\sigma^-)_R &= \tfrac{1}{2}x^- + \tfrac{1}{2}\alpha' p^- \sigma^- + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{n}\, \alpha_n^-\, e^{-in\sigma^-}\,, \end{aligned} \tag{C.2.7a}$$

where the x^- (along with the x^i) describes the string's centre of mass. The mode expansions are given by

$$\begin{aligned} \tilde{\alpha}_n^- &= \frac{1}{\sqrt{2\alpha'}}\frac{1}{p^+} \sum_m \sum_{i=1}^{D-2} \tilde{\alpha}_{n-m}^i \cdot \tilde{\alpha}_m^i \\ \alpha_n^- &= \frac{1}{\sqrt{2\alpha'}}\frac{1}{p^+} \sum_m \sum_{i=1}^{D-2} \alpha_{n-m}^i \cdot \alpha_m^i\,, \end{aligned} \tag{C.2.8a}$$

where the p^+ (along with the p^i) describes the string's momentum. In the lightcone gauge, the bosonic string theory is expressed in terms of $2(D-2)$ transverse oscillators only and so a string with $c = 26$ has only transverse oscillations. Note that the transverse oscillators are not regarded as the transverse excitations of the string. As in the case of the classical string, it is found that the zero modes satisfy

$\tilde{\alpha}_0^- = \alpha_0^- = \sqrt{\frac{\alpha'}{2}}p^-$. Hence, applying the constraints of Eq. (3.2.15) it is found that:

$$\frac{\alpha'}{2}p^- = \frac{1}{2p^+}\sum_{i=1}^{D-2}\left(\tfrac{1}{2}\alpha' p^i \cdot p^i + \sum_{n\neq 0}\tilde{\alpha}_n^i \cdot \tilde{\alpha}_{-n}^i\right) = \frac{1}{2p^+}\sum_{i=1}^{D-2}\left(\tfrac{1}{2}\alpha' p^i \cdot p^i + \sum_{n\neq 0}\alpha_n^i \cdot \alpha_{-n}^i\right). \quad \text{(C.2.9)}$$

Moving on to the quantisation procedure, it is not surprising to find that some equal-time commutation relations are the same as the ones defined for the covariant quantisation in Eq. (C.1.2). In fact, this occurrence is natural as some of the physical d.o.f in the lightcone quantisation are the same as those in the covariant quantisation. A distinct difference of lightcone quantisation from covariant quantisation is that the former has additional fields, the x^+ and p^-, which must be promoted to operators. As a result the equal-time commutation relations for the quantisation of a bosonic closed string in the lightcone gauge are given by

$$\begin{aligned}
&[\hat{x}^i, \hat{p}^j] = i\,\delta^{ij} \\
&[\hat{\tilde{\alpha}}_m^i, \hat{\tilde{\alpha}}_n^j] = [\hat{\alpha}_m^i, \hat{\alpha}_n^j] = n\,\delta^{ij}\,\delta_{m+n,0} \\
&[\hat{\tilde{\alpha}}_m^i, \hat{\alpha}_n^j] = 0 \\
&[\hat{x}^+, \hat{p}^-] = i\,, \qquad [\hat{x}^-, \hat{p}^+] = -i\,.
\end{aligned} \quad \text{(C.2.10)}$$

Identically to the covariant quantisation method, the string vacuum and physical states are defined as

$$\hat{\tilde{a}}_n^i\,|0\rangle = \hat{a}_n^i\,|0\rangle = 0 \qquad \text{and} \qquad p^\mu\,|0;k\rangle = k^\mu\,|0;k\rangle \qquad \forall\, n > 0\,, \quad \text{(C.2.11)}$$

whereas the Fock space is built up by acting on the physical states with the creation operators $\hat{\tilde{\alpha}}_{-n}^i$ or $\hat{\alpha}_{-n}^i$. It should be noted that since the spatial index takes values $i = 1, \ldots, D-2$, the Hilbert space of the theory is positive definite. This clearly demonstrates that said theory has no unphysical states. Thence the expressions in Eq. (C.2.9) are modified in the sense that they obey the normal ordering condition. From the resulting normal ordered expressions one is able to derive the *mass-shell* *level-matching condition for closed strings in lightcone gauge*:

$$m_S^2 = 2p^+ \cdot p^- - \sum_{i=1}^{D-2} p^i p^i = \frac{4}{\alpha'}\left(\sum_{i=1}^{D-2}\sum_{n>0} \hat{\hat{\alpha}}^i_{-n} \cdot \hat{\hat{\alpha}}^i_n - z\right) = \frac{4}{\alpha'}(\hat{\hat{N}} - z)$$
$$= \frac{4}{\alpha'}\left(\sum_{i=1}^{D-2}\sum_{n>0} \hat{\alpha}^i_{-n} \cdot \hat{\alpha}^i_n - z\right) = \frac{4}{\alpha'}(\hat{N} - z)\,. \tag{C.2.12}$$

The $\hat{\hat{N}}$ and $\hat{N}$ are the number operators of the left- and right-moving transverse oscillator modes respectively. Naturally, the next step required for the completion of quantisation is to fix the value of the constant z. In Ref. [107], there is a very detailed derivation of the value, which is achieved when one employs tools from conformal field theory. It turns out then that the value of the constant is directly proportional to the number of background spacetime dimensions (D), *i.e.*

$$z = \frac{D-2}{24}\,. \tag{C.2.13}$$

To overcome the hurdle of not having a manifest Lorentz invariance, a constraint needs to be imposed on the theory. This means that z and hence D can each take a specific value. Through a series of calculations one derives $z \equiv 1$, thus $D = 26$ which is equivalent to the value obtained for the central charge of this theory. Substituting Eq. (C.2.13) into the formula in Eq. (C.2.12) it is straightforward to determine the mass spectrum of said theory:

- In the absence of oscillator modes, the ground state $|0\rangle = |0; k^\mu\rangle$ has mass-squared equal to $-4(\alpha')^{-1}$. It does not actually make much sense for a state to have negative mass-squared, thus the state is deemed unphysical. Such states give rise to particles which are commonly known as *tachyons*. Tachyons in the bosonic string theory contribute extra terms in the potential resulting to the destabilisation of vacuum. Therefore, tachyons are unwanted states; they make the theory look extremely problematic as it is not in toto stable.

- For the first excited states there is one left- and one right-moving oscillator mode such that, $|\Omega^{ij}\rangle = \alpha^i_{-1}\tilde{\alpha}^j_{-1}|0;k^\mu\rangle$[1]. This is a tensor product between two *massless* vectors, one left- and one right-moving, and the total number of states is $24^2 = 576$. The first excited states give rise to *massless bosonic* particles which fill out the $\mathbf{24} \otimes \mathbf{24}$ representation of the little group $SO(24)$. This representation is further decomposed into three irreducible representations and the corresponding particles are associated with a massless field in spacetime. The irrepresentation which is symmetric in (i, j) transforms under $SO(24)$ as a massless spin-2 particle; this is the graviton $(g_{\mu\nu})$. The irrepresentation which is antisymmetric in (i, j) transforms under the little group as a massless tensor; this is also called a "2-form" $(B_{\mu\nu})$. Finally, the trace irrepresentation transforms as a massless scalar, which is the dilaton (ϕ).

- Higher excited states are infinite in number and all of them are massive. Such states give rise to *massive bosonic* particles which fill the representations of the $SO(25)$ Lorentz group.

So far, it has been made clear that both methods utilised to quantise the string have their advantages and disadvantages. Covariant quantisation ensures that the theory is manifestly Lorentz invariant. However, it is difficult to identify the physical states and it allows negative norm states in the mass spectrum of the theory. On the other hand, lightcone quantisation ensures that the mass spectrum of the theory has no unphysical states and is relatively easy to identify the physical states. The drawback of this method is that the theory is no longer manifestly Lorentz invariant. The third method of quantisation meets both of the previous methods halfway. BRST quantisation is manifestly Lorentz invariant and it is relatively easy to identify the physical states, but there are also unphysical ones that must be eliminated.

[1] The hat is dropped on this notation for a matter of convenience.

C.3 BRST quantisation

Revisiting the Polyakov action in Eq. (3.2.5), it has already been mentioned that besides the global and local gauge symmetries it is also invariant under another symmetry which is characterised by conformal transformations, *i.e.*

$$\delta h_{\alpha\beta} = D_\alpha \xi_\beta + D_\beta \xi_\alpha + 2\omega\, h_{\alpha\beta} \tag{C.3.1a}$$

$$\delta X^\mu = \xi^\alpha \partial_\alpha X^\mu\,, \tag{C.3.1b}$$

with ξ_α and ω being bosonic parameters. In BRST quantisation there are a number of steps that one must follow, as listed below:

1. Firstly, it is necessary to introduce fermionic ghost fields $c_{\alpha,\beta}$ and c_ω that correspond to each of the bosonic parameters respectively.

2. Secondly, one introduces gauge fixing terms $F^A(\Phi^I)$, two for the bosonic parameters $\xi_{\alpha,\beta}$ and one for the bosonic parameter ω. The gauge fixing terms are $F_{\alpha\beta} = h_{\alpha\beta} - \delta_{\alpha\beta}$, where $\delta_{\alpha\beta}$ is the two-dimensional Euclidean metric.

3. Thirdly, one includes an anti-ghost field $b^{\alpha\beta}$ and an auxiliary field $B^{\alpha\beta}$ for each of the gauge fixing terms $F_{\alpha\beta}$.

4. The fourth step requires a modification of the Polyakov action, adding two terms: the gauge-fixing action S_2 and the ghost action S_3 which are given by

$$S_2 \;=\; \int d\tau\, d\sigma\ \sqrt{h}\,[-iB^{\alpha\beta}(h_{\alpha\beta} - \delta_{\alpha\beta})] \tag{C.3.2a}$$

$$S_3 \;=\; \int d\tau\, d\sigma\, b^{\alpha\beta}\,(D_\alpha c_\beta + D_\beta c_\alpha + 2c_\omega h_{\alpha\beta})\,. \tag{C.3.2b}$$

5. Finally, the last step is the quantisation itself. This is realised through the definition of the partition function

$$Z = \int \mathcal{D}X^\mu\, \mathcal{D}h_{\alpha\beta}\, \mathcal{D}B^{\alpha\beta}\, \mathcal{D}b^{\alpha\beta}\, \mathcal{D}c^\alpha\, \mathcal{D}c_\omega\ e^{-(S_P+S_2+S_3)}\,, \tag{C.3.3}$$

which describes a bosonic string propagating in a twenty-six-dimensional background spacetime.

The overall action $S_P + S_2 + S_3$ in the partition function has a global symmetry, known as BRST. This symmetry is defined by the following set of transformations:

$$\begin{aligned} \delta_B h_{\alpha\beta} &= -i\kappa\,(D_\alpha\,\xi_\beta + D_\beta\,\xi_\alpha + 2\omega\,h_{\alpha\beta}), & \delta_B X^\mu &= -i\kappa\,(c^\alpha\partial_\alpha X^\mu) \\ \delta_B c^\alpha &= -i\kappa\,(c^\beta\partial_\beta c^\alpha), & \delta_B c_\omega &= -i\kappa\,(c^\alpha\partial_\alpha c_\omega) \\ \delta_B b^{\alpha\beta} &= \kappa\,B^{\alpha\beta}, & \delta_B B^{\alpha\beta} &= 0\,; \end{aligned} \tag{C.3.4}$$

where κ is an arbitrary global parameter with the same statistics as the ghost fields c_α. The partition function is linear in both the ghost field c_ω and the auxiliary field $B^{\alpha\beta}$, hence both fields can be integrated out. Integrating out the ghost fields introduces a Dirac δ-function which removes the trace of the anti-ghost field $b^{\alpha\beta}$. Integrating out the auxiliary fields introduces another Dirac δ-function which causes the dynamical metric $h_{\alpha\beta}$ to be flat, *i.e.* $h_{\alpha\beta} = \delta_{\alpha\beta}$. Taking these into account and realising that the worldsheet now becomes Euclidean, *i.e.* $D_\alpha \mapsto \partial_\alpha$, the partition function simplifies to

$$Z = \int \mathcal{D}X^\mu\,\mathcal{D}b^{\alpha\beta}\,\mathcal{D}c^\alpha\;\; e^{-S'}\,; \tag{C.3.5}$$

$$S' = \int d\tau d\sigma \left[\frac{1}{4\pi\alpha'}\,\partial_\alpha X^\mu \partial^\alpha X_\mu + b^{\alpha\beta}(\partial_\alpha c_\beta + \partial_\beta c_\alpha)\right]. \tag{C.3.6}$$

Due to the worldsheet being Euclidean there is the benefit of $X^\mu(\tau,\sigma) \mapsto X^\mu(z,\bar{z})$ and hence the partition function can be mapped to the complex plane

$$Z = \int \mathcal{D}X\,\mathcal{D}b\,\mathcal{D}c\;\; e^{-S''}\,; \tag{C.3.7}$$

$$S'' = \frac{1}{2\pi\alpha'}\int dz\,d\bar{z}\;(\partial_z X^\mu \partial_{\bar{z}} X_\mu) + \frac{1}{2\pi}\int dz\,d\bar{z}\,(b_{zz}\,\partial_{\bar{z}}c^z + b_{\bar{z}\bar{z}}\,\partial_z c^{\bar{z}})\,. \tag{C.3.8}$$

The mapping to the complex plane gauge fixes the BRST transformations of Eq. (C.3.4) so that the action in Eq. (C.3.8) remains invariant. The gauge fixed transformations are given by

$$\delta_B c^z = -i\kappa\,(c^z\,\partial_z c^z), \qquad \delta_B c^{\bar{z}} = -i\kappa\,(c^{\bar{z}}\,\partial_{\bar{z}} c^{\bar{z}})$$
$$\delta_B b^{zz} = -i\kappa\,T_{S-E}, \qquad \delta_B b_{\overline{zz}} = -i\kappa\,\bar{T}_{S-E}$$
$$\delta_B X^\mu = -i\kappa\,(c^z\,\partial_z X^\mu + c^{\bar{z}}\,\partial_{\bar{z}} X^\mu)\,, \tag{C.3.9}$$

where T_{S-E} is the stress-energy tensor of the theory. It is given by the sum of both the matter and ghost contributions from the action in Eq. (C.3.8):

$$T_{S-E} = T^M + T^{gh}\,;$$
$$T^M = -\frac{1}{\alpha'}\,(\partial_z X)^2 \quad \text{and} \quad T^{gh} = -2 : b_{zz}\,\partial_z c^z : + : c^z \partial_z b_{zz} : \quad . \tag{C.3.10}$$

For the derivation of the mass spectrum of the BRST theory, there is a different procedure to be followed than the other two quantisation methods. This stems from the fact that the BRST theory is basically split into two parts, one part that includes the matter fields and another part that includes the ghost fields. The vacuum of the theory, $|0\rangle_T$ is the tensor product of the matter vacuum and the ghost vacuum. However, the ghost vacuum is split into two states, $|\uparrow\rangle$ and $|\downarrow\rangle$ that satisfy

$$b_0\,|\uparrow\rangle = |\downarrow\rangle\,, \qquad c_0\,|\uparrow\rangle = 0$$
$$b_0\,|\downarrow\rangle = 0\,, \qquad c_0\,|\downarrow\rangle = |\uparrow\rangle$$
$$b_n, c_n\,|\uparrow\rangle = 0\,, \qquad b_n, c_n\,|\downarrow\rangle = 0 \qquad \forall\, n \geq 1\,. \tag{C.3.11a}$$

It is evident that due to the zero modes, the vacuum state of ghost fields is doubly degenerate. However, only one of the two states is necessary for the formation of the BRST vacuum state which is defined as $|0\rangle_T = |0\rangle \otimes |\downarrow\rangle$ [126]. The physical states are obtained by acting on the vacuum with the BRST charge operator Q_B:

$$Q_B = \sum_m (L^X_{-m} - \delta_{m,0}) c_m - \sum_{m,n} (m-n) : c_{-m} \cdot c_{-n} \cdot b_{m+n} :\,, \tag{C.3.12}$$

$$Q_B\,|0\rangle_T = [(L_0^X - 1)\,|0\rangle]\,[c_0|\downarrow\rangle] + \sum_{m>0}[L_m^X\,|0\rangle]\,[c_{-m}|\downarrow\rangle]\,, \tag{C.3.13}$$

with L_m^X being the Virasoro operators obtained from the matter fields. If the Q_B annihilates the vacuum then $(L_0^X - 1)\,|0\rangle = L_m^X\,|0\rangle = 0$. This condition results in the existence of tachyons in the theory. The full mass spectrum of the BRST theory can be found in Ref. [107] but it should be noted that it is in agreement with the mass spectrum obtained from the previous two methods of quantisation.

Appendix D

Theta-function notation and partition function conventions

The basic η function is defined as

$$\eta(\tau) \equiv q^{1/24} \prod_{n=1}^{\infty} (1-q^n) = \sum_{n=-\infty}^{\infty} (-1)^n \, q^{3(n-1/6)^2/2} \,, \tag{D.0.1}$$

and the ϑ functions are defined as

$$\begin{aligned}
\vartheta_1(\tau) &\equiv -i \sum_{n=-\infty}^{\infty} (-1)^n q^{(n+1/2)^2/2} \\
\vartheta_2(\tau) &\equiv 2q^{1/8} \prod_{n=1}^{\infty} (1+q^n)^2(1-q^n) = \sum_{n=-\infty}^{\infty} q^{(n+1/2)^2/2} \\
\vartheta_3(\tau) &\equiv \prod_{n=1}^{\infty} (1+q^{n-1/2})^2(1-q^n) = \sum_{n=-\infty}^{\infty} q^{n^2/2} \\
\vartheta_4(\tau) &\equiv \prod_{n=1}^{\infty} (1-q^{n-1/2})^2(1-q^n) = \sum_{n=-\infty}^{\infty} (-1)^n q^{n^2/2} \,.
\end{aligned} \tag{D.0.2}$$

Here $q \equiv \exp(2\pi i\tau)$, with $\tau_{1,2}$ denoting $\mathrm{Re}\,\tau$ and $\mathrm{Im}\,\tau$ respectively. These functions satisfy the identities

$$\vartheta_3{}^4 = \vartheta_2{}^4 + \vartheta_4{}^4 \tag{D.0.3a}$$

$$\vartheta_2\vartheta_3\vartheta_4 = 2\eta^3 \,. \tag{D.0.3b}$$

Note that $\vartheta_1(q)$ has a vanishing q-expansion and is modular invariant; the coefficient of its infinite-product representation vanishes and is thus omitted. Nonetheless, this function needs to be taken into consideration because within string partition functions it can often play a role when determining the chirality of fermionic states, as discussed below.

In order to simplify and unify the notation, and also in order to be able to handle more complicated systems, it is more efficient to introduce several generalisations of these functions. First, the more general theta-function of two arguments is defined as follows:

$$\begin{aligned}\vartheta(z,\tau) &\equiv \sum_{n=-\infty}^{\infty} \xi^n\, q^{n^2/2} \\ &= q^{-1/24}\,\eta(\tau)\prod_{m=1}^{\infty}(1+\xi q^{m-1/2})\,(1+\xi^{-1}q^{m-1/2})\,,\end{aligned} \tag{D.0.4}$$

where $\xi \equiv e^{2\pi i z}$. Similarly, the ϑ-functions with characteristics are defined as

$$\begin{aligned}\vartheta\left[\begin{array}{c} a \\ b \end{array}\right](z,\tau) &\equiv \sum_{n=-\infty}^{\infty} e^{2\pi i\,(n+a)(z+b)}\, q^{(n+a)^2/2} \\ &= e^{2\pi i\,ab}\,\xi^a\, q^{a^2/2}\,\vartheta(z+a\tau+b,\tau)\,;\end{aligned} \tag{D.0.5}$$

of course these latter functions have a certain redundancy, depending only on $z+b$ rather than z and b separately. For $a,b \in \{0,1/2\}$, a common "shorthand" for these functions is given by

$$\begin{aligned}\vartheta_{00} \equiv \vartheta\left[\begin{array}{c} 0 \\ 0 \end{array}\right] = \vartheta_3 \qquad\qquad \vartheta_{10} \equiv \vartheta\left[\begin{array}{c} 1/2 \\ 0 \end{array}\right] = \vartheta_2 \\ \vartheta_{01} \equiv \vartheta\left[\begin{array}{c} 0 \\ 1/2 \end{array}\right] = \vartheta_4 \qquad\qquad \vartheta_{11} \equiv \vartheta\left[\begin{array}{c} 1/2 \\ 1/2 \end{array}\right] = -\vartheta_1\,.\end{aligned} \tag{D.0.6}$$

Note that there are no restrictions on the (z,τ) arguments. Hence the expressions in Eq. (D.0.6) implicitly define two-argument Jacobi functions $\vartheta_i(z,\tau)$ for $i,\ldots,4$.

In general, the functions in Eq.(D.0.5) have modular transformations

$$\begin{aligned}
\vartheta\begin{bmatrix} a \\ b \end{bmatrix}(z,-\tau^{-1}) &= \sqrt{-i\,\tau}\, e^{2\pi i\, ab} e^{i\pi\tau z^2}\vartheta\begin{bmatrix} -b \\ a \end{bmatrix}(-z\tau,\tau)\,, \\
\vartheta\begin{bmatrix} a \\ b \end{bmatrix}(z,\tau+1) &= e^{-i\pi(a^2+a)}\vartheta\begin{bmatrix} a \\ a+b+1/2 \end{bmatrix}(z,\tau)\,.
\end{aligned} \tag{D.0.7}$$

Moreover, in the $\tau_2 \gg 1$ (or $|q| \ll 1$) limit, these functions have the leading behaviours

$$\begin{aligned}
\eta(\tau) &\sim q^{1/24} + \ldots \\
\vartheta_{00}(0|\tau) &\sim 1 + 2q^{1/2} + \ldots \\
\vartheta_{01}(0|\tau) &\sim 1 - 2q^{1/2} + \ldots \\
\vartheta_{10}(0|\tau) &\sim 2q^{1/8} + \ldots \\
\vartheta_{11}(0|\tau) &= 0\,.
\end{aligned} \tag{D.0.8}$$

Worldsheet bosons and fermions give rise to partition-function contributions which can be expressed in terms of these functions. For those worldsheet bosons which are spacetime coordinates (which is always the case for the string constructions presented in this work), the partition-function contributions also depend on the spacetime compactification metric. In general, a single complex extra dimension has a metric which is conventionally parametrized as

$$G_{ij} = \frac{T_2}{U_2}\begin{pmatrix} 1 & U_1 \\ U_1 & |U|^2 \end{pmatrix}; \qquad B_{ij} = \begin{pmatrix} 0 & -T_1 \\ T_1 & 0 \end{pmatrix}, \tag{D.0.9}$$

where $T \equiv T_1 + iT_2$ and $U \equiv U_1 + iU_2$. In this study, the compactification metrics used are all *diagonal*, *i.e.* metrics with $T_1 = U_1 = 0$. For $U_1 = 0$, the corresponding Poisson resummed partition function for the compactified complex boson is given by

$$Z_{\rm B}\begin{bmatrix} 0 \\ 0 \end{bmatrix}(\tau) = \mathcal{M}^2 \frac{T_2}{\tau_2|\eta(\tau)|^4}\sum_{\jmath,\ell}\exp\left\{-\frac{\pi}{\tau_2}\frac{T_2}{U_2}|\ell_1 + \jmath_1\tau|^2 - \frac{\pi}{\tau_2}T_2U_2|\ell_2 + \jmath_2\tau|^2\right\}. \tag{D.0.10}$$

From this, it is straightforward to identify $R_1 = \sqrt{T_2 U_2^{-1}}$ and $R_2 = \sqrt{T_2 U_2}$. Conversely, $T_2 = R_1 R_2$ is a volume modulus while $U_2 = R_2 R_1^{-1}$ is a complex-structure modulus. By contrast, the contribution to the total partition function from a single complex fermion with worldsheet boundary conditions $v \equiv \overline{\alpha V_i}$ and $u \equiv \beta V_i$ is given by

$$
\begin{aligned}
Z_u^v &= \mathrm{Tr}\left[q^{\hat{H}_v} e^{-2\pi i u \hat{N}_v}\right] \\
&= q^{\frac{1}{2}(v^2 - \frac{1}{12})} \prod_{n=1}^{\infty} (1 + e^{2\pi i (v\tau - u)} q^{n-\frac{1}{2}})(1 + e^{-2\pi i (v\tau - u)} q^{n-\frac{1}{2}}) \\
&= e^{2\pi i uv} \frac{1}{\eta(\tau)} \vartheta\left[\begin{array}{c} v \\ -u \end{array}\right](0,\tau)\,.
\end{aligned}
\tag{D.0.11}
$$

Appendix E

Breaking of supersymmetry by discrete torsion

In Chapter 6, it is stated that the choice of structure constants k_{ij} that define a theory is of great importance. Indeed, in the free fermionic formulation, a "wrong" choice of structure constants can result in the breaking of spacetime SUSY. This occurs when some combination of boundary condition phases *not overlapping the gravitinos* depend on these structure constants, and the end result is that the gravitinos are projected from the physical spectrum of the theory. The breaking of SUSY via this option is called *discrete torsion*.

To see this, the first thing to note is that in $4D$ any $\mathcal{N} = 1$ model in the fermionic formulation can be written without loss of generality in terms of the following vectors:

$$\begin{aligned} V_0 &= -\frac{1}{2}\left[1\,(111)^3\,|\,(1)^{22}\right] \\ V_1 &= -\frac{1}{2}\left[1\,(100)^3\,|\,(0)^{22}\right] \\ V_{i\geq 2} &= \ldots \end{aligned} \tag{E.0.1}$$

This basis is always possible because the V_0 vector must always be present for modular invariance, and because there must be gravitinos in the supersymmetric model. The sector in which these appear can be taken to define the V_1 sector. In addition, there is the assumption that the right-movers have only 0 and $-\frac{1}{2}$

boundary conditions. Therefore, since the lowest possible vacuum energy on the right-moving side is $-\frac{1}{2}$, a tachyon can appear *only* if there are no right-moving **NS** excitations.

The $V_{0,1}$ projections on the gravitinos are determined by

$$\begin{aligned} V_0 \cdot N + \frac{1}{4}(1-\Gamma) &= k_{01} + \frac{1}{2} - V_0 \cdot V_1 \mod (1) \\ V_1 \cdot N + \frac{1}{4}(1-\Gamma) &= k_{11} + \frac{1}{2} - V_1 \cdot V_1 \mod (1)\,, \end{aligned} \tag{E.0.2}$$

where $\Gamma = \Gamma_{V_1} = \Gamma_{V_0}$ are the chirality projections when the vectors overlap with the **R** states; for the gravitinos they are necessarily degenerate. By inspection, the massless gravitinos have no excitations, thus $V_0 \cdot N = V_1 \cdot N = 0$. These equations are compatible if and only if $k_{01} + k_{11} = 0 \mod (1)$, which must be true since $k_{10} + k_{01} = k_{11} + k_{10} = 0$ by the relations in Eq. (4.1.7). However, an incompatibility for these states can occur if there is an additional vector (or combination of vectors) that does not overlap with V_1. This is because if there was an overlap, then any projection would simply fix the definition of chirality in a subset of the spinors.

To be more explicit, this additional vector V_X is defined as

$$V_X \cdot N = k_{X1} \mod (1)\,. \tag{E.0.3}$$

This leads to a general conclusion: If $k_{X1} = \frac{1}{2}$ then the gravitinos are projected out and spacetime SUSY is broken. By contrast, if there is a combination of vectors that did not overlap with V_1, then it would be the corresponding linear combination of k_{i1} that would have to sum to $\frac{1}{2}$ in order to achieve spacetime SUSY breaking. Moreover, a V_X completely overlapping with V_1 could also be incompatible, but this is again equivalent to a new vector $V_X \to \overline{V_X + V_0}$ that has no overlap with V_1. Given this, one can then prove the following: *tachyons can be present in the resulting non-supersymmetric model only if there are sectors including V_X that have negative vacuum energy*.

Proof: To see this, it is assumed that there exists a would-be tachyonic sector $\overline{\alpha V}$ in the theory. Normally, this would be just the **NS**-**NS** sector; however, it could also involve a linear combination of other V_i sectors. In the supersymmetric theory, these states are absent as they are projected out by the V_1 sector which is the supersymmetric generator in the theory. Hence tachyons are absent if and only if

$$\sum_{i \in V_1 = \mathbf{R}} \frac{1}{4}(1 - \Gamma_i) \neq \alpha_i k_{1i} + \frac{1}{2} - V_1 \cdot \overline{\alpha V} \qquad \forall\, \Gamma \bmod (1)\,. \tag{E.0.4}$$

Note that $k_{X1} \equiv k_{1X}$ appears in this equation only if $\alpha_X \neq 0$. Thus, in sectors without V_X, the generalised GSO projections of the gravitinos are independent of the projections of the tachyons. Since the supersymmetric theory is tachyon-free, it follows that the theories with discrete torsion which have the “wrong” choice of k_{X1} also have no tachyons in these sectors, and are thus also tachyon-free.

It remains to consider sectors that *do* contain V_X, such as $V_X + \hat{\alpha}V$. The overlap of V_X with V_1 is zero, so the left side of Eq. (E.0.4) is the same as it is for the sector $\hat{\alpha}V$. Likewise the right side of this equation is $k_{1X} + \hat{\alpha}_i k_{1i} + \frac{1}{2} - V_1 \cdot \overline{\hat{\alpha}V}$, which differs only by k_{1X} from the version without V_X. Therefore, since there are no tachyons in any $\hat{\alpha}V$ sector without V_X, the “wrong” choice $k_{X1} = \frac{1}{2}$ may be consistent with tachyons in any sector that *does* contain V_X, provided there is negative vacuum energy (on both left and right moving sides). □

In general, it is not difficult to exploit these observations in order to generate non-supersymmetric string models whose tree-level spectra are tachyon-free. A particularly large collection of such models is presented and analysed in Refs. [30, 43].

Bibliography

[1] H. Georgi and S. L. Glashow, "Unity of All Elementary Particle Forces," Phys. Rev. Lett. **32** (1974) 438–441.

[2] S. Dimopoulos, S. A. Raby, and F. Wilczek, "Unification of couplings," Phys. Today **44N10** (1991) 25–33.

[3] H. Georgi, H. R. Quinn, and S. Weinberg, "Hierarchy of Interactions in Unified Gauge Theories," Phys. Rev. Lett. **33** (1974) 451–454.

[4] H. Georgi, "The State of the Art—Gauge Theories," AIP Conf. Proc. **23** (1975) 575–582.

[5] H. Fritzsch and P. Minkowski, "Unified Interactions of Leptons and Hadrons," Annals Phys. **93** (1975) 193–266.

[6] F. Gursey, P. Ramond, and P. Sikivie, "A Universal Gauge Theory Model Based on E6," Phys. Lett. **B60** (1976) 177–180.

[7] E. Witten, "Quest for unification," in Supersymmetry and unification of fundamental interactions. Proceedings, 10th International Conference, SUSY'02, Hamburg, Germany, June 17-23, 2002, pp. 604–610. 2002. hep-ph/0207124.

[8] a) P. Ramond, "Dual Theory for Free Fermions," Phys. Rev. **D3** (1971) 2415–2418

b) A. Neveu and J. H. Schwarz, "Factorizable dual model of pions," Nucl. Phys. **B31** (1971) 86–112.

[9] a) J. Wess and B. Zumino, "A Lagrangian Model Invariant Under Supergauge Transformations," Phys. Lett. **B49** (1974) 52

b) J. Wess and B. Zumino, "Supergauge Transformations in Four-Dimensions," Nucl. Phys. **B70** (1974) 39–50.

[10] a) P. Nath and R. L. Arnowitt, "Generalized Supergauge Symmetry as a New Framework for Unified Gauge Theories," Phys. Lett. **B56** (1975) 177–180

b) R. L. Arnowitt, P. Nath, and B. Zumino, "Superfield Densities and Action Principle in Curved Superspace," Phys. Lett. **B56** (1975) 81–84

c) D. Z. Freedman, P. van Nieuwenhuizen, and S. Ferrara, "Progress Toward a Theory of Supergravity," Phys. Rev. **D13** (1976) 3214–3218

d) S. Deser and B. Zumino, "Consistent Supergravity," Phys. Lett. **B62** (1976) 335

e) D. Z. Freedman and P. van Nieuwenhuizen, "Properties of Supergravity Theory," Phys. Rev. **D14** (1976) 912

f) E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello, and P. van Nieuwenhuizen, "Spontaneous Symmetry Breaking and Higgs Effect in Supergravity Without Cosmological Constant," Nucl. Phys. **B147** (1979) 105

g) J. A. Bagger, "Coupling the Gauge Invariant Supersymmetric Nonlinear Sigma Model to Supergravity," Nucl. Phys. **B211** (1983) 302.

[11] F. Gliozzi, J. Scherk, and D. I. Olive, "Supersymmetry, Supergravity Theories and the Dual Spinor Model," Nucl. Phys. **B122** (1977) 253–290.

[12] a) D. J. Gross, J. A. Harvey, E. J. Martinec, and R. Rohm, "The Heterotic String," Phys. Rev. Lett. **54** (1985) 502–505

b) D. J. Gross, J. A. Harvey, E. J. Martinec, and R. Rohm, "Heterotic String Theory. 1. The Free Heterotic String," Nucl. Phys. **B256** (1985) 253

c) D. J. Gross, J. A. Harvey, E. J. Martinec, and R. Rohm, "Heterotic String Theory. 2. The Interacting Heterotic String," Nucl. Phys. **B267** (1986) 75.

[13] M. B. Green and J. H. Schwarz, “Anomaly Cancellation in Supersymmetric D=10 Gauge Theory and Superstring Theory,” Phys. Lett. **B149** (1984) 117–122.

[14] P. Candelas, G. T. Horowitz, A. Strominger, and E. Witten, “Vacuum Configurations for Superstrings,” Nucl. Phys. **B258** (1985) 46–74.

[15] S. Abel, K. R. Dienes, and E. Mavroudi, “Towards a nonsupersymmetric string phenomenology,” Phys. Rev. **D91** (2015), no. 12, 126014, 1502.03087.

[16] a) S. Abel, K. R. Dienes, and E. Mavroudi, “Setting the Stage for a Non-Supersymmetric UV-Complete String Phenomenology,” in Proceedings, 18th International Conference From the Planck Scale to the Electroweak Scale (Planck 2015). 2016. 1603.05195

b) S. Abel, K. R. Dienes, and E. Mavroudi, “Towards a Phenomenology for the non-supersymmetric Heterotic String,” PoS **PLANCK2015** (2016) 001.

[17] a) W. Fischler and L. Susskind, “Dilaton Tadpoles, String Condensates and Scale Invariance,” Phys. Lett. **B171** (1986) 383

b) W. Fischler and L. Susskind, “Dilaton Tadpoles, String Condensates and Scale Invariance. 2.,” Phys. Lett. **B173** (1986) 262.

[18] D. Lust, “COMPACTIFICATION OF THE O(16) x O(16) HETEROTIC STRING THEORY,” Phys. Lett. **B178** (1986) 174.

[19] E. Dudas and J. Mourad, “Brane solutions in strings with broken supersymmetry and dilaton tadpoles,” Phys. Lett. **B486** (2000) 172–178, hep-th/0004165.

[20] a) K. R. Dienes, “Modular invariance, finiteness, and misaligned supersymmetry: New constraints on the numbers of physical string states,” Nucl. Phys. **B429** (1994) 533–588, hep-th/9402006

b) K. R. Dienes, “How strings make do without supersymmetry: An Introduction to misaligned supersymmetry,” in PASCOS ’94: Proceedings, 4th

International Symposium on Particles, Strings and Cosmology, Syracuse, New York, USA, May 19-24, 1994, pp. 0234–243. 1994. hep-th/9409114

c) K. R. Dienes, "Space-time properties of (1,0) string vacua," in Future perspectives in string theory. Proceedings, Conference, Strings'95, Los Angeles, USA, March 13-18, 1995. 1995. hep-th/9505194.

[21] a) K. R. Dienes, M. Moshe, and R. C. Myers, "String theory, misaligned supersymmetry, and the supertrace constraints," Phys. Rev. Lett. **74** (1995) 4767–4770, hep-th/9503055

b) K. R. Dienes, M. Moshe, and R. C. Myers, "Supertraces in string theory," in Future perspectives in string theory. Proceedings, Conference, Strings'95, Los Angeles, USA, March 13-18, 1995. 1995. hep-th/9506001.

[22] K. R. Dienes, "Solving the hierarchy problem without supersymmetry or extra dimensions: An Alternative approach," Nucl. Phys. **B611** (2001) 146–178, hep-ph/0104274.

[23] a) L. Alvarez-Gaume, P. H. Ginsparg, G. W. Moore, and C. Vafa, "An O(16) x O(16) Heterotic String," Phys. Lett. **B171** (1986) 155–162

b) L. J. Dixon and J. A. Harvey, "String Theories in Ten-Dimensions Without Space-Time Supersymmetry," Nucl. Phys. **B274** (1986) 93–105.

[24] R. Rohm, "Spontaneous Supersymmetry Breaking in Supersymmetric String Theories," Nucl. Phys. **B237** (1984) 553–572.

[25] a) V. P. Nair, A. D. Shapere, A. Strominger, and F. Wilczek, "Compactification of the Twisted Heterotic String," Nucl. Phys. **B287** (1987) 402

b) P. H. Ginsparg and C. Vafa, "Toroidal Compactification of Nonsupersymmetric Heterotic Strings," Nucl. Phys. **B289** (1987) 414.

[26] H. Itoyama and T. R. Taylor, "Supersymmetry Restoration in the Compactified O(16) x O(16)-prime Heterotic String Theory," Phys. Lett. **B186** (1987) 129.

[27] H. Itoyama and T. R. Taylor, "Small Cosmological Constant in String Models," in International Europhysics Conference on High-energy Physics Uppsala, Sweden, June 25-July 1, 1987. 1987. .

[28] G. W. Moore, "ATKIN-LEHNER SYMMETRY," Nucl. Phys. **B293** (1987) 139 [Erratum: Nucl. Phys.B299,847(1988)].

[29] a) J. Balog and M. P. Tuite, "The Failure of Atkin-lehner Symmetry for Lattice Compactified Strings," Nucl. Phys. **B319** (1989) 387–414

b) K. R. Dienes, "GENERALIZED ATKIN-LEHNER SYMMETRY," Phys. Rev. **D42** (1990) 2004–2021.

[30] K. R. Dienes, "New string partition functions with vanishing cosmological constant," Phys. Rev. Lett. **65** (1990) 1979–1982.

[31] D. Kutasov and N. Seiberg, "Number of degrees of freedom, density of states and tachyons in string theory and CFT," Nucl. Phys. **B358** (1991) 600–618.

[32] a) S. Kachru, J. Kumar, and E. Silverstein, "Vacuum energy cancellation in a nonsupersymmetric string," Phys. Rev. **D59** (1999) 106004, hep-th/9807076

b) S. Kachru and E. Silverstein, "On vanishing two loop cosmological constants in nonsupersymmetric strings," JHEP **01** (1999) 004, hep-th/9810129.

[33] a) J. A. Harvey, "String duality and nonsupersymmetric strings," Phys. Rev. **D59** (1999) 026002, hep-th/9807213

b) S. Kachru and E. Silverstein, "Selfdual nonsupersymmetric type II string compactifications," JHEP **11** (1998) 001, hep-th/9808056

c) R. Blumenhagen and L. Gorlich, "Orientifolds of nonsupersymmetric asymmetric orbifolds," Nucl. Phys. **B551** (1999) 601–616, hep-th/9812158

d) C. Angelantonj, I. Antoniadis, and K. Forger, "Nonsupersymmetric type I strings with zero vacuum energy," Nucl. Phys. **B555** (1999) 116–134, hep-th/9904092

e) M. R. Gaberdiel and A. Sen, "Nonsupersymmetric D-brane configurations with Bose-Fermi degenerate open string spectrum," JHEP **11** (1999) 008, hep-th/9908060.

[34] G. Shiu and S. H. H. Tye, "Bose-Fermi degeneracy and duality in non-supersymmetric strings," Nucl. Phys. **B542** (1999) 45–72, hep-th/9808095.

[35] R. Iengo and C.-J. Zhu, "Evidence for nonvanishing cosmological constant in nonSUSY superstring models," JHEP **04** (2000) 028, hep-th/9912074.

[36] a) E. D'Hoker and D. H. Phong, "Two loop superstrings 4: The Cosmological constant and modular forms," Nucl. Phys. **B639** (2002) 129–181, hep-th/0111040

b) E. D'Hoker and D. H. Phong, "Lectures on two loop superstrings," Conf. Proc. **C0208124** (2002) 85–123, hep-th/0211111[,85(2002)].

[37] A. E. Faraggi and M. Tsulaia, "Interpolations Among NAHE-based Supersymmetric and Nonsupersymmetric String Vacua," Phys. Lett. **B683** (2010) 314–320, 0911.5125.

[38] C. Angelantonj, M. Cardella, S. Elitzur, and E. Rabinovici, "Vacuum stability, string density of states and the Riemann zeta function," JHEP **02** (2011) 024, 1012.5091.

[39] a) O. Bergman and M. R. Gaberdiel, "A Nonsupersymmetric open string theory and S duality," Nucl. Phys. **B499** (1997) 183–204, hep-th/9701137

b) O. Bergman and M. R. Gaberdiel, "Dualities of type 0 strings," JHEP **07** (1999) 022, hep-th/9906055

c) R. Blumenhagen and A. Kumar, "A Note on orientifolds and dualities of type 0B string theory," Phys. Lett. **B464** (1999) 46–52, hep-th/9906234.

[40] J. D. Blum and K. R. Dienes, "Duality without supersymmetry: The Case of the SO(16) x SO(16) string," Phys. Lett. **B414** (1997) 260–268, hep-th/9707148.

[41] J. D. Blum and K. R. Dienes, "Strong / weak coupling duality relations for nonsupersymmetric string theories," Nucl. Phys. **B516** (1998) 83–159, hep-th/9707160.

[42] A. E. Faraggi and M. Tsulaia, "On the Low Energy Spectra of the Nonsupersymmetric Heterotic String Theories," Eur. Phys. J. **C54** (2008) 495–500, 0706.1649.

[43] K. R. Dienes, "Statistics on the heterotic landscape: Gauge groups and cosmological constants of four-dimensional heterotic strings," Phys. Rev. **D73** (2006) 106010, hep-th/0602286.

[44] K. R. Dienes, M. Lennek, and M. Sharma, "Strings at Finite Temperature: Wilson Lines, Free Energies, and the Thermal Landscape," Phys. Rev. **D86** (2012) 066007, 1205.5752.

[45] a) E. Alvarez and M. A. R. Osorio, "Cosmological Constant Versus Free Energy for Heterotic Strings," Nucl. Phys. **B304** (1988) 327[Erratum: Nucl. Phys.B309,220(1988)]

b) E. Alvarez and M. A. R. Osorio, "Duality Is an Exact Symmetry of String Perturbation Theory," Phys. Rev. **D40** (1989) 1150

c) M. A. R. Osorio, "Quantum fields versus strings at finite temperature," Int. J. Mod. Phys. **A7** (1992) 4275–4292.

[46] J. J. Atick and E. Witten, "The Hagedorn Transition and the Number of Degrees of Freedom of String Theory," Nucl. Phys. **B310** (1988) 291–334.

[47] a) M. McGuigan, "Finite Temperature String Theory and Twisted Tori," Phys. Rev. **D38** (1988) 552

b) I. Antoniadis and C. Kounnas, "Superstring phase transition at high temperature," Phys. Lett. **B261** (1991) 369–378

c) I. Antoniadis, J. P. Derendinger, and C. Kounnas, "Nonperturbative temperature instabilities in N=4 strings," Nucl. Phys. **B551** (1999) 41–77, hep-th/9902032.

[48] C. Kounnas and B. Rostand, "Coordinate Dependent Compactifications and Discrete Symmetries," Nucl. Phys. **B341** (1990) 641–665.

[49] a) M. J. Bowick and L. C. R. Wijewardhana, "Superstrings at High Temperature," Phys. Rev. Lett. **54** (1985) 2485

b) S. H. H. Tye, "The Limiting Temperature Universe and Superstring," Phys. Lett. **B158** (1985) 388–392

c) B. Sundborg, "Thermodynamics of Superstrings at High-energy Densities," Nucl. Phys. **B254** (1985) 583–592

d) E. Alvarez, "STRINGS AT FINITE TEMPERATURE," Nucl. Phys. **B269** (1986) 596–620

e) E. Alvarez and M. A. R. Osorio, "Superstrings at Finite Temperature," Phys. Rev. **D36** (1987) 1175

f) E. Alvarez and M. A. R. Osorio, "THERMAL HETEROTIC STRINGS," Physica **A158** (1989) 449–459[Erratum: PhysicaA160,119(1989)]

g) E. Alvarez and M. A. R. Osorio, "Thermal Strings in Nontrivial Backgrounds," Phys. Lett. **B220** (1989) 121

h) M. Axenides, S. D. Ellis, and C. Kounnas, "Universal Behavior of D-dimensional Superstring Models," Phys. Rev. **D37** (1988) 2964

i) Y. Leblanc, "Cosmological Aspects of the Heterotic String Above the Hagedorn Temperature," Phys. Rev. **D38** (1988) 3087

j) B. A. Campbell, J. R. Ellis, S. Kalara, D. V. Nanopoulos, and K. A. Olive, "Phase transitions in QCD and string theory," Phys. Lett. **B255** (1991) 420–426.

[50] S. Ferrara, C. Kounnas, and M. Porrati, "Superstring Solutions With Spontaneously Broken Four-dimensional Supersymmetry," Nucl. Phys. **B304** (1988) 500.

[51] S. Ferrara, C. Kounnas, and M. Porrati, "$N = 1$ Superstrings With Spontaneously Broken Symmetries," Phys. Lett. **B206** (1988) 25.

[52] S. Ferrara, C. Kounnas, M. Porrati, and F. Zwirner, "Superstrings with Spontaneously Broken Supersymmetry and their Effective Theories," Nucl. Phys. **B318** (1989) 75.

[53] E. Kiritsis and C. Kounnas, "Perturbative and nonperturbative partial supersymmetry breaking: N=4 —> N=2 —> N=1," Nucl. Phys. **B503** (1997) 117–156, hep-th/9703059.

[54] C. A. Scrucca and M. Serone, "On string models with Scherk-Schwarz supersymmetry breaking," JHEP **10** (2001) 017, hep-th/0107159.

[55] M. Borunda, M. Serone, and M. Trapletti, "On the quantum stability of IIB orbifolds and orientifolds with Scherk-Schwarz SUSY breaking," Nucl. Phys. **B653** (2003) 85–108, hep-th/0210075.

[56] C. Angelantonj, M. Cardella, and N. Irges, "An Alternative for Moduli Stabilisation," Phys. Lett. **B641** (2006) 474–480, hep-th/0608022.

[57] J. Scherk and J. H. Schwarz, "Spontaneous Breaking of Supersymmetry Through Dimensional Reduction," Phys. Lett. **B82** (1979) 60–64.

[58] W. Lerche, D. Lust, and A. N. Schellekens, "Ten-dimensional Heterotic Strings From Niemeier Lattices," Phys. Lett. **B181** (1986) 71.

[59] W. Lerche, D. Lust, and A. N. Schellekens, "Chiral Four-Dimensional Heterotic Strings from Selfdual Lattices," Nucl. Phys. **B287** (1987) 477.

[60] A. H. Chamseddine, J. P. Derendinger, and M. Quiros, "NONSUPERSYMMETRIC FOUR-DIMENSIONAL STRINGS," Nucl. Phys. **B311** (1988) 140.

[61] A. Font and A. Hernandez, "Nonsupersymmetric orbifolds," Nucl. Phys. **B634** (2002) 51–70, hep-th/0202057.

[62] M. Blaszczyk, S. Groot Nibbelink, O. Loukas, and S. Ramos-Sanchez, "Non-supersymmetric heterotic model building," JHEP **10** (2014) 119, 1407.6362.

[63] C. Angelantonj, I. Florakis, and M. Tsulaia, "Universality of Gauge Thresholds in Non-Supersymmetric Heterotic Vacua," Phys. Lett. **B736** (2014) 365–370, 1407.8023.

[64] a) C. Bachas, "A Way to break supersymmetry," hep-th/9503030

b) J. G. Russo and A. A. Tseytlin, "Magnetic flux tube models in superstring theory," Nucl. Phys. **B461** (1996) 131–154, hep-th/9508068

c) A. A. Tseytlin, "Closed superstrings in magnetic field: Instabilities and supersymmetry breaking," Nucl. Phys. Proc. Suppl. **49** (1996) 338–349, hep-th/9510041

d) H.-P. Nilles and M. Spalinski, "Generalized string compactifications with spontaneously broken supersymmetry," Phys. Lett. **B392** (1997) 67–76, hep-th/9606145

e) I. Shah and S. Thomas, "Finite soft terms in string compactifications with broken supersymmetry," Phys. Lett. **B409** (1997) 188–198, hep-th/9705182.

[65] A. Sagnotti, "Some properties of open string theories," in Supersymmetry and unification of fundamental interactions. Proceedings, International Workshop, SUSY 95, Palaiseau, France, May 15-19, 1995. 1995. hep-th/9509080.

[66] A. Sagnotti, "Surprises in open string perturbation theory," Nucl. Phys. Proc. Suppl. **56B** (1997) 332–343, hep-th/9702093.

[67] C. Angelantonj, "Nontachyonic open descendants of the 0B string theory," Phys. Lett. **B444** (1998) 309–317, hep-th/9810214.

[68] R. Blumenhagen, A. Font, and D. Lust, "Tachyon free orientifolds of type 0B strings in various dimensions," Nucl. Phys. **B558** (1999) 159–177, hep-th/9904069.

[69] S. Sugimoto, "Anomaly cancellations in type I D-9 - anti-D-9 system and the USp(32) string theory," Prog. Theor. Phys. **102** (1999) 685–699, hep-th/9905159.

[70] G. Aldazabal, L. E. Ibanez, and F. Quevedo, "Standard - like models with broken supersymmetry from type I string vacua," JHEP **01** (2000) 031, hep-th/9909172.

[71] C. Angelantonj, "Nonsupersymmetric open string vacua," PoS **trieste99** (1999) 015, hep-th/9907054.

[72] K. Forger, "On nontachyonic Z(N) x Z(M) orientifolds of type 0B string theory," Phys. Lett. **B469** (1999) 113–122, hep-th/9909010.

[73] S. Moriyama, "USp(32) string as spontaneously supersymmetry broken theory," Phys. Lett. **B522** (2001) 177–180, hep-th/0107203.

[74] C. Angelantonj and I. Antoniadis, "Suppressing the cosmological constant in nonsupersymmetric type I strings," Nucl. Phys. **B676** (2004) 129–148, hep-th/0307254.

[75] C. Angelantonj, "Open strings and supersymmetry breaking," AIP Conf. Proc. **751** (2005) 3–14, hep-th/0411085[,3(2004)].

[76] E. Dudas and C. Timirgaziu, "Nontachyonic Scherk-Schwarz compactifications, cosmology and moduli stabilization," JHEP **03** (2004) 060, hep-th/0401201.

[77] B. Gato-Rivera and A. N. Schellekens, "Non-supersymmetric Tachyon-free Type-II and Type-I Closed Strings from RCFT," Phys. Lett. **B656** (2007) 127–131, 0709.1426.

[78] I. Antoniadis, C. Bachas, D. C. Lewellen, and T. N. Tomaras, "On Supersymmetry Breaking in Superstrings," Phys. Lett. **B207** (1988) 441.

[79] I. Antoniadis, "A Possible new dimension at a few TeV," Phys. Lett. **B246** (1990) 377–384.

[80] I. Antoniadis, C. Munoz, and M. Quiros, "Dynamical supersymmetry breaking with a large internal dimension," Nucl. Phys. **B397** (1993) 515–538, hep-ph/9211309.

[81] I. Antoniadis and M. Quiros, "Large radii and string unification," Phys. Lett. **B392** (1997) 61–66, hep-th/9609209.

[82] K. Benakli, "Phenomenology of low quantum gravity scale models," Phys. Rev. **D60** (1999) 104002, hep-ph/9809582.

[83] C. P. Bachas, "Scales of string theory," Class. Quant. Grav. **17** (2000) 951–959, hep-th/0001093.

[84] E. Dudas, "Theory and phenomenology of type I strings and M theory," Class. Quant. Grav. **17** (2000) R41–R116, hep-ph/0006190.

[85] S. G. Nibbelink, "Model building with the non-supersymmetric heterotic SO(16)xSO(16) string," J. Phys. Conf. Ser. **631** (2015), no. 1, 012077, 1502.03604.

[86] I. Florakis, "Universality of radiative corrections to gauge couplings for strings with spontaneously broken supersymmetry," J. Phys. Conf. Ser. **631** (2015), no. 1, 012079, 1502.07537.

[87] J. M. Ashfaque, P. Athanasopoulos, A. E. Faraggi, and H. Sonmez, "Non-Tachyonic Semi-Realistic Non-Supersymmetric Heterotic String Vacua," Eur. Phys. J. **C76** (2016), no. 4, 208, 1506.03114.

[88] M. Blaszczyk, S. G. Nibbelink, O. Loukas, and F. Ruehle, "Calabi-Yau compactifications of non-supersymmetric heterotic string theory," JHEP **10** (2015) 166, 1507.06147.

[89] S. G. Nibbelink, O. Loukas, and F. Ruehle, "(MS)SM-like models on smooth Calabi-Yau manifolds from all three heterotic string theories," Fortsch. Phys. **63** (2015) 609–632, 1507.07559.

[90] C. Angelantonj, I. Florakis, and M. Tsulaia, "Generalised universality of gauge thresholds in heterotic vacua with and without supersymmetry," Nucl. Phys. **B900** (2015) 170–197, 1509.00027.

[91] S. Weinberg, "The Making of the standard model," Eur. Phys. J. **C34** (2004) 5–13, hep-ph/0401010[,99(2005)].

[92] **CMS** Collaboration, V. Khachatryan et al., "Precise determination of the mass of the Higgs boson and tests of compatibility of its couplings with the standard model predictions using proton collisions at 7 and 8 TeV," Eur. Phys. J. **C75** (2015), no. 5, 212, 1412.8662.

[93] **Particle Data Group** Collaboration, J. Beringer et al., "Review of Particle Physics (RPP)," Phys. Rev. **D86** (2012) 010001.

[94] a) J. M. Cornwall, D. N. Levin, and G. Tiktopoulos, "Uniqueness of spontaneously broken gauge theories," Phys. Rev. Lett. **30** (1973) 1268–1270 [Erratum: Phys. Rev. Lett.31,572(1973)]

b) J. M. Cornwall, D. N. Levin, and G. Tiktopoulos, "Derivation of Gauge Invariance from High-Energy Unitarity Bounds on the s Matrix," Phys. Rev. **D10** (1974) 1145[Erratum: Phys. Rev.D11,972(1975)].

[95] C. H. Llewellyn Smith, "High-Energy Behavior and Gauge Symmetry," Phys. Lett. **B46** (1973) 233–236.

[96] a) B. W. Lee, C. Quigg, and H. B. Thacker, "The Strength of Weak Interactions at Very High-Energies and the Higgs Boson Mass," Phys. Rev. Lett. **38** (1977) 883–885

b) B. W. Lee, C. Quigg, and H. B. Thacker, "Weak Interactions at Very High-Energies: The Role of the Higgs Boson Mass," Phys. Rev. **D16** (1977) 1519.

[97] **ATLAS, CMS** Collaboration, G. Aad et al., "Combined Measurement of the Higgs Boson Mass in pp Collisions at $\sqrt{s} = 7$ and 8 TeV with the ATLAS and CMS Experiments," Phys. Rev. Lett. **114** (2015) 191803, 1503.07589.

[98] J. Ellis, "The Physics Landscape after the Higgs Discovery at the LHC," Nucl. Part. Phys. Proc. **267-269** (2015) 3–14, 1504.03654.

[99] U. Amaldi, W. de Boer, and H. Furstenau, "Comparison of grand unified theories with electroweak and strong coupling constants measured at LEP," Phys. Lett. **B260** (1991) 447–455.

[100] H. Baer and X. Tata, Weak scale supersymmetry: From superfields to scattering events. Cambridge University Press, 2006.

[101] D. I. Kazakov, "Beyond the standard model: In search of supersymmetry," in 2000 European School of high-energy physics, Caramulo, Portugal, 20 Aug-2 Sep 2000: Proceedings, pp. 125–199. 2000. hep-ph/0012288.

[102] G. 't Hooft, "Naturalness, chiral symmetry, and spontaneous chiral symmetry breaking," NATO Sci. Ser. B **59** (1980) 135.

[103] L. Susskind, "Dynamics of Spontaneous Symmetry Breaking in the Weinberg-Salam Theory," Phys. Rev. **D20** (1979) 2619–2625.

[104] **CMS** Collaboration, C. Collaboration, "Determination of the top-quark mass from the m(lb) distribution in dileptonic ttbar events at sqrt(s) = 8 TeV,".

[105] K. G. Wilson, "The Renormalization Group and Strong Interactions," Phys. Rev. **D3** (1971) 1818.

[106] S. Dawson, "The MSSM and why it works," in Supersymmetry, supergravity and supercolliders. Proceedings, Theoretical Advanced Study Institute in elementary particle physics, TASI'97, Boulder, USA, June 2-27, 1997, pp. 261–339. 1997. hep-ph/9712464.

[107] J. Polchinski, String theory. Vol. 1: An introduction to the bosonic string. Cambridge University Press, 2007.

[108] P. Ramond, "Neutrinos: A Glimpse beyond the standard model," Nucl. Phys. Proc. Suppl. **77** (1999) 3–9, hep-ph/9809401.

[109] a) M. Gell-Mann, P. Ramond, and R. Slansky, "Complex Spinors and Unified Theories," Conf. Proc. **C790927** (1979) 315–321, 1306.4669

b) T. Yanagida, "HORIZONTAL SYMMETRY AND MASSES OF NEUTRINOS," Conf. Proc. **C7902131** (1979) 95–99

c) T. Yanagida, "Horizontal Symmetry and Masses of Neutrinos," Prog. Theor. Phys. **64** (1980) 1103.

[110] G. Bauer, "In hot pursuit of CP violation," CERN Cour. **39N5** (1999) 22–25.

[111] S. R. Coleman and J. Mandula, "All Possible Symmetries of the S Matrix," Phys. Rev. **159** (1967) 1251–1256.

[112] R. Haag, J. T. Lopuszanski, and M. Sohnius, "All Possible Generators of Supersymmetries of the s Matrix," Nucl. Phys. **B88** (1975) 257.

[113] a) C. Bouchiat, J. Iliopoulos, and P. Meyer, "An Anomaly Free Version of Weinberg's Model," Phys. Lett. **B38** (1972) 519–523

b) H. Georgi and S. L. Glashow, "Gauge theories without anomalies," Phys. Rev. **D6** (1972) 429

c) D. J. Gross and R. Jackiw, "Effect of anomalies on quasirenormalizable theories," Phys. Rev. **D6** (1972) 477–493

d) L. Alvarez-Gaume and E. Witten, "Gravitational Anomalies," Nucl. Phys. **B234** (1984) 269.

[114] S. P. Martin, "A Supersymmetry primer," hep-ph/9709356[Adv. Ser. Direct. High Energy Phys.18,1(1998)].

[115] L. O'Raifeartaigh, "Spontaneous Symmetry Breaking for Chiral Scalar Superfields," Nucl. Phys. **B96** (1975) 331–352.

[116] A. Signer, "ABC of SUSY," J. Phys. **G36** (2009) 073002, 0905.4630.

[117] a) M. Dine and A. E. Nelson, "Dynamical supersymmetry breaking at low-energies," Phys. Rev. **D48** (1993) 1277–1287, hep-ph/9303230

b) M. Dine, A. E. Nelson, and Y. Shirman, "Low-energy dynamical supersymmetry breaking simplified," Phys. Rev. **D51** (1995) 1362–1370, hep-ph/9408384

c) M. Dine, A. E. Nelson, Y. Nir, and Y. Shirman, "New tools for low-energy dynamical supersymmetry breaking," Phys. Rev. **D53** (1996) 2658–2669, hep-ph/9507378.

[118] a) H. P. Nilles, "Dynamically Broken Supergravity and the Hierarchy Problem," Phys. Lett. **B115** (1982) 193

b) R. Barbieri, S. Ferrara, and C. A. Savoy, "Gauge Models with Spontaneously Broken Local Supersymmetry," Phys. Lett. **B119** (1982) 343

c) L. E. Ibanez, "Locally Supersymmetric SU(5) Grand Unification," Phys. Lett. **B118** (1982) 73–78

d) A. H. Chamseddine, R. L. Arnowitt, and P. Nath, "Locally Supersymmetric Grand Unification," Phys. Rev. Lett. **49** (1982) 970

e) E. Cremmer, P. Fayet, and L. Girardello, "Gravity Induced Supersymmetry Breaking and Low-Energy Mass Spectrum," Phys. Lett. **B122** (1983) 41

f) P. Nath, R. L. Arnowitt, and A. H. Chamseddine, "Gauge Hierarchy in Supergravity Guts," Nucl. Phys. **B227** (1983) 121–133

g) H. P. Nilles, M. Srednicki, and D. Wyler, "Weak Interaction Breakdown Induced by Supergravity," Phys. Lett. **B120** (1983) 346.

[119] H. E. Haber, "The Status of the minimal supersymmetric standard model and beyond," Nucl. Phys. Proc. Suppl. **62** (1998) 469–484, hep-ph/9709450.

[120] P. Fayet, "Spontaneously Broken Supersymmetric Theories of Weak, Electromagnetic and Strong Interactions," Phys. Lett. **B69** (1977) 489.

[121] S. Ferrara, J. R. Ellis, and P. Van Nieuwenhuizen, eds., UNIFICATION OF THE FUNDAMENTAL PARTICLE INTERACTIONS. PROCEEDINGS, EUROPHYSICS STUDY CONFERENCE, ERICE, ITALY, MARCH 17-24, 1980 1980.

[122] E. Cremmer, S. Ferrara, L. Girardello, and A. Van Proeyen, "Yang-Mills Theories with Local Supersymmetry: Lagrangian, Transformation Laws and SuperHiggs Effect," Nucl. Phys. **B212** (1983) 413.

[123] W. Lerche, "Recent developments in string theory," in Lepton photon interactions. Proceedings, 18th International Symposium, LP'97, Hamburg, Germany, July 28-August 1, 1997. 1997. hep-th/9710246.

[124] E. Witten, "Reflections on the fate of space-time," Phys. Today **49N4** (1996) 24–30.

[125] J. Louis, T. Mohaupt, and S. Theisen, "String theory: An overview," Lect. Notes Phys. **721** (2007) 289–323.

[126] K. Becker, M. Becker, and J. H. Schwarz, String theory and M-theory: A modern introduction. Cambridge University Press, 2006.

[127] I. Antoniadis and G. Ovarlez, "An Introduction to perturbative and nonperturbative string theory," hep-th/9906108[PoScorfu98,013(1998)].

[128] M. B. Green and J. H. Schwarz, "Supersymmetrical String Theories," Phys. Lett. **B109** (1982) 444–448.

[129] J. H. Schwarz, "Status of Superstring and M-Theory," Int. J. Mod. Phys. **A25** (2010) 4703–4725, 0812.1372[Subnucl. Ser.46,335(2011)].

[130] J. H. Schwarz, "The power of M theory," Phys. Lett. **B367** (1996) 97–103, hep-th/9510086.

[131] J. H. Schwarz, "Lectures on superstring and M theory dualities: Given at ICTP Spring School and at TASI Summer School," Nucl. Phys. Proc. Suppl. **55B** (1997) 1–32, hep-th/9607201.

[132] I. Antoniadis, C. P. Bachas, and C. Kounnas, "Four-Dimensional Superstrings," Nucl. Phys. **B289** (1987) 87.

[133] I. Antoniadis and C. Bachas, "4-D Fermionic Superstrings with Arbitrary Twists," Nucl. Phys. **B298** (1988) 586–612.

[134] H. Kawai, D. C. Lewellen, and S. H. H. Tye, "Construction of Fermionic String Models in Four-Dimensions," Nucl. Phys. **B288** (1987) 1.

[135] H. Kawai, D. C. Lewellen, J. A. Schwartz, and S. H. H. Tye, "The Spin Structure Construction of String Models and Multiloop Modular Invariance," Nucl. Phys. **B299** (1988) 431–470.

[136] J. Bagger, D. Nemeschansky, N. Seiberg, and S. Yankielowicz, "Bosons, Fermions and Thirring Strings," Nucl. Phys. **B289** (1987) 53–86.

[137] a) J. H. Schwarz, ed., SUPERSTRINGS. THE FIRST 15-YEARS OF SUPERSTRING THEORY. VOL. 1. 1986

b) J. H. Schwarz, ed., SUPERSTRINGS. THE FIRST 15-YEARS OF SUPERSTRING THEORY. VOL. 2. 1986.

[138] M. B. Green and D. Gross, eds., UNIFIED STRING THEORIES. PROCEEDINGS, WORKSHOP, SANTA BARBARA, USA, JULY 29 - AUGUST 16, 19851986.

[139] D. Friedan, E. J. Martinec, and S. H. Shenker, "Conformal Invariance, Supersymmetry and String Theory," Nucl. Phys. **B271** (1986) 93–165.

[140] E. J. Martinec, "Nonrenormalization Theorems and Fermionic String Finiteness," Phys. Lett. **B171** (1986) 189.

[141] a) G. H. Hardy and S. Ramanujan, "Asymptotic formulae in combinatory analysis," Proc. London Math. Soc. (2) **17** (1918) 75–115.

b) I. Kani and C. Vafa, "Asymptotic Mass Degeneracies in Conformal Field Theories," Commun. Math. Phys. **130** (1990) 529–580.

[142] R. Hagedorn, "Statistical thermodynamics of strong interactions at high-energies," Nuovo Cim. Suppl. **3** (1965) 147–186.

[143] I. Antoniadis, J. R. Ellis, J. S. Hagelin, and D. V. Nanopoulos, "The Flipped SU(5) x U(1) String Model Revamped," Phys. Lett. **B231** (1989) 65–74.

[144] I. Antoniadis, G. K. Leontaris, and J. Rizos, “A Three generation SU(4) x O(4) string model,” Phys. Lett. **B245** (1990) 161–168.

[145] A. E. Faraggi, “Hierarchical top - bottom mass relation in a superstring derived standard - like model,” Phys. Lett. **B274** (1992) 47–52.

[146] A. E. Faraggi, “A New standard - like model in the four-dimensional free fermionic string formulation,” Phys. Lett. **B278** (1992) 131–139.

[147] A. E. Faraggi, “Construction of realistic standard - like models in the free fermionic superstring formulation,” Nucl. Phys. **B387** (1992) 239–262, hep-th/9208024.

[148] A. E. Faraggi, “Custodial nonAbelian gauge symmetries in realistic superstring derived models,” Phys. Lett. **B339** (1994) 223–231, hep-ph/9408333.

[149] K. R. Dienes and A. E. Faraggi, “Making ends meet: String unification and low-energy data,” Phys. Rev. Lett. **75** (1995) 2646–2649, hep-th/9505018.

[150] K. R. Dienes and A. E. Faraggi, “Gauge coupling unification in realistic free fermionic string models,” Nucl. Phys. **B457** (1995) 409–483, hep-th/9505046.

[151] A. H. Chamseddine, J. P. Derendinger, and M. Quiros, “A Unified Formalism for Strings in Four-dimensions,” Nucl. Phys. **B326** (1989) 497–543.

[152] S. E. M. Nooij, Classification of the chiral Z(2) x Z(2) heterotic string models. PhD thesis, Oxford U., Theor. Phys., 2004. hep-th/0603035. .

[153] H. Murayama, Y. Nomura, S. Shirai, and K. Tobioka, “Compact Supersymmetry,” Phys. Rev. **D86** (2012) 115014, 1206.4993.

[154] a) S. Dimopoulos, K. Howe, and J. March-Russell, “Maximally Natural Supersymmetry,” Phys. Rev. Lett. **113** (2014) 111802, 1404.7554

b) I. García García and J. March-Russell, “Rare Flavor Processes in Maximally Natural Supersymmetry,” JHEP **01** (2015) 042, 1409.5669.

[155] a) Y. Shadmi and P. Z. Szabo, "Flavored Gauge-Mediation," JHEP **06** (2012) 124, 1103.0292

b) N. Craig, M. McCullough, and J. Thaler, "Flavor Mediation Delivers Natural SUSY," JHEP **06** (2012) 046, 1203.1622.

[156] R. Davies, J. March-Russell, and M. McCullough, "A Supersymmetric One Higgs Doublet Model," JHEP **04** (2011) 108, 1103.1647.

[157] E. Caceres, V. S. Kaplunovsky, and I. M. Mandelberg, "Large volume string compactifications, revisited," Nucl. Phys. **B493** (1997) 73–100, hep-th/9606036.

[158] E. Kiritsis, C. Kounnas, P. M. Petropoulos, and J. Rizos, "Solving the decompactification problem in string theory," Phys. Lett. **B385** (1996) 87–95, hep-th/9606087.

[159] E. Kiritsis, C. Kounnas, P. M. Petropoulos, and J. Rizos, "String threshold corrections in models with spontaneously broken supersymmetry," Nucl. Phys. **B540** (1999) 87–148, hep-th/9807067.

[160] I. Antoniadis and K. Benakli, "Large dimensions and string physics in future colliders," Int. J. Mod. Phys. **A15** (2000) 4237–4286, hep-ph/0007226.

[161] A. E. Faraggi, C. Kounnas, and H. Partouche, "Large volume susy breaking with a solution to the decompactification problem," Nucl. Phys. **B899** (2015) 328–374, 1410.6147.

[162] K. R. Dienes, E. Dudas, and T. Gherghetta, "GUT precursors and nontrivial fixed points in higher dimensional gauge theories," Phys. Rev. Lett. **91** (2003) 061601, hep-th/0210294.

[163] K. R. Dienes, E. Dudas, and T. Gherghetta, "GUT precursors and fixed points in higher-dimensional theories," Pramana **62** (2004) 219–228.

[164] E. Kiritsis and C. Kounnas, "Infrared regulated string theory and loop corrections to coupling constants," in Future perspectives in string theory.

Proceedings, Conference, Strings'95, Los Angeles, USA, March 13-18, 1995. 1995. hep-th/9507051.

[165] E. Kiritsis, C. Kounnas, P. M. Petropoulos, and J. Rizos, "Universality properties of N=2 and N=1 heterotic threshold corrections," Nucl. Phys. **B483** (1997) 141–171, hep-th/9608034.

[166] S. Abel, K. R. Dienes, and E. Mavroudi, "GUT precursors and Entwined SUSY: The phenomenology of Stable Non-Supersymmetric Strings," To be submitted (arXiV 2017).

[167] a) K. R. Dienes, E. Dudas, and T. Gherghetta, "Extra space-time dimensions and unification," Phys. Lett. **B436** (1998) 55–65, hep-ph/9803466

b) K. R. Dienes, E. Dudas, and T. Gherghetta, "Grand unification at intermediate mass scales through extra dimensions," Nucl. Phys. **B537** (1999) 47–108, hep-ph/9806292

c) K. R. Dienes, E. Dudas, and T. Gherghetta, "TeV scale GUTs," in 1st European Meeting From the Planck Scale to the Electroweak Scale (Planck 1998) Kazimierz, Poland, May 24-30, 1998, pp. 613–620. 1998. hep-ph/9807522[,613(1998)].

[168] Z. Kakushadze and T. R. Taylor, "Higher loop effects on unification via Kaluza-Klein thresholds," Nucl. Phys. **B562** (1999) 78–102, hep-th/9905137.

[169] S. Abel, A. Ben, and E. Mavroudi, "On the interpolation from Non-Supersymmetric to Supersymmetric String Theories," To be submitted (arXiV 2016).

[170] S. L. Parameswaran, S. Ramos-Sanchez, and I. Zavala, "On Moduli Stabilisation and de Sitter Vacua in MSSM Heterotic Orbifolds," JHEP **01** (2011) 071, 1009.3931.

[171] a) G. Lopes Cardoso, D. Lust, and T. Mohaupt, "Moduli spaces and target space duality symmetries in (0,2) Z(N) orbifold theories with continuous Wilson lines," Nucl. Phys. **B432** (1994) 68–108, hep-th/9405002

b) G. Lopes Cardoso, D. Lust, and T. Mohaupt, "Duality symmetries and supersymmetry breaking in string compactifications," in 27th International Conference on High-energy Physics (ICHEP 94) Glasgow, Scotland, July 20-27, 1994. 1994. hep-th/9409095.

[172] I. Antoniadis, E. Gava, K. S. Narain, and T. R. Taylor, "Effective mu term in superstring theory," Nucl. Phys. **B432** (1994) 187–204, hep-th/9405024.

[173] A. Brignole, L. E. Ibanez, C. Munoz, and C. Scheich, "Some issues in soft SUSY breaking terms from dilaton / moduli sectors," Z. Phys. **C74** (1997) 157–170, hep-ph/9508258.

[174] A. Brignole, L. E. Ibanez, and C. Munoz, "Orbifold induced mu term and electroweak symmetry breaking," Phys. Lett. **B387** (1996) 769–774, hep-ph/9607405.

[175] Z. G. Berezhiani and G. R. Dvali, "Possible solution of the hierarchy problem in supersymmetrical grand unification theories," Bull. Lebedev Phys. Inst. **5** (1989) 55–59[Kratk. Soobshch. Fiz.5,42(1989)].

[176] G. R. Dvali and S. Pokorski, "The Role of the anomalous U(1)-A for the solution of the doublet - triplet splitting problem," Phys. Rev. Lett. **78** (1997) 807–810, hep-ph/9610431.

[177] a) A. Hebecker, A. K. Knochel, and T. Weigand, "A Shift Symmetry in the Higgs Sector: Experimental Hints and Stringy Realizations," JHEP **06** (2012) 093, 1204.2551

b) A. Hebecker, A. K. Knochel, and T. Weigand, "The Higgs mass from a String-Theoretic Perspective," Nucl. Phys. **B874** (2013) 1–35, 1304.2767.

[178] H. Luo and F. Zwirner, "Geometrical hierarchies in classical supergravity," Phys. Rev. Lett. **113** (2014) 021801, 1403.4942.

[179] L. E. Ibanez, F. Marchesano, and I. Valenzuela, "Higgs-otic Inflation and String Theory," JHEP **01** (2015) 128, 1411.5380.

[180] S. Abel and R. J. Stewart, "Shift-Symmetries at Higher Order," JHEP **02** (2016) 182, 1511.02880.

[181] I. Simonsen, "A Review of minimal supersymmetric electroweak theory," hep-ph/9506369.